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A Rotation-free Isogeometric Analysis for Composite Sandwich Thin Plates

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Abstract In this paper, a rotation-free isogeometric formulation for static analysis of composite sandwich plates is presented. The idea relies on a combination of isogeometric analysis with a classical laminate plate theory (CLPT). Isogeometric analysis (IGA) based on non-uniform rational B-spline(NURBS) basic function was recently proposed to preserve exact geometries and to enhance very significantly the accuracy of the traditional finite elements. B-splines basis functions (or NURBS) is used to represent for both geometric and field variable approximations, which provide a flexible way to make refinement and degree elevation. They enable us to achieve easily the smoothness with arbitrary continuity order compared with the traditional FEM. CLPT ignores the transverse shear deformation so it is only applied for thin plates. In our formulation, only deflection variables (without rotational degrees of freedom (dof)) are used for each control point. Essential displacements and rotations boundary conditions can be satisfied strongly by assigning control variable values on the boundary and these adjacent to the boundary, respectively. Several numerical examples are illustrated to demonstrate the performance of the present method in comparison with other published methods.

Keywords NURBS, Isogeometric analysis, Rotation-free isogeometric formulation, Composite sandwich plates

1. Introduction

Sandwich structures have been widely used in various engineering such as aircrafts, aerospace, vehicles, buildings, etc. Sandwich structures are made of three layers (two face sheets and a core) with different materials stacked together to achieve desired properties (e.g. high stiffness and strength-to-weight ratios, long fatigue life, wear resistance, lightweight, etc). For the analysis of sandwich plate, the exact elasticity solution first has been proposed by Pagano[1] to predict accurately of static behavior. Elasticity solution three-dimensional (3D) can become very expensive when the complex structures are modeled. Generally, computational costs are reduced when two-dimensional model is used. Using two-dimensional model, several plate theories using equivalent single layer have been developed to analyze laminated composite sandwich plates. The classical laminate plate theory (CLPT)[3] can only give good results to thin plates because it ignores the transverse shear deformation. The first-order shear deformation theory (FSDT)[2] can be applied for both moderately thick and thin plates. This theory assumes that transverse shear stresses are constant through the thickness and a shear correction factor is needed to take into account the non-linear distribution of shear stresses. To bypass the limitations of the FSDT, the higher-order shear deformation theories (HSDT) have been developed by Kant *et al*.[2] for the static analysis of composite sandwich plates based on analytical methods (Navier's solution). Analytical methods have available for benchmark problems. Thanks to advanced numerical approaches such as finite elements [4, 5], smoothed finite elements (SFEM) [6, 7, 8], meshfree methods [9, 10, 11] and extended meshfree methods [12, 13, 14], we can solve effectively more complicated problems in practice. For illustration of this work, finite element analysis for composite sandwich plates is given by Tran *et al*.[15] based on HSDT. In addition, two-dimensional model based on zigzag theory is also used to calculate the composite sandwich such as: the static analysis of composite sandwich plate with soft-core by Pandit *et al*.[16], C^0 finite element model for the analysis of sandwich laminates with general layup by Singh *et al*.[17] and an improved C^0 finite element model for the analysis of laminated sandwich plate with soft-core by Chalak *et al*.[19], etc.

In the traditional FE method, a discretized geometry obtained through the so-called meshing process is required. This process often leads to geometrical errors even using the higher-order FEM. Also, the communication of the geometry model and the mesh generation during an analysis process that aims to provide the desired accuracy for the solution is always needed and this constitute a time-consuming part in the overall analysis-design process, especially for industrial

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problems[20]. To overcome this disadvantage, Hughes *et al.* [20] have recently proposed a NURBS-based isogeometric analysis to bridge the gap between Computer Aided Design (CAD) and Finite Element Analysis (FEA). In contrast to the standard FEM with Lagrange polynomial basis, isogeometric approach utilized more general basis functions such as Non-Uniform Rational B-splines (NURBS) that are common in CAD approaches. Isogeometric analysis is thus very promising because it can directly use CAD data to describe both exact geometry and approximate solution. For structural mechanics, isogeometric analysis has been extensively studied for structural vibrations[21], the Reissner-Mindlin composite plate[24], the composite plate based on HSDT [25], laminated composite layerwise plates [28], the Reissner-Mindlin shell[22] and Kirchhoff-Love shell[23, 27] and further developments[26], etc. The plates are commonly employed in engineering applications as thin plates. So, CLPT is utilized in this paper to reduce computational costs. We focus on NURBS elements using a rotation-free isogeometric formulation for static analysis of composite sandwich plates.

The paper is arranged as follows: a brief of the B-spline and NURBS surface is described in section 2. Section 3 describes an isogeometric approximation for composite sandwich plates. Several numerical examples are illustrated in section 4. Finally we close our paper with some concluding remarks.

2. Nurbs-Based Isogeometric Analysis Fundamentals

2.1. Knot Vectors and Basis Functions

Let be a nondecreasing $\Xi=\{\xi_1,\xi_2...,\xi_{n+p+1}\}$ sequence of parameter values, $\xi_i < \xi_{i+1}, i=1,...,n+p$. The ξ_i is called knots, and Ξ is the set of coordinates in the parametric space. If all knots are equally spaced the knot vector is called uniform. If the first and the last knots are repeated $p+1$ times, the knot vector is described as open. A B-Spline basis function is C^{∞} continuous inside a knot span and C^{p-1} continuous at a single knot. A knot value can appear more than once and is then called a multiple knot. At a knot of multiplicity k the continuity is C^{p-k}.

Given a knot vector, the B-spline basis functions $N_{i,p}(\xi)$ of order p = 0 are defined recursively on the corresponding knot vector as follows

$$N_{i,0}(\xi)=\begin{cases}1 & if \quad \xi_i \le \xi \le \xi_{i+1} \\ 0 & \text{otherwise}\end{cases} \tag{1}$$

The basis functions of p>1 are defined by the following recursion formula

$$N_{i,p}(\xi)=\frac{\xi-\xi_i}{\xi_{i+p}-\xi_i}N_{i,p-1}(\xi) + \frac{\xi_{i+p+1}-\xi}{\xi_{i+p+1}-\xi_{i+1}}N_{i+1,p-1}(\xi) \tag{2}$$

For p = 0 and 1 the basis functions of isogeometric analysis are identical to those of standard piecewise constant and linear finite elements, respectively. However, they are different for $p \ge 2$. In this study, we consider basis functions with $p \ge 2$.

2.2. NURBS Surface

The B-spline curve is defined as:

$$\mathbf{C}(\xi)=\sum_{i=1}^{n} N_{i,p}(\xi)\mathbf{P}_i \tag{3}$$

where $\mathbf{P}_i$ are the control points and $N_{i,p}(\xi)$ is the pth-degree B-spline basis function defined on the open knot vector.

The B-spline surfaces are defined by the tensor product of basis functions in two parametric dimensions ξ and η with two knot vectors $\Xi=\{\xi_1,\xi_2...,\xi_{n+p+1}\}$and $\mathbf{H}=\{\eta_1,\eta_2...,\eta_{m+q+1}\}$ are expressed as follows:

$$\mathbf{S}(\xi,\eta)=\sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi)M_{j,q}(\eta)\mathbf{P}_{i,j} \tag{4}$$

where $\mathbf{P}_{i,j}$ is the bidirectional control net, $N_{i,p}(\xi)$ and $M_{j,q}(\eta)$ are the B-spline basis functions defined on the knot vectors over an n×m net of control points $\mathbf{P}_{i,j}$. Similarly to notations used in finite elements, we identify the logical coordinates (i, j) of the B-spline surface with the traditional notation of a node A[22]. Eq.(4) can be rewritten in the following form:

$$\mathbf{S}(\xi,\eta)=\sum_{A}^{nxm} N_A(\xi,\eta)\mathbf{P}_A \tag{5}$$

where $N_A(\xi,\eta) = N_{i,p}(\xi)M_{j,q}(\eta)$ is the shape function associated with node A.

Similar to B-Splines, a NURBS surface is defined as

$$\mathbf{S}(\xi,\eta)=\sum_{A=1}^{nxm} R_A(\xi,\eta)\mathbf{P}_A \quad \text{where} \quad R_A=\frac{N_A w_A}{\sum_{A}^{nxm} N_A w_A} \tag{6}$$

where w_A is the weight function.

3. A Rotation-Free Isogeometric Formulation for Kirchhoff Plate Model

Let Ω be the domain in R^2 occupied by the mid-plane of the plate and u, v and w denote the displacement components in the x, y and z directions, respectively. Using the Kirchhoff model[3], the displacements of any point in the plate can be expressed as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\theta_x(x,y) \\ v(x,y,z) &= v_0(x,y) - z\theta_y(x,y) \\ w(x,y,z) &= w(x,y) \end{aligned} \tag{7}$$

where

$$\theta_x = \frac{\partial w}{\partial x} \text{ and } \theta_y = \frac{\partial w}{\partial y} \tag{8}$$

In-plane strains through the following equation:

$$\boldsymbol{\varepsilon} = [\varepsilon_{xx}\ \ \varepsilon_{yy}\ \ \gamma_{xy}]^T = \boldsymbol{\varepsilon}_0 + z\boldsymbol{\kappa} \tag{9}$$

where $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\kappa}$ are the in-plane deformations and curvatures of the middle surface, respectively:

$$\boldsymbol{\varepsilon}_0 = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix}\mathbf{u} \text{ and } \boldsymbol{\kappa} = \begin{bmatrix} 0 & 0 & -\frac{\partial^2}{\partial x^2} \\ 0 & 0 & -\frac{\partial^2}{\partial y^2} \\ 0 & 0 & -2\frac{\partial^2}{\partial x \partial y} \end{bmatrix}\mathbf{u} \tag{10}$$

and $\mathbf{u} = \{u_0\ \ v_0\ \ w\}^T$ is displacement components at the middle surface.

The Hook's law for an arbitrary layer k, the stress in plane is expressed as

$$\begin{Bmatrix} \sigma_1^{(k)} \\ \sigma_2^{(k)} \\ \tau_{12}^{(k)} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_1^{(k)} \\ \varepsilon_2^{(k)} \\ \gamma_{12}^{(k)} \end{Bmatrix} \tag{11}$$

where subscripts 1 and 2 are the directions of the fiber and in-plane normal to fiber, respectively, subscript 3 indicates the direction normal to the plate; and the reduced stiffness components, $Q_{ij}^{(k)}$ are given by

$$Q_{11}^{(k)} = \frac{E_1^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}}, Q_{12}^{(k)} = \frac{\nu_{12}^{(k)}E_2^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}},$$

$$Q_{22}^{(k)} = \frac{E_2^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}}, Q_{33}^{(k)} = G_{12}^{(k)}$$

in which $E_1^{(k)}$, $E_2^{(k)}$, $G_{12}^{(k)}$, $\nu_{12}^{(k)}$ and $\nu_{21}^{(k)}$ are independent material properties for each layer.

The laminate is usually made of several orthotropic layers. Each layer must be transformed into the laminate coordinate system (x, y, z). The stress - strain relationship is given as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}^{(k)} \tag{12}$$

where $\bar{Q}_{ij}$ is the transformed material constant matrix[3].

A weak form of the static model for composite sandwich plates can be briefly expressed as:

$$\int_\Omega \delta\boldsymbol{\varepsilon}^T \bar{\mathbf{D}}\boldsymbol{\varepsilon}\,\mathrm{d}\Omega = \int_\Omega \delta w p\,\mathrm{d}\Omega \tag{13}$$

where $\boldsymbol{\varepsilon}$ and w are the strains and the deflection and the material matrix $\bar{\mathbf{D}}$:

$$\bar{\mathbf{D}} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix},\ \boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon}_0 \\ \boldsymbol{\kappa} \end{bmatrix} \tag{14}$$

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} \left(1, z\ , z^2\right) Q_{ij}\,\mathrm{d}z \quad i,j = 1,2,6$$

Using the same NURBS basis functions, both the description of the geometry (or the physical

point) and the displacement field are expressed as

$$\mathbf{x}^h(\xi,\eta) = \sum_A^{nxm} \mathbf{R}_A(\xi,\eta)\mathbf{P}_A \text{ and}$$

$$\mathbf{u}^h(\mathbf{x}(\xi,\eta)) = \sum_A^{nxm} \mathbf{R}_A(\xi,\eta)\mathbf{q}_A \tag{15}$$

where $n \times m$ is the number basis functions, $\mathbf{x}^T = (x\quad y)$ is the physical coordinates vector. $\mathbf{R}_A(\xi,\eta)$ is rational basic functions and $\mathbf{q}_A = [u_A\ v_A\ w_A]^T$ is the degrees of freedom of $\mathbf{u}^h$ associated to control point A.

The strains in Eq. (14) can be expressed to following nodal displacements as:

$$[\boldsymbol{\varepsilon}_0\quad \boldsymbol{\kappa}]^T = \sum_{A=1}^{nm} [\mathbf{B}_A^m\ \ \mathbf{B}_A^b]^T \mathbf{q}_A \tag{16}$$

where

$$\mathbf{B}_A^m = \begin{bmatrix} R_{A,x} & 0 & 0 \\ 0 & R_{A,y} & 0 \\ R_{A,y} & R_{A,x} & 0 \end{bmatrix} \text{ and}$$

$$\mathbf{B}_A^b = \begin{bmatrix} 0 & 0 & -R_{A,xx} \\ 0 & 0 & -R_{A,yy} \\ 0 & 0 & -2R_{A,xy} \end{bmatrix} \tag{17}$$

$\mathbf{B}_A^m$ and $\mathbf{B}_A^b$ are membrane and bending strain-displacement matrices gained from derivative of shape functions, respectively.

The IGA formulation of composite sandwich plates can then be obtained for static analysis:

$$\mathbf{Kq} = \mathbf{f} \tag{18}$$

where the global stiffness matrix is

$$\mathbf{K} = \int_\Omega \left\{ \begin{bmatrix} \mathbf{B}^m \\ \mathbf{B}^b \end{bmatrix}^T \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{B}^m \\ \mathbf{B}^b \end{bmatrix} \right\} \mathrm{d}\Omega \tag{19}$$

and $\mathbf{f}$ is the global force matrix:

$$\mathbf{f} = \int_{\Omega} p\mathbf{R}\mathrm{d}\Omega \tag{20}$$

where $\mathbf{q}$ are the global displacements matrix

4. Numerical Results

In this section, several numerical studies using a rotation-free isogeometric analysis are presented. For all numerical examples, quadratic, cubic and quartic NURBS elements integrated with $nG = (p+1)(q+1)$ Gauss points are used . The material parameters are assumed as:

Material I: $E_1 = 25E_2$; $G_{12} = G_{13} = 0.5E_2$; $G_{23} = 0.2E_2$; $v_{12} = 0.25$

Material II:

Face sheets : $E_1 = 172.4$ GPa; $E_2 = 6.89$ GPa; $G_{12} = G_{13} = 3.45$ GPa; $G_{23} = 1.378$ GPa; $v_{12} = 0.25$ Core: $E_1 = E_2 = 0.276$ GPa; $G_{12} = 0.1104$ GPa; $G_{13} = G_{23} = 0.414$ GPa; $v_{12} = 0.25$

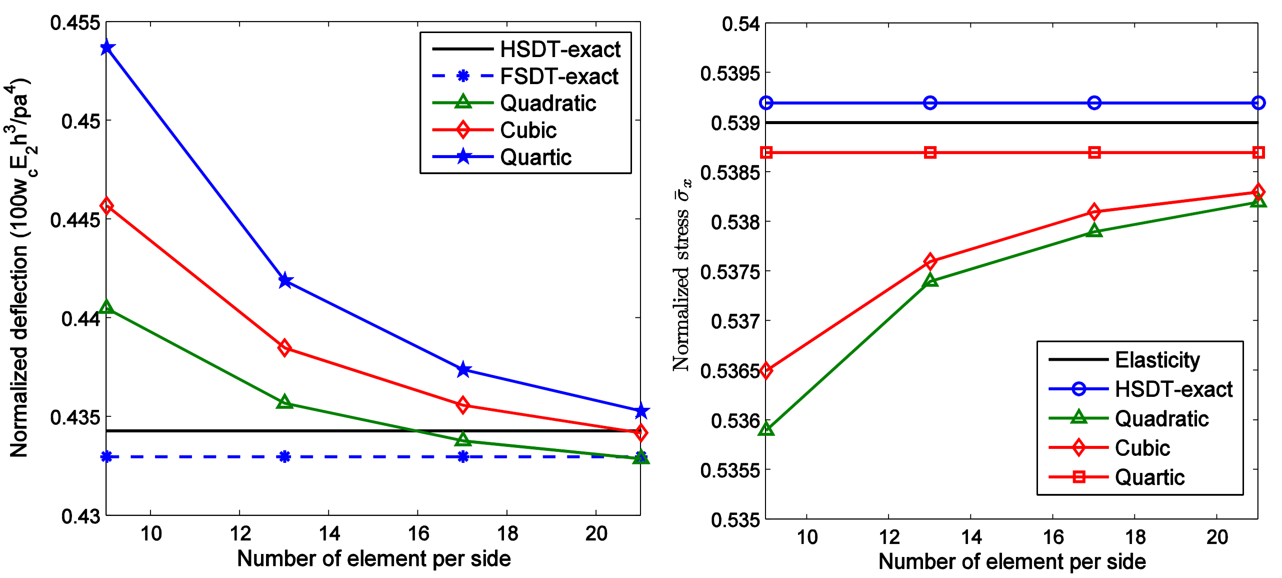
The normalized displacement and in-plane stresses of composite sandwich plate are defined as: $\bar{w} = \dfrac{10^2 wE_2h^3}{q_0a^4}$,

$$\bar{\sigma}_x = \frac{\sigma_x h^2}{q_0 a^2},\ \bar{\sigma}_y = \frac{\sigma_y h^2}{q_0 a^2} \text{ and } \bar{\sigma}_{xy} = \frac{\sigma_{xy} h^2}{q_0 a^2}.$$

4.1. Three Layer ($0^0/90^0/0^0$) Square Laminated Plate Under Sinusoidally Distributed Load

Let us consider a simply supported square laminated plate subjected to a sinusoidal load $q = q_0 \sin\left(\dfrac{\pi x}{a}\right)\sin\left(\dfrac{\pi y}{b}\right)$. The length to width ratios is a/b=1 and the length to thickness ratios is a/h=100. Material I described is use. The plate is modeled by 9x9, 13x13, 17x17 and 21x21 B-spline elements. The convergence of normalized displacement and in-plane stresses are given in Figure 1. It can be seen that, the obtained results is very closed with analytical solutions by Kant[2] based on the third shear deformation plate theory and the elasticity solution 3D by Pagano[1].

In order to compare the results, we calculate the normalized displacement and in-plane stresses of the sandwich square plate using 21x21 B-spline elements, as given in Table 1. Obtained results are compared with the several other methods including the close form solution (CFS) based on the exponential shear deformation plate theory (ESDT) by Aydogdu[18], the elasticity solution given in Pagano[1] and analytical solutions based on Navier's technique by Kant[2]. In[2], there are three-solutions such as: the fully third shear deformation plate theory using 12 dof/node (Kant 1), the third shear deformation plate theory of Reddy using 5 dof/node(Kant 2) and the first shear deformation plate theory 5 dof/node (Kant 3). It is observed that for deflection and stresses the results of the present method agrees well with published results. Figure 2 plots the distribution of stresses through the thickness of the plate. The obtained results are in good agreement with those reported by Kant[2].



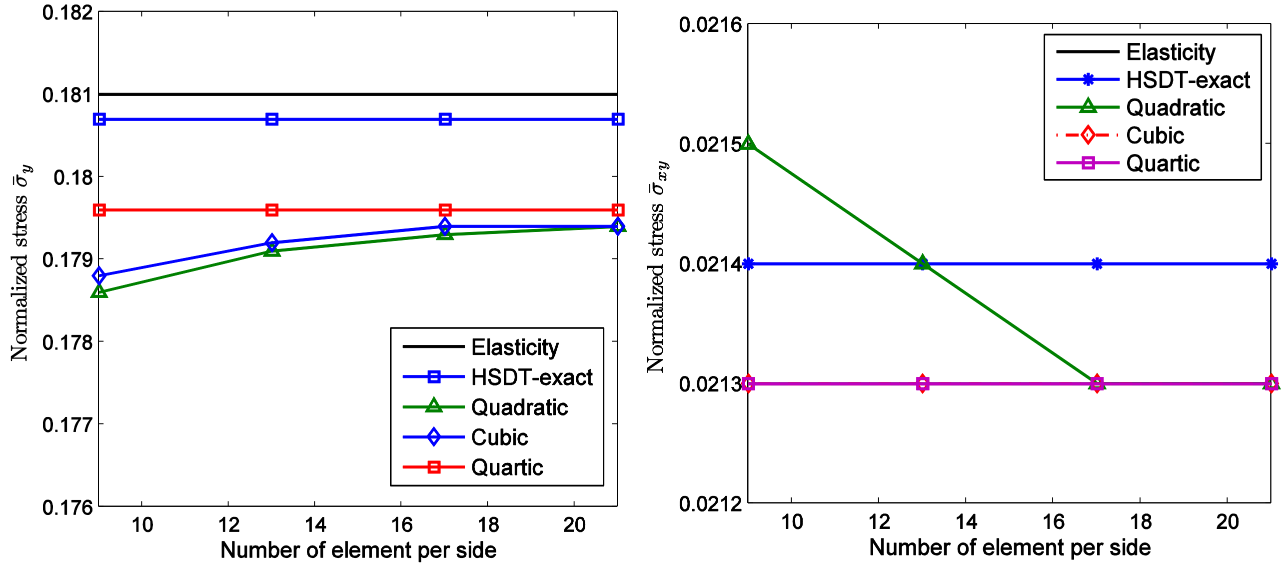


Figure 1. The in-plane normal and shear stresses of the three-layer composite ($0^0/90^0/0^0$) simple supported square plates

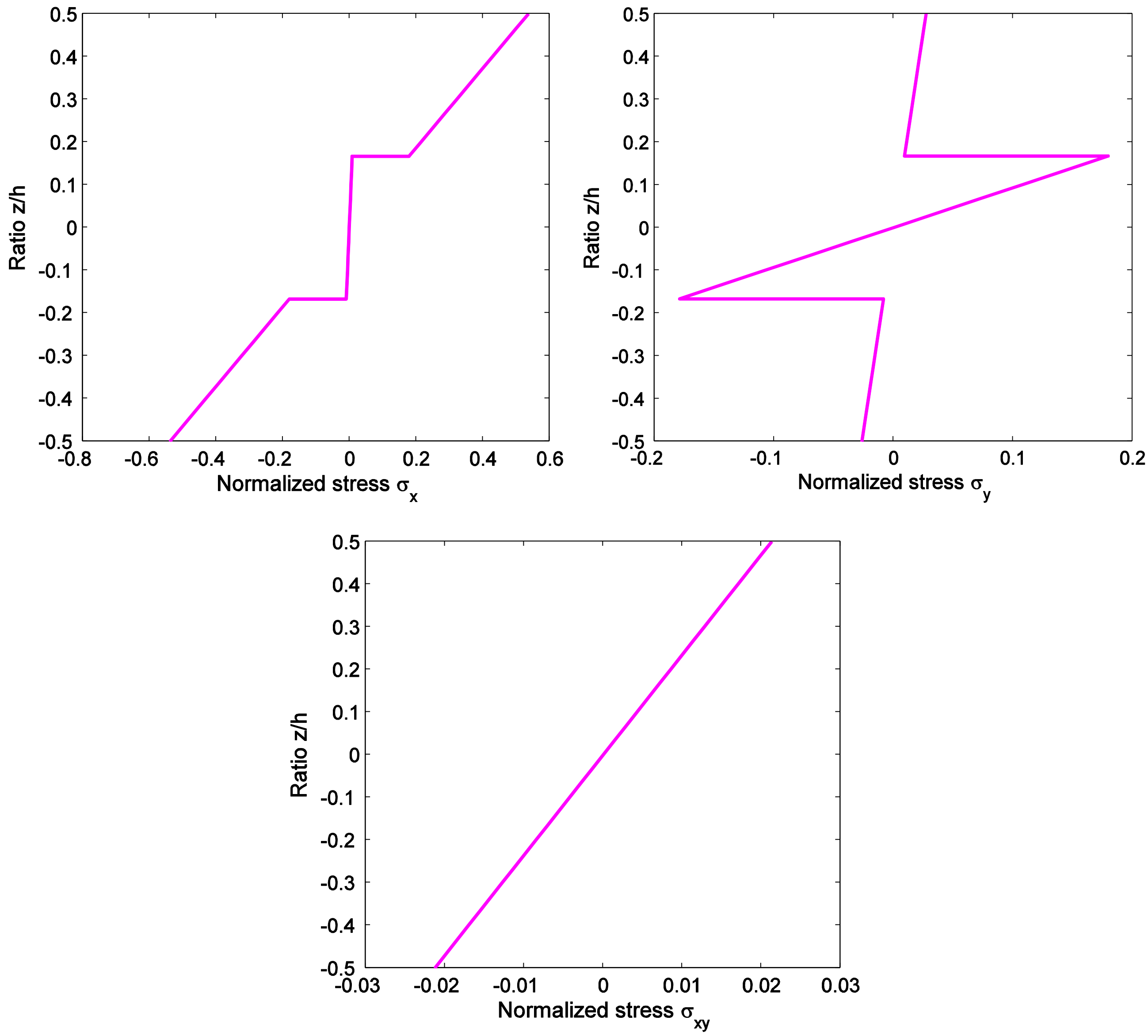


Figure 2. The in-plane normal and shear stresses of the three-layer composite ($0^0/90^0/0^0$) simple supported square plates

Table 1. The normalized displacement and the stresses in a three-layer ($0^0/90^0/0^0$) simply supported square laminate under sinusoidal transverse load

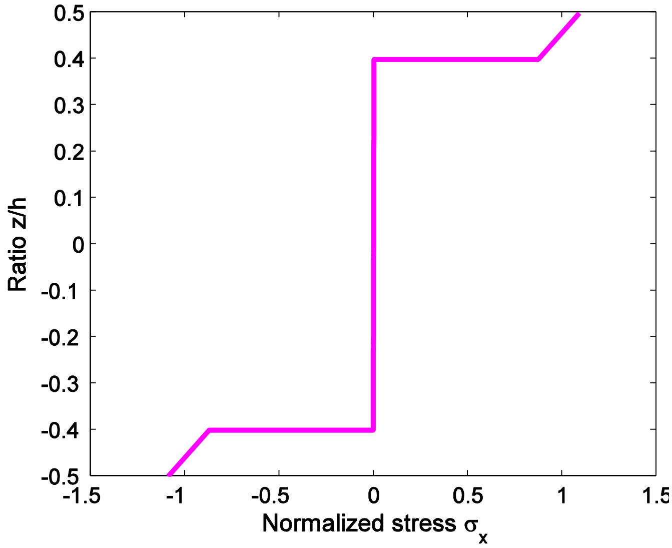
Authors & methods	$\bar{w}(\frac{a}{2},\frac{b}{2},0)$	$\bar{\sigma}_x(\frac{a}{2},\frac{b}{2},\frac{h}{2})$	$\bar{\sigma}_y(\frac{a}{2},\frac{b}{2},\frac{h}{2})$	$\bar{\sigma}_{xy}(0,0,\frac{h}{2})$
Kant 1[2] (HSDT)	0.4343	0.5392	0.1807	0.0214
Kant 2[2] (HSDT)	0.4342	0.5390	0.1806	0.0214
Aydogdu[18] (ESDT)	0.4350	0.5389	0.1806	0.0214
Kant 3[2] (FSDT)	0.4337	0.5384	0.1804	0.0213
Elasticity[1]	-	0.5390	0.1810	0.0213
Quadratic (CLPT)	0.4329	0.5383	0.1794	0.0213
Cubic (CLPT)	0.4342	0.5382	0.1794	0.0213
Quartic (CLPT)	0.4353	0.5387	0.1796	0.0213

4.2. The Sandwich (0^0/core/0^0) Square Plate under Sinusoidally Distributed Load

We consider the sandwich (0^0/core/0^0) simply supported square plate subjected to sinusoidally distributed load with the thickness of each face sheet equal $h/10$. Material II is used. The plate is modeled by 21x21 B-spline element. The normalized transverse displacement and normalized stresses are reported Table 2. The obtained results are compared with the exact elasticity solution by[1], the analytical solution by[2], FEM solutions based on the higher order zigzag plate theory (HOZT) by[16, 17] and and FEM solutions based on the third shear deformation plate theory by[15]. It is found that the results of present method shown good agreements with those solutions. The distribution of stresses through the thickness of the plate is illustrated in Figure 3.

Table 2. The normalized displacement and the stresses in a three-layer (0^0/core/0^0) simply supported square sandwich under sinusoidal transverse load

Author & method	$\bar{w}(\frac{a}{2},\frac{b}{2},0)$	$\bar{\sigma}_x(\frac{a}{2},\frac{b}{2},\frac{h}{2})$	$\bar{\sigma}_y(\frac{a}{2},\frac{b}{2},\frac{h}{2})$	$\bar{\sigma}_{xy}(0,0,\frac{h}{2})$
Kant 1[2]	0.8913	1.0990	0.0560	0.0436
Kant 2[2]	0.8908	1.0973	0.0549	0.0436
Kant 3[2]	0.8852	1.0964	0.0546	0.0435
Elasticity[1]	-	1.0980	0.0550	0.0437
Singh *et al.*[17]	0.9017	1.1020	-	0.0453
Pandit *et al.* [16]	0.8917	1.1093	0.0547	0.0434
Tran *et al.* [15]	0.8919	1.1069	0.0573	0.0432
Quadratic	0.8816	1.0962	0.0542	0.0434
Cubic	0.8842	1.0965	0.0542	0.0433
Quartic	0.8864	1.0970	0.0543	0.0433



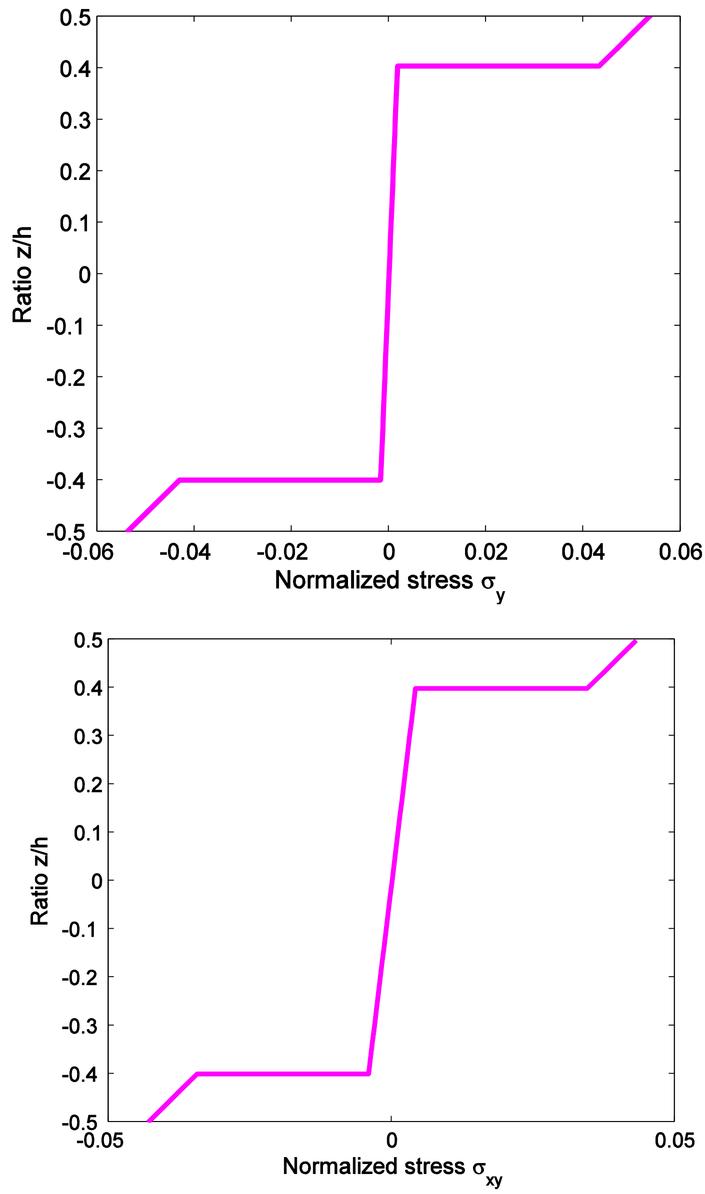


Figure 3. The in-plane normal and shear stresses of the sandwich (0/core/0) simple supported square plates

4.3. An-symmetry the Sandwich ($0^0/90^0$/core/$0^0/90^0$) Square Plate under Sinusoidally Load

Table 3. The normalized displacement and the stresses in a five-layer ($0^0/90^0$/core/$0^0/90^0$) SCSC and CCCC square sandwich under sinusoidal transverse load

Boundary conditions	Method	$\bar{w}(\frac{a}{2},\frac{b}{2},0)$	$\bar{\sigma}_x(\frac{a}{2},\frac{b}{2},\frac{h}{2})$	$\bar{\sigma}_y(\frac{a}{2},\frac{b}{2},\frac{h}{2})$
SCSC	Pandit *et al.*[16]	0.3453	0.4077	0.0326
	Singh *et al.*[17]	0.3920	0.5986	–
	Chalak *et al.*[19]	0.3430	0.4250	0.0366
	Quadratic	0.3328	0.3969	0.0327
	Cubic	0.3358	0.3990	0.0327
	Quartic	0.3369	0.3994	0.0328
CCCC	Pandit *et al.*[16]	0.2286	0.4270	0.0228
	Singh *et al.*[17]	0.2260	0.4283	–
	Chalak *et al.*[19]	0.2267	0.4371	0.0259
	Quadratic	0.2200	0.4300	0.0229
	Cubic	0.2221	0.4302	0.0229
	Quartic	0.2231	0.4305	0.0229

In order to study the stretching-bending coupling effect, the an-symmetry five-layer sandwich plate ($0^0/90^0$/core/$0^0/90^0$) is considered. Material II is also used. The core has a thickness of 0.8*h* while the two laminated face -sheets are of 0.1*h*. The plate has supported (S) and clamped (C) boundary conditions. When 21x21 element mesh, the normalized displacement and stresses derived from the present method of a five-layer sandwich plate with various boundary conditions are given in Table 3. For comparison, other methods based on C^0 higher order zigzag plate theory by Chalak *et al*.[19], Singh *et al*.[17] and Pandit *et al*. [16] are cited. It is observed that the present results are in good agreement with published ones for both SCSC and CCCC boundary conditions.

5. Conclusions

An isogeometric formulation has been developed for static analysis of the composite sandwich plates using a rotation-free isogeometric formulation of CLPT. Weak form of the static model for composite sandwich plates using CLPT was derived. The present method only used three degrees of freedom per node (3 dof/node), and the obtained results are in very good agreement with analytical solution by Kant 1[2] using 12 dof/node, Kant 2[2] using 5 dof/node, FEM solutions using 11 dof/node[16, 17, 19] and FEM solutions using 9 dof/node[15]. The distribution of stresses through the thickness of the sandwich plates are in very good agreement with those of other existing methods.

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