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Estimation of Coda Wave Attenuation Quality Factor from Digital Seismogram Using Statistical Approach

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Abstract The main objective of the paper is to estimate seismic wave attenuation from the decay rate of coda wave amplitudes of digital seismogram of local earthquakes (epicentral distance < 100 km) following statistical approach. Three earthquake events have been selected for estimation of coda wave attenuation quality factor (Q_c). The coda wave of 9 seismograms from 3 local earthquakes recorded digitally by three seismic stations in the region have been analyzed for this purpose at ten central frequencies (1, 1.5, 2, 3, 4, 6, 8, 12, 16 and 18Hz) and three lapse time windows of 20, 30 and 40 seconds duration. Single backscattering model proposed by Aki (1969) and extended by Aki and Chouet (1975) and Sato (1977) is employed to determine Q_c values. The mean values of the estimated Q_c vary from 31$\pm$12 (at 1Hz) to 1974$\pm$211 (at 18Hz) for 20 seconds coda window length. For 30 seconds coda window length Q_c vary from 46$\pm$16 (at 1Hz) to 1977$\pm$ 256(at 18Hz). Similarly for 40 seconds coda window length Q_c vary from 97$\pm$ 22 to 2552 $\pm$312. It is observed that Q_c value for the study area is frequency dependent and increase with increase in frequency. Moreover the observed Q_c values increase with increasing lapse time at all frequency bands. The estimated Q_c values show a frequency dependent relationship of the form $Q_c=Q_o f^n$, where Q_0 is Q_c at 1Hz and n represents degree of frequency dependence. The frequency dependent empirical attenuation relationship for 20, 30 and 40 seconds window lengths are obtained as $Q_c = 21.49+1.17 f^{1.48+0.08}$, $Q_c = 48.6+1.11 f^{1.29+0.06}$ and $Q_c = 88.86+1.12 f^{1.19+0.06}$ respectively.

Keywords Earthquake, Seismogram, Coda Waves, Central Frequency, Epicentral Distance, Local Earthquake

1. Introduction

Attenuation is one of the fundamental properties of seismic waves from which the material and physical conditions in the Earth's interior can be inferred (Aki, 1980). The decay of seismic wave amplitude with distance defines the attenuation of the medium. Seismic attenuation is usually considered to be a combination of two mechanisms – intrinsic absorption and scattering loss. As seismic wave propagate through the Earth's interior and finally arrives at seismic stations on the surface, its energy (amplitude) decays due to conversion of elastic energy to heat or other forms of energy (intrinsic attenuation) as well as energy redistribution in the heterogeneities present in the lithosphere (scattering). The measurement and interpretation of elastic wave attenuation is important for studying the medium through which seismic wave propagates.

Attenuation of seismic wave is described by a dimensionless parameter called the quality factor Q (Knopoff, 1964) that expresses the wave amplitude decay that occurs when a wave propagates through a medium the inverse of the quality factor (Q^{-1}) is known as the attenuation factor, which is proportional to decay of amplitude with passages of time or source- receiver distance. The attenuation quality factor (Q) is a combination of intrinsic quality factor, Q_i and scattering quality factor Q_s.

There are various methods to estimate the attenuation quality factor of seismic wave in a region. It is difficult to measure attenuation from a high frequency seismogram using a deterministic approach, because large number of parameters is required to adequately explain a high frequency seismogram. In order to overcome this difficulty Aki (1969) initiated the application of a statistical approach to study of high frequency seismic waves. All portions of a seismogram cannot be treated entirely statistically, because their characters are determined by a particular nature of the path between the source and station (Aki, 1969). Aki (1969), developed a model for the rate of coda decay and suggested that the seismic coda waves of local earthquakes are backscattered body waves from numerous randomly distributed heterogeneities in the earth's crust and may be treated by a statistical method. The longer the wave travels the greater the variety of heterogeneities they encountered. The later portions of a seismogram therefore may be considered as a result of some kind of averaging over many samples of heterogeneities, thus suggesting a statistical treatment in which a small number of parameters characterize the average prop-

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erties of the heterogeneous medium (Aki and Chouet, 1975). One of the most promising means of studying seismic scattering and attenuation in the crust and lithosphere is analysis of the coda of local earthquake (Aki, 1969). Coda is the end part of a seismogram of locally recorded earthquake following the body and surface waves containing a very complex and gradually decaying signal. The coda wave amplitude on a seismogram is explained by two backscattering models (Aki and Chouet, 1975).

The first is the single scattering model, according to which waves are backscattered S waves and generated when S wave encounters different heterogeneity present in the propagating media. This model assumes that the scattering wave field is weak and does not produce secondary scattering when it encounters another scatterer. The law of energy conservation is violated because of oversimplification of this model. This model assumes that earthquake source and the seismic station are located at the same point in an unbounded, homogenous, and isotropic medium containing a random but uniform spatial distribution of heterogeneities. This model was further modified by Sato (1977), considering a finite distance separating the source and receiver.

The second model is the multiple scattering models (Aki and Chouet, 1975; Gao et al., 1983a), which assumes secondary scattering and consider the seismic energy transfer as a diffusion process. Aki (1969) suggested that coda Q_c^{-1} is more closely related to intrinsic rather than scattering attenuation. Within the frame work of the single scattering theory Q_c^{-1} represents an effective attenuation, including the contribution of both intrinsic and scattering loss (Akinci et al., 2000). Several authors (e.g. Aki and Chouet, 1975; Sato, 1977; Rautian and Khalturin, 1978; Hermann, 1098; Roecker et al., 1982; Frankel, 1991; Woodgold, 1992; Gupta, 1998) observed that the coda wave model of Aki (1969), extended by Aki and Chouet (1975) and Sato (1977) is the easiest way to estimate the attenuation, backscattering and source spectrum.

In the present study, Q_c is estimated following single scattering model of Aki and Chouet (1975) extended by Sato (1977). The events used in this study are listed in Table 1. Q_c is measured as a function of frequency using the digital waveform data of local earthquakes. A study is made of elastic wave attenuation to develop an attenuation relationship. The coda Q_c values are calculated for different lapse time windows of coda envelope to investigate the lapse time dependence of Q_c as well as the possibility of depth dependence of seismic attenuation.

2. Data

In this study, we selected only 3 earthquakes given in the Table 1 recorded by local seismic network during the three local seismic stations namely Tezpur (TZR), Dokmok (DMK) and Seijusa (SJA).The broadband seismic stations are equipped with CMG-3T / CMG-3ESP seismometer (3-component) and REFTEK 72A series data acquisition system. The sampling rate of the signal is 100 samples per second. The locations of the seismic stations as well as the epicenters of the earthquake events are shown in Figure 1. The selection of the data set is made on the basis of the following criteria:year 2001. These events are recorded by

i. Epicentral distance of the events less than 100 km
ii. Higher signal to noise ratio.
iii. Clear record of coda wave

Out of two horizontal components of seismograms only one component is utilized for this purpose for which S-wave arrival is earlier compared to the other. Total 9 horizontal component seismograms are selected for this purpose. The hypocentral parameters of the selected events are listed in Table 1.

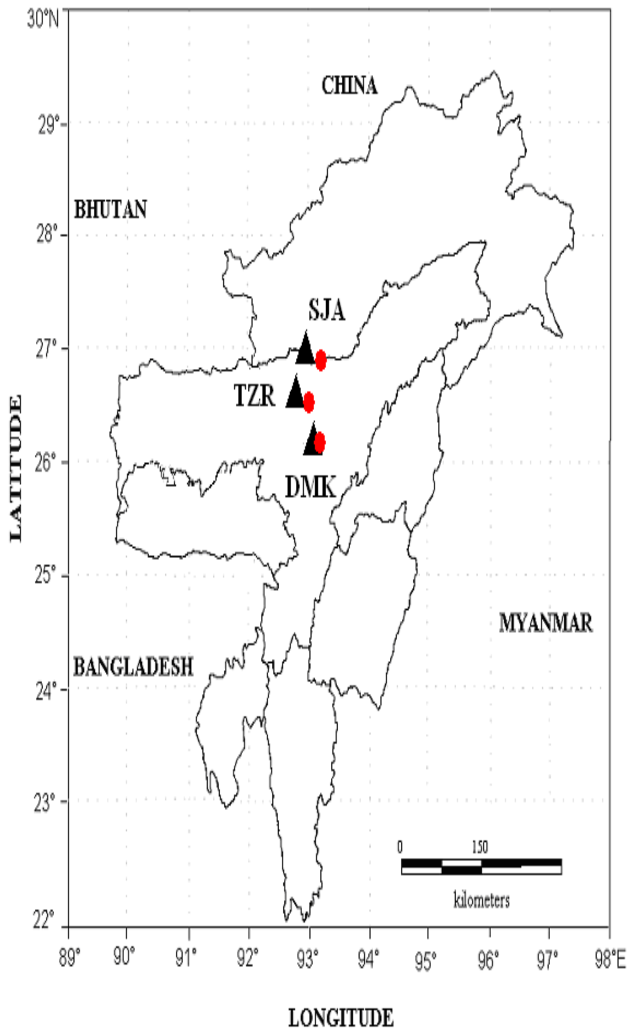

Figure 1. Figure showing location of 3 earthquakes (Red balls) used in the present study and the recording seismograph stations (black triangles) namely Tezpur (TZR), Dokmok (DMK) and Seijusa (SJA)

The signal to noise ratio of each event is estimated. For this purpose the noise and signal power spectra are determined using Fast Fourier Transformation (FFT) algorithm using about 2 seconds long time series before and after the P-wave first arrival. An example for MND station is shown in Figure 2a and Figure 2b.

Table 1. Hypocentral parameters of the events used in this study

Sl. No	Date	Origin Time			Location		Focal Depth	Magnitude
		Hr.	Mu.	Sec.	Lat.^{0}N	Long.^{0}N	(Km)	(M_DA)
1	20011120	21	35	04.12	26.80	93.20	9.8	3.0
2	20011122	11	53	55.98	26.37	93.11	10	2.1
3	20011118	19	37	26.30	26.19	93.11	35	3.4

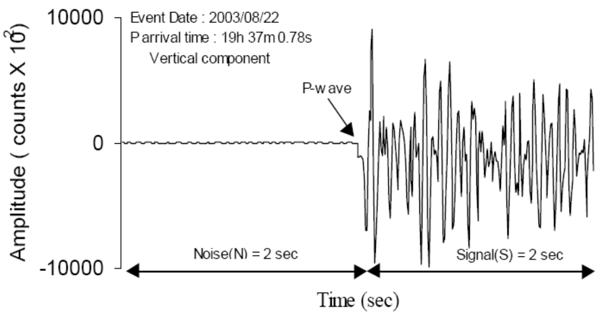

Figure 2a. shows the original seismogram and two time windows (2 sec before and after P arrival) used in the evaluation

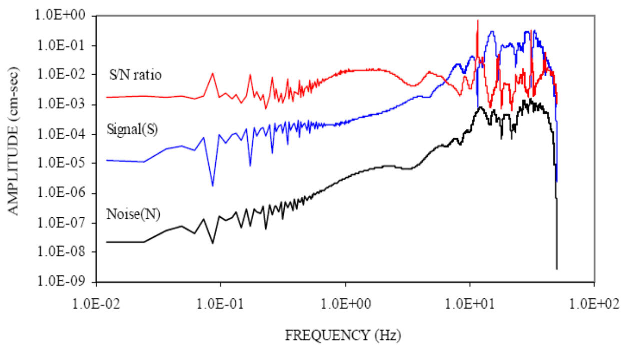

Figure 2b. Shows the corresponding frequency spectra designated as Noise (N)[black], Signal (S)[blue] and S/N ratio [red]

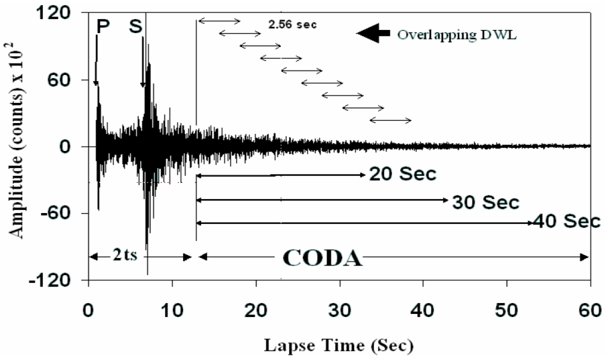

Figure 3. Figure showing a digital seismogram recorded by DMK seismic station. Different coda window length used in this study are shown along with the discrete window length of 2.56 sec used for smoothening the coda envelope by calculating RMS values of coda wave amplitudes with a sliding window of half of the discrete window length i.e. 1.28 seconds

3. Method and Data Analysis

The coda Q method was introduced by Aki(1969) and has been extended by different researchers e.g. Aki and Chouet (1975), Sato (1977) and Rautian and Khalturin (1978). We have utilized the single back scattering model developed by Aki (1969) and extended by Aki and Chouet (1975) and Sato (1977) for estimation of coda wave attenuation quality factor, Q_c.

This model is based on the following assumptions:

i. Scattering is a weak process and doesn't produce any secondary (multiple) when it encounters anther scatter. This so call Born approximation, which violates the energy conservation law but has been accepted in various physical problems and used successfully in high frequency seismic waves analysis.

ii. As the coda waves arrive long time after arrival of all direct waves (p, s and surface waves) the source and receiver are assumed to be placed at one point (for coincident).

According to Aki (1969), Aki & Chouet (1975) and Sato (1977), the time dependence of root mean square coda wave amplitude, A(ω,t), on a bandpass-filtered seismogram can be written as

$$A(\omega,t) = C(\omega).\ t^{-1}\exp(-\omega t/2Q_c) \quad (1)$$

where Q_c is the attenuation quality factor as a function of frequency, t^{-1} is a correction factor for the geometrical spreading , and C(ω) takes into account these terms of source and site amplification. This model is believed to be more appropriate for small local earthquake than multiple – scattering model (Ibanez et al., 1990).

Sato (1977) developed the model where root mean square (rms) coda wave amplitude at lapse time t may be written as

$$A(r, \omega, t) = C(\omega)[K(r, x)] \exp(-\omega t/2Q_c \quad (2)$$

where, $x = t/t_s$ (t_s is the travel time of S wave) and r is station-source distance; K(r, x) is a function of x and r, defined as

$$K(r, x) = 1/r.1/x.\ \ln(x+1/x-1) \quad (3)$$

By taking the natural logarithms of equation (2) and rearranging terms, we obtain the following equation:

$$\ln[A(r, \omega,t)/ [K(r,x)]] = \ln[C(\omega)]-(\omega t/2Q_c) \quad (4)$$

For narrow bandpass-filtered seismograms, C(ω) is constant. Therefore, by using a linear regression of terms on the left side of equation (4) vs t, Q_c can be determined from the slope of the fit, which is equal to $-\omega t/2Q_c$

Table 2. Parameters of band-pass filter showing central frequencies with respective low and high cut off frequencies

Low cutoff (Hz)	Central frequency (Hz)	High cutoff (Hz)
0.67	1.0	1.33
1.00	1.5	2.00
1.33	2.0	2.67
2.00	3.0	4.00
2.67	4.0	5.33
4.00	6.0	8.00
5.33	8.0	10.67
8.00	12.0	16.00
10.67	16.0	21.33
12.00	18.0	24.00

In order to study the frequency and lapse time dependence of Q_c, we used following scheme to analyze the data:

i. The seismograms are filtered for narrow frequency bands centered at f_c = 1, 1.5, 2, 3, 4, 6, 8, 12, 16 and 18 Hz respectively using 8 – pole Butterworth band pass filter. The ten frequency bands (bandwidth $0.67f_c$, where f_c is the central frequency) with filter parameters are given in Table 2.

ii. The beginning of coda amplitude starts at $2t_s$, where t_s is the travel time of S – wave measured from the origin time of the earthquake (Rautian and Khalturin, 1978).

iii. The filtered coda amplitudes are smoothed by determining Root Mean Squire (RMS) amplitude for sliding window of 2.56 seconds in steps of half of the time window i.e. 1.28 seconds. The RMS amplitude values are assigned to the center point of corresponding window Figure 3.

iv. Once the set of A(r, ω, t)/[K(r,x)] and the coda intervals are obtained beginning at 2ts, Q_c could be easily estimated from the slope of the linear fit of equation (4), ln [A(r, ω,t)/ [K(r,x)] vs. t, the lapse time.

In the next step, the lapse time dependence of Q_c is observed by increasing the coda duration step by step, measured from origin time. Each seismogram falling in the lapse time range of 20-90 seconds is analyzed starting at $2t_s$. Figure 4a shows an example of original and band pass filtered seismogram (event no. 2 in Table 1) of a local earthquake recorded by the DMK (Dokmok) station and Figure 4b shows the plot of ln [A(r, ω,t)/ [K(r,x)] vs. t, the lapse time for 30 sec coda window.

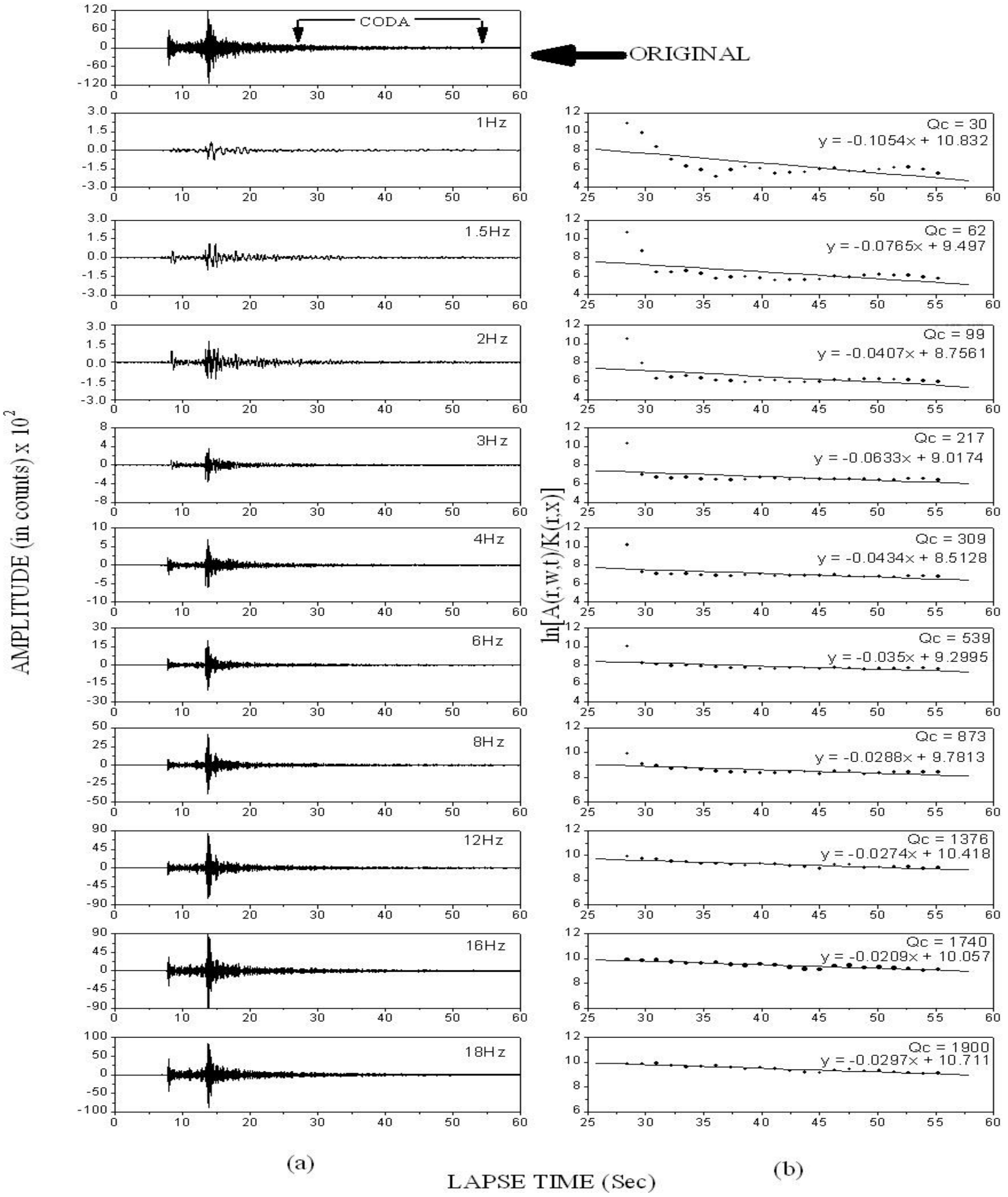

Figure 4 (a). Shows an example of original and band pass filtered seismogram (event no. 2 in Table 1) of a local earthquake recorded by the DMK (Dokmok) station and **(b)** Shows the plot of ln [A(r, ω,t)/ [K(r,x)] vs. t, the lapse time for 30 sec coda window

4. Results and Discussion

The Q_c values are estimated filtering the coda waves of 9 waveforms in frequency band centered at 1, 1.5, 2, 3, 4, 6, 8, 12, 16 and 18Hz for lapse time window length of 20, 30 and 40 seconds. Total 270 Q_c measurements are obtained for 20, 30 and 40 seconds coda window length estimated from the linear regression of the $\ln[A(f, t)/K(r, x)]$ vs t plot. The Q_c values for all lapse time and frequency ranges are listed in Table 3a, Table 3b and Table 3c

The Q_c measurements estimated from 20 sec coda window length vary from 7 to 92 at frequency 1Hz and 562 to 5492 at 18Hz. The distributions of Q_c values with frequency are shown in Figure 5a. The mean value of Q_c observations (filled circles) using 20 seconds coda window length for all the stations are also shown in the Figure 5a which vary from 31 ± 14 at 1Hz to 1974 ± 209 at 18Hz. It is observed from the general trend (Figure 5a) that Q_c values follows a power law of the from $Q_c = Q_0 f^n$, where Q_0 is the quality factor at 1Hz and n is the frequency dependent coefficient. For 20 seconds coda window Q_0 and n are 21.49 ± 1.11 and 1.48 ± 0.08 respectively and we obtain the frequency dependent attenuation relation as $Q_c = 21.49 \pm 1.11 f^{1.48 \pm 0.08}$.

For 30 sec coda window length Q_c measurements vary from 14 to 115 at frequency 1Hz and 1065 to 3400 at 18Hz. The distributions of Q_c values with frequency are shown in Figure 5b. The mean value of Q_c observations (shown by dark filled circles in Figure 5b) vary from 46 ± 22 at 1Hz to 1977 ± 225 at 18Hz. The empirical attenuation relationship for 30 seconds coda window length is obtained as $Q_c = 69.92 \pm 1.11 f^{123 \pm 0.058}$.

Table 3a. The Q_c values for the three at different stations and lapse time coda window length of event no.1.

Frequency (Hz)	Qc values at different stations and lapse time coda window length								
	TZR			DMK			SJA		
	20Sec	30Sec	40Sec	20Sec	30Sec	40Sec	20Sec	30Sec	40Sec
1	7	14	24	15	37	80	32	115	80
1.5	14	33	67	27	79	178	59	332	178
2	25	70	148	45	153	370	119	599	370
3	58	184	354	120	354	834	345	600	834
4	99	305	718	206	474	1197	497	655	1197
6	149	379	1157	263	789	1349	545	637	1349
8	189	451	1065	382	830	1643	603	760	1643
12	325	675	1407	614	1205	1813	1065	1314	1813
16	482	938	2372	827	1552	2206	2036	1957	2206
18	562	1065	3143	921	1704	2377	2254	2103	2377

Table 3b. The Q_c values for the three at different stations and lapse time coda window length of event no.2.

Frequency (Hz)	Qc values for different stations and lapse time coda window								
	TZR			DMK			SJA		
	20Sec	30Sec	40Sec	20Sec	30Sec	40Sec	20Sec	30Sec	40Sec
1	7	16	30	13	30	56	91	90	165
1.5	18	46	88	23	62	111	131	117	168
2	35	97	192	40	99	183	273	200	203
3	81	233	499	97	217	334	396	326	314
4	118	310	655	147	309	437	283	345	414
6	174	392	702	242	539	704	423	725	1136
8	249	528	822	388	873	1061	783	1360	1369
12	446	941	1253	804	1376	1661	1831	2170	2071
16	734	1327	1586	1148	1740	2149	2592	2313	3291
18	883	1513	1773	1277	1905	2338	4480	3400	3701

Table 3c. The Q_c values for the three at different stations and lapse time coda window length of event no.3

Frequency (Hz)	Qc values for different stations at different lapse time coda window								
	TZR			DMK			SJA		
	20Sec	30Sec	40Sec	20Sec	30Sec	40Sec	20Sec	30Sec	40Sec
1	7	14	25	13	35	60	92	64	357
1.5	15	35	64	27	74	125	139	130	536
2	30	69	116	46	126	206	224	234	551
3	63	112	237	106	239	37	439	446	693
4	94	117	328	174	332	433	635	454	990
6	141	278	414	362	577	612	1217	659	1193
8	205	400	640	699	1056	895	1722	982	1316
12	368	664	1003	1429	1485	1305	2357	1539	1964
16	548	976	1425	1876	1876	1783	3104	2296	2941
18	632	1157	1697	2050	2135	2006	3697	2814	3558

Similarly, the distribution of Q_c estimates using 40 Sec window length is shown in Figure 5c. The mean value of Q_c estimates at 40 seconds vary from 97 $\pm$ 34 at 1Hz to 2552 $\pm$ 685 at 18Hz. The empirical attenuation relationship from these Q_c values are obtained as $Q_c = 88.86 \pm 1.12 f^{1.19 \pm 0.06}$.

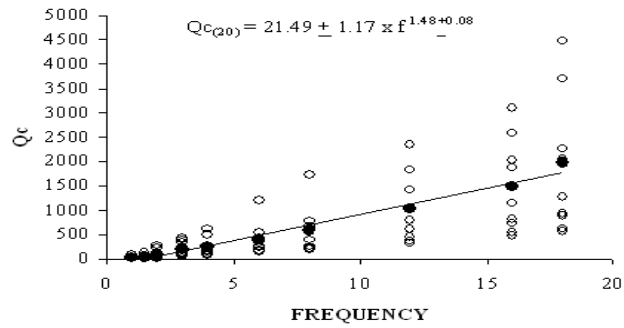

Figure 5a. Shows the distributions of Q_c values with frequency for coda window lengths 20 seconds

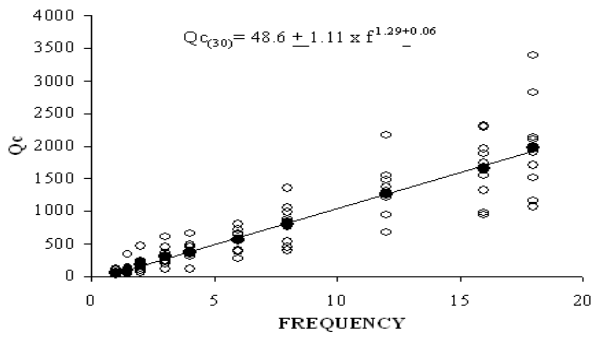

Figure 5b. Shows the distributions of Q_c values with frequency for coda window lengths 30 seconds

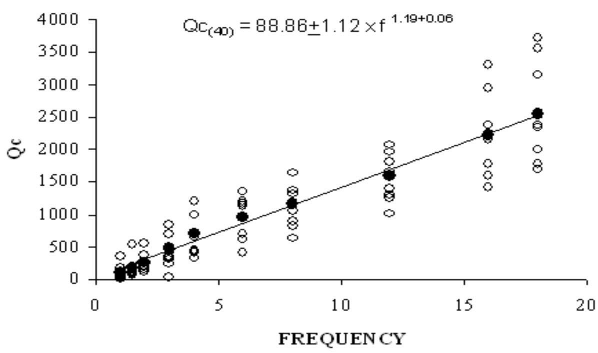

Figure 5c. Shows the distributions of Q_c values with frequency for coda window lengths 40 seconds

From the above results it is observed that Q_c values obtained for the seismograms are highly frequency dependent. The Q_c values increases with increase in frequency. The high frequency dependent characteristics of the Q_c values may be due to different heterogeneity present in the propagating media. Aki(1980) observed that only highly fractured media can generate frequency dependent Q_c values. Moreover it is observed that Q_c value increases with increase in coda window length. The variation Q_c values with lapse time is plotted in Figure 6. The degree of frequency dependence 'n' is almost constant whereas Q_o values are increasing with increase in lapse time coda window length. The higher lapse time of coda window samples larger area of the earth's crust covering the deeper part. The Q_c value increases with depth implies that attenuation is decreasing with depth. This may be due to the fact that homogeneity increases with depth.

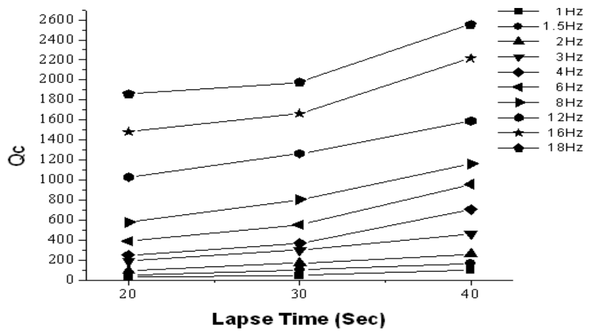

Figure 6. Figure showing the variation of Qc values with lapse time of coda window length

5. Conclusions

This study suggests that Q_c values are frequency dependence in the media through which it propagates. The value of frequency dependence n (> 1) suggests that the media is not homogeneous. Since the media is non-homogeneities, intrinsic and scattering mechanism takes place causing the attenuation of the coda wave amplitudes. Attenuation decreases with depth may indicating that deeper part of the earth's crust is comparatively homogeneous than the upper most crust.

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