A Method in Computing Successive Interference Canceller

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Abstract Successive Interference Canceller is a multistage approach that sequentially recovers each user from the received signal. Each stage provides an estimate using the MF detector. The re-modulated signal is subtracted from the original signal and the difference signal becomes the input. Bit Error Rate expressions are derived to evaluate performance of the SIC and its algorithm.

Keywords DS/CDMA, SIC, BER, PIC, Error probability, MAI, Optimum Detector

1. Introduction

Multiuser Detection is developed for CDMA uplink systems and other environments in which the receiver has knowledge of the signature waveforms for K users. Subspace techniques are used to estimate the interference from the unknown users and the interference estimate is subtracted from the received signal. It can be difficult to recover CDMA signals, primary because the received power levels may be spread over a wide range among the users. We focus on a multistage algorithm called the successive interference canceller (SIC) which uses a single antenna element to sequentially recover CDMA signals according to their power levels[2]. Bit error rate (BER) expressions are derived to evaluate the performance of the SIC for near-far scenarios[5]. We also examine a parallel interference canceller (PIC) which is similar to the SIC except that all users are estimated simultaneously in each stage.

2. Multiuser Detection

Consider a DS/CDMA system where K active users transmit their information asynchronously over a common additive white Gaussian noise (AWGN) channel[4]. The received signal impinging on a single antenna element at the base station can be modeled as

\[ r(t) = \sum_{j=1}^{K} \sum_{i=-\infty}^{\infty} A_j b_j(i) s_j(t - iT_j) + n(t) \]  

(2.1)

Where \( A_j \) = received amplitude of the jth user, \( b_j(i) \) = transmitted symbol (±1) of the jth user, \( s_j(t) \) = signature waveform of the jth user, \( T = \) symbol interval, \( \tau_j = \) time delay of the jth user, and \( n(t) = \) AWGN with zero mean and a two-sided power spectral density.

A signature waveform can be represented by

\[ s_j(t) = \sum_{i=0}^{N_j} s_{j,i} P_j(t - iT_j) \]  

(2.2)

where

\[ P_j(t) = \begin{cases} 1/\sqrt{T_j}, & t \in [0, T_j] \\ 0, & \text{otherwise} \end{cases} \]

Multiple access interference (MAI) is caused by nonzero cross-correlations between the signature waveforms; it is the limiting factor in a DS/CDMA system[13]. The \((k,j)\)th component of the cross-correlation matrix \( R(l) \) for time-shifted versions of the signature waveforms of the \( k \)th and \( j \)th users is given by

\[ R_{kj}(l) = \int_{-\infty}^{\infty} s_k(t - \tau_k) s_j(t + lT - \tau_j) dt = \rho_{kj}(l) \]  

(2.3)

3. Conventional and Optimum Detector

The sampled output of the matched filter of the \( j \)th user in the \( m \)th symbol interval is

\[ r_j(m) = \sum_{l=-\infty}^{\infty} r(t) s_j(t - mT - \tau_j) dt = \sum_{l=-\infty}^{\infty} A_j b_j(i) s_j(t - iT_j) + n(t) \]  

(3.1)

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where
\[ n_j(m) = \int_{mT+\tau_j}^{(m+1)T+\tau_j} n(t) s_j(t-mT-\tau_j)dt \quad(3.2) \]

The optimum detector employs maximum likelihood sequence estimation (MLSE) to jointly estimate all users (JMLSE)[14]. The Viterbi algorithm replaces the conventional decision devices, which uses the signal cross-correlations to choose the globally optimal transmitted data sequence for each user[2]. It can be useful to employ JMLSE as a benchmark to evaluate the performance of suboptimum approaches.

4. Successive Interference Canceller (SIC)

Successive interference cancellation is a multistage approach that sequentially recovers each user from the received signal[15]. Each stage provides an estimate of one source by using the conventional MF detector[14]. The transmitted symbol of the \( j \)th user in the \( m \)th symbol interval can be determined by
\[ \hat{b}_j(m) = \text{sgn} \left[ r_j(m) - \sum_{k<j} \hat{A}_k b_k(m) \rho_{jk}(0) \right] \quad(4.1) \]

Where \( \hat{A}_k \) is the estimated amplitude of the \( k \)th user. For the case of synchronous channels,
\[ \hat{b}_j(m) = \text{sgn} \left[ r_j(m) - \sum_{k<j} \hat{A}_k b_k(m) \rho_{jk}(0) \right] \quad(4.1) \]

5. Parallel Interference Canceller (PIC)

The PIC is another multistage nonlinear receiver that can be used to separate and demodulate co-channel signals in a DS/CDMA system[11,12].

The decision variables in the \( m \)th stage can be written as:
\[ r_i^{(m)} = A_d i + \sum_{k \neq j} \rho_{ki} A_j (d_k - \hat{d}_k) + n_i \]

Consider the estimate in the first stage:
\[ \hat{d}_i = \text{sgn} \left[ Ad_i + \sum_{k \neq j} \rho_{ki} A_j (d_k + n_k) \right] \quad(5.1) \]

The probability of error in decoding the user in the first stage is:
\[ P(d_i \neq \hat{d}_i | d_j, ..., d_j) = \begin{cases} P(n_i^{(1)} < -A - \sum_{k \neq j} \rho_{ki} A_j d_k), d_i = 1 \\ P(n_i^{(1)} > -A - \sum_{k \neq j} \rho_{ki} A_j d_k), d_i = -1 \end{cases} \]

where
\[ Q\left( (A + \sum_{k \neq j} \rho_{ki} A_j d_k) / \sigma, d_i = 1 \right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt \quad(5.3) \]

6. Group-Blind Detector for Asynchronous CDMA

The received signal due to user \( k \) (1 < \( k \) < \( K \)) is given by
\[ y_k(t) = A_k \sum_{i=0}^{M-1} c_i(t-jT_i-iT_d) \quad(6.1) \]

The total received signal is given by
\[ r(t) = \sum_{k=1}^{K} y_k(t) + v(t) \quad(6.2) \]

The \( n \)th matched filter output during the \( i \)th symbol interval is
\[ r[i,n] = \int_{T'nT'}^{T'nT'} r(t) s(t-iT-nT_d) dt \]

Substitutes (5.1) into (5.3), then we have
\[ r[i,n] = h[i] + \sum_{j=1}^{r-1} h[j,n] b_j(i-j) + \sum_{k \neq k} y[k,i,n] + v[i,n] \quad(6.4) \]

with
\[ h[k,p,n] = A_k \sum_{j=0}^{r-1} c_j s(t-jT_i + nT_d + pT - d_i) dt \]

5.5. Numerical Results

We consider a DS/CDMA system with ten users and spreading codes of length \( N=64 \). Ten million events were generated using Monte-Carlo simulations to estimate the error probabilities.

The below table shows the Correlation Values \( \rho_{ij} \) of the spreading codes for a Ten-User DS / CDMA system.

<table>
<thead>
<tr>
<th>Correlation Values</th>
<th>( \rho_{ij} )</th>
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<tbody>
<tr>
<td>1</td>
<td>-0.06</td>
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<tr>
<td>-0.06</td>
<td>0.19</td>
</tr>
<tr>
<td>0.06</td>
<td>-0.19</td>
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<tr>
<td>-0.19</td>
<td>0.31</td>
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<tr>
<td>0.31</td>
<td>0.13</td>
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<td>0.13</td>
<td>0.06</td>
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<tr>
<td>0.06</td>
<td>-0.13</td>
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<tr>
<td>-0.13</td>
<td>0.19</td>
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<td>0.19</td>
<td>-0.13</td>
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<tr>
<td>-0.13</td>
<td>0.25</td>
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As a result, estimates of the error probabilities on the order of $10^{-5}$ (or higher) are quite reliable. Note that most of the cross-correlation coefficients are in the range (-0.2, 0.2); the noise variables are approximately uncorrelated.

8. Conclusions

In this paper, we described SIC and PIC algorithms for DS/CDMA signals, and analyzed their BER performance using conditional probability methods. An analysis of the BER performance of the SIC for a two-user DS/CDMA system illustrated the effects of erroneous cancellation and amplitude mismatch. With reference to single path, a robust multiuser detector for non-coherent detection signals has been implemented. A novel blind multiuser detector is derived for joint multiuser detection and differential decoding, which can be efficiently implemented using Monte Carlo procedure.

REFERENCES


