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An Analysis of the Stability and the Efficiency of a Cooperative Network among Companies, Universities and Local Government

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Abstract We build a model of cooperative economic network to study the relations among three types of agents: companies, universities, and local government or federal agents. First, we study the pairwise stability and efficiency of the network without restricting the research capacity of member universities. Under these circumstances, a network including only the "best" member universities ("good" in the sense of research capabilities) is pairwise stable. The number of universities included in the network depends on the strength of return for member companies. However, as long as the minimum effective return for government is large enough, an *incomplete* pairwise stable network does *not* necessarily have maximum efficiency. The network exhibits "tension" between stability and efficiency. Secondly, as a natural extension, we study a more refined model with restrictions on research capacities of member universities. Optimization methods are used to study the resource allocation of member companies to universities in order to maximize the utility function of member companies. We are able to show that by taking into account research capacity, "weaker" universities could be included in the network when "stronger" universities are over their capacities and a possibly greater efficiency of the entire network could thus be obtained.

Keywords Social (Economic) Network, Cooperative Economic Network, Game Theory, Optimization, Stability, Efficiency

1. Introduction

Network structures play an important role in organizations having strong social or economic interactions. Networks include the relationships among companies regarding with whom and how they conduct their business. The place of a company in the network affects not only its own productivity, but also its bargaining position within the network, which can be reflected in the structure of such organizations. Such networks may also include the relationships among relatives and friends with whom we share information in our daily life. A cooperative economic network through which relevant information is shared among its members can be formed by a group of companies that are of similar type or of supplementary type. Such collaborations bear important significance on the overall productivity of the group. In the era of global business, more and more cooperative networks are formed among all kind of business types.

In particular, cooperative economic networks among companies, universities, and local governments (CUG) have shown its potential power in the area of advanced manufacturing. Such network provides a platform for collaborations between manufacturers and research institutions and aims at delivering “production ready” solutions to manufacturing factories. It is a game changer in that it provides a faster and more affordable approach to turn new finding and new ideas in the academic forefront into cutting edge technologies that brings in business profits. In the US, President Obama announced a proposal to create a National Network for Manufacturing Innovation (NNMI) consisting of up to 15 Institutes for Manufacturing Innovation around the country. The Institutes will bring together industry and academia, research universities and community colleges, federal agencies and state organizations to accelerate innovations by investing in industrially-relevant manufacturing technologies that have broad applications. The President’s Budget for Fiscal Year 2013 proposes a 1 billion, one-time investment to create the new National Network for Manufacturing Innovation (See program information at http://www.manufacturing.gov/amp/nnmi.html).

It is possible to conduct a scientific study of such networks because of the similar regularities among different network structures across applications. It is necessary to

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make such study because of the profound impact that the networks have nowadays.

Regarding the network formation, there are two major approaches: the random graph approach and the more economic focused game theoretic approach. The random graph approach explains how links in a network are built either through a stochastic process where links appear random and satisfy certain distributions, or through specific algorithms, while the game theoretic approach aims at analysing the equilibrium networks where links are formed at the discretion of agents who are in control of nodes. A key feature of the game theory approach is that the agents derive utility functions from the network, and thus by incorporating costs and benefits into the analysis of networks, the question of whether a right network is formed in the sense of maximizing the overall societal welfare could be answered. Another key feature of the game theory approach is that the participating agents can control the nodes, therefore, the ultimate formation of the network could be predicted through (various) notions of equilibrium. This integrated equilibrium/stability analysis enables one to analyse and understand potential conflicts that arise between networks due to the choices of involved parties and networks, which are optimal from a societal perspective. In a nutshell, the "random" approach answers the question of "*how*" while the "game theory" approach answers the "*why*".

In this paper we investigate the network structure of CUG mainly focusing on the game theoretical approach. We wish to find in which situations the pair-wise stability or efficiency of this CUG network model can be achieved. The study of stability and efficiency of social or economic networks via game theory methods has drawn a lot of attentions since the pioneer work of Jackson and Wolinsky [9], in which the tension between stability and efficiency of a network was established. Following this work, the subsequent[5],[6],[7] continue the study on the tension between the member incentives and the overall network efficiency in several contexts and under different definitions of network efficiency.[8] extends the above results from static case to dynamic process, that is, it places networks in a dynamic framework and studies the pairwise stability in this scenario. Our approach in the CUG network model is close to[9],[5],[6],[7]. We adapt the notion of pair-wise stability to the CUG model and discuss when the network will be stable and how to achieve the overall efficiency of the network. Other important literature regarding the formation (stability) and efficiency of network structure including Currarini and Morelli[4], Mutuswami and Winter[11], both solve the tention between individual member incentive and overall network efficiency under certain conditions (compromise among members). Slikker and van den Nouweland[12] more focus on the formation and efficiency of cooperative network while Bala and Goyal[1] on noncooperative network structures.

This paper is organized as follows: In the remaining part of Section 1, we introduce the CUG model and propose our problem of interest. In Section 2, the main results on stability and efficiency of the CUG model is presented; In Section 3, a detailed proof of stability and efficiency of the CUG model using game theoretic approach is given; In Section 4, we slighted modified the assumptions in Section 2 and 3 by incorporate the consideration that research institutions are not unlimited in their research capacities. In this case, some interesting results are obtained regarding maximizing overall network utility, or specifically, the optimal allocation of funding from business companies to universities.

1.1. The Structure of CUG Model

In a CUG model, three types of key agents are involved: manufacturing companies, local research universities, and local government. Each one of these three types of key agents plays a different role in the network formed among them. The main source of funding comes from manufacturing companies. Member companies pool their R&D money together to fund projects solving problems too big for one single company to tackle. Depending on research specialty, equipment availability, etc..., a member university is then selected to complete a project. The result of a project is then shared among member companies. By pooling resources and sharing results, member companies could benefit from reducing the risk of wasting money on a failed project and obtaining more advanced manufacturing technologies. Companies can share access to cutting-edge capabilities and some rare-use but key equipments. Member universities on the other hand, gain experiences from the research on project, which should help further advanced research. Students get new skills and trainings on state-of-the art equipments. Local government also has strong interest in supporting this kind of research centers. By providing tax incentives or start-up fund, the local government is hoping that the state-of-art technology developed in the center could ultimately help the member companies to keep and create jobs in the US.

One interesting part of the CUG research center is that the companies may have different level of competition and the universities may be in different tiers. Will all the agents in the network benefit from participating in the network? Will the difference within the universities or companies stabilize the network or destabilize the network? How about efficiency? The model in this paper will take into consideration the difference.

1.2. Goal of the Project

The main goal of this paper is to study the network of relations within the CUG research center. We want to present some understandings of the stability of the network, when self-interested industries choose to form new cooperation with a new university or to serve existing cooperation. We also check the efficiency of the network and address the balance of the stability and efficiency.

1.3. Methods for Studying a Social (economic) Network

There are basically two different approaches to study a social (economic) network: the random graph approach and the game theory approach[6]. The former largely answers "how" a network is formed, while the latter answers the "why". "Hybrid" models, the so called evolutionary network, (an example,[8]) could be also built. Although the formation of the network is a very interesting and important question to investigate, we here focus on the game theory approach on the existing CUG research centers.

1.4. Preliminary Concepts

Jackson introduces the concept of "pairwise stability", which helps to apply cooperative game theory in the framework of social (economic) network. A social (economic) network is pairwise stable if for any linked pair of agents in the network, neither will have an increased utility if either a link to one of their neighbors is severed or a link to an agent who is not one of their neighbors is added. This is a key point of distinguishing an equilibrium of a strategic network game from a traditional Nash equilibrium. It makes sense because the formation of a link in the network needs the consent of both parties. An interesting observation made by Jackson and Wolinsky[9] is that there is tension between stability and efficiency, which means a (pairwise) stable network might not generate the maximum total utility and a network which generates the maximum total utility might not be (pairwise) stable. This might be an interesting point to investigate for the network model we want to build.

2. Model and Main Results

2.1. Analysis/ideas on how to Construct the Utility Functions

In the model to be built, agents in the network are divided into three groups: companies, universities and government or federal agents.

Assumption 2.1: Links are formed between a company and a school only. No links are formed between different companies or different schools.

This assumption might seem a little bit too simple at first, which means University A could not take part in a project given to University B. However, if two universities collaborate on one project proposed by a company, the project can be ideally divided into two parts and links are only formed between the company and the two universities. Thus, it makes sense not to consider links between schools. Simplicity of this assumption might open the door for a possible improvement of stability.

Suppose there are a total of m companies and n universities. By Assumption 2.1, an m by n 0-1 matrix V could be constructed to save information on connections between a company and a university. If company i cooperates with university j, then the ij - th entry $\boldsymbol{v_{ij}}$ in V is 1. Otherwise, $\boldsymbol{v_{ij}} = \boldsymbol{0}$. Let $\boldsymbol{V^{(i)}}$ denote the i - th row of V which records the cooperations of universities with company i. Let $\boldsymbol{V_{(j)}}$ denote the j - th column of V which records the cooperation of companies with university j. Superscripts and subscripts are associated with companies and universities respectively.

The utility function $\boldsymbol{u^i(g)}$ for company i in network G:

Utility function of a company i is determined by three parts:

1. Funding to all connected schools;
2. Tax incentives received from the government;
3. Return from funded projects;

An m by n matrix $\boldsymbol{C = (c_{ij})_{m\times n}}$ stores the contribution of member companies, where $\boldsymbol{c_{ij}}$ is the contribution of company i to fund projects done by university j. Let $\boldsymbol{C^{(i)}}$ be the i -th row of C. Then,

Cost of company i is $[\boldsymbol{C^{(i)}}]^{\boldsymbol{T}} \cdot \boldsymbol{V^{(i)}}$

Let an m by 1 vector $\boldsymbol{T = (t_i)_{m\times 1}}$ be the tax incentive given by the government. Then,

Tax incentive received by company i is $\boldsymbol{t_i}$

As mentioned, member universities might belong to different tiers. In the model, we will use an n by 1 vector $\boldsymbol{P = (p_j)_{n\times 1}}$ to reflect this fact. $\boldsymbol{p_j}$ is a number between 0 and 1 representing the success rate of projects done by a member school j. $\boldsymbol{p_j}$ may also represent the cooperation preference of the companies to school j. Depending on the research strength of a member school, $\boldsymbol{p_j}$'s should have different values.

To model the return for company i from projects done by school j, we have the following assumption:

Assumption 2.2: The return for company i from successfully completed projects done by school j is modelled by $\boldsymbol{rp_j}$, where r is a constant uniform for all companies and all schools.

We assume that r is uniform for all companies, because the result of a project is shared among all member companies. Then,

Return of company i from funded projects is $\boldsymbol{rP \cdot V^{(i)}}$ ssemble all three parts together, the utility function for company i in network G is given by

$$\boldsymbol{u^i(g) = rP \cdot V^{(i)} + t^i - C^{(i)} \cdot [V^{(i)}]^T}$$

- The utility function $\boldsymbol{u_j(g)}$ for university j in network G:

Utility function for a school j is determined by two parts:

1. Funding received from all connected companies;
2. Costs generated from participating in activities conducted in the center;

For school j, the funding it receives is the sum of all $\boldsymbol{c_{ij}}$'s from all connected companies. The cost for school j could be modeled by an n by 1 vector: $\boldsymbol{D = (d_{ij})_{n\times 1}}$

Let $\boldsymbol{C_{(j)}}$ be the j -th column of C. Then, the utility function $\boldsymbol{u_j(g)}$ for university j in network G is given by

$$\boldsymbol{u_j(g) = [C_{(j)}]^T \cdot V_{(j)} - d_j}$$

- The utility function $\boldsymbol{u_l(g)}$ for government in network G:

Utility function for the government is determined by two parts:

1. Return from funded projects;
2. Tax incentives giving to the member companies;

The return of government from successfully completed projects could be measured by a positive constant *a*, uniform for all schools and all companies. This constant, however, should be different from the constant *r*, since *a* should model the broader impacts to the entire society such as newly created jobs, taxes from the companies who bring profits back to local etc.

Let $\vec{\mathbf{1}}$ be a 1 by *m* vector where any entry of $\vec{\mathbf{1}}$ is a constant 1. Then, the utility function $\boldsymbol{u_l(g)}$ for government in network *G* could be modeled as

$$\boldsymbol{u_l(g) = a\vec{1} \cdot V \cdot P - \vec{1} \cdot T = a\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}p_i - \sum_{i=1}^{m} t_i}$$

which is the difference between the total return received by the government from the funded projects and the total tax incentives the government distributed to the member companies.

• Value function $\boldsymbol{v(g)}$:

The efficiency function which accounts for the total utility of this network is given by

$$\boldsymbol{v(g)} = \sum_{i=1}^{m} \boldsymbol{u^i(g)} + \sum_{j=1}^{n} \boldsymbol{u_j(g)} + \boldsymbol{u_l(g)}$$

$$= \boldsymbol{(r+a)}\sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{v_{ij}p_j} - \sum_{j=1}^{n} \boldsymbol{d_j}$$

$$= \boldsymbol{(r+a)\vec{1} \cdot V \cdot P - \vec{1} \cdot D}$$

which is the sum of the total benefits received by the government and member companies minus the net cost for companies to fund the projects. Note that the tax incentives terms are cancelled out.

2.2. Main Results

Before we give our main results, we provide two definitions on pairwise stability and efficiency of a network. Let $\boldsymbol{u_i(g)}$ denote the utility that agent $\boldsymbol{i}$ receives under the network $\boldsymbol{g}$, inclusive of all costs and benefits.

Definition 2.1. A network $\boldsymbol{g}$ is pairwise stable if

For all $\boldsymbol{ij \in g, u_i(g) \geq u_i(g - ij)}$ and $\boldsymbol{u_j(g) \geq u_j(g - ij)}$, and

For all $\boldsymbol{ij \notin g}$, if $\boldsymbol{u_i(g + ij) > u_i(g)}$, then

$$\boldsymbol{u_j(g + ij) < u_j(g)}.$$

Definition 2.2. A network $\boldsymbol{g}$ is efficient if $\boldsymbol{v(g)}$ is maximized relative to $\boldsymbol{v}$ among all possible networks that could be formed.

We want to investigate the following problems for the model mentioned in the previous section:

• Which networks are pairwise stable?

• Does a pairwise stable network also have maximum efficiency?

To obtain useful observations to these two problems, in addition to Assumption 2.1-2.2, we assume:

Assumption 2.3. Let $\boldsymbol{n_j(g)}$ denote the number of companies that school j connects to and let $\boldsymbol{n^i(g)}$ denote the number of schools that company i connects to.

- $\boldsymbol{c_{ij}}$ is a constant c>0, for all $\boldsymbol{1 \leq i \leq m, 1 \leq j \leq n}$.
- $\boldsymbol{d_j = d \times n_j(g)}$ for all $\boldsymbol{1 \leq j \leq n}$.
- $\boldsymbol{t^i = t \times n^i(g)}$ where $\boldsymbol{t} > 0$ is a constant.
- c>t+d.

The last assumption in Assumption 2.3 is very natural, which asserts that the contribution a member company makes in maintaining the relation with a member school should outweigh the tax incentives it receives from the government and the cost for a member school to maintain a relation with that company combined. This avoids any arbitrage opportunity in the network.

Without loss of generality, the school can be rearranged as its research strength. So school 1 is the strongest, while school n is the weakest in terms of research ability.

Assumption 2.4. $\boldsymbol{p_1 > p_2 > \cdots > p_n}$.

In our model, the contribution made by member companies (measured by c), the tax incentives (measured by t), the success rate P (measured by $\boldsymbol{p_j}$) and the cost of member schools (measured by d) could all be predetermined. The major factors uncontrollable in the model are the return for member companies (measured by r) and for the government (measured by a) from funded projects.

With regard to the pairwise stability problem, we will show below that a member company has the ultimate power to determine if it wants to connect to a member school. Therefore, the pairwise stability result depends mainly on the strength of r. We will show that if r is large, a complete network with each member company connecting to all member schools is pairwise stable. As r becomes smaller than certain thresholds determined by c, t and $\boldsymbol{p_j}$, however, in order to form a pairwise stable network, the weakest schools will be excluded from the network. A member company maximizes its utility in a pairwise stable network by maintaining connections with the strongest schools.

A subsequent problem to investigate is if total utility of the entire network (efficiency) is maximized for a pairwise stable network. We discover that if the smallest effective return, $\boldsymbol{p_n a}$ for government from the weakest member school exceeds the cost d, then a complete network always obtain maximum efficiency. Therefore, only if r is large enough to support a complete pairwise stable network, an incomplete pairwise stable network does not have the maximum efficiency.

A natural question to ask next is how to lower the threshold for r to obtain a complete pairwise stable network which also has a maximum efficiency. Our result shows that a possible way is through government regulation. The government could encourage the member company to keep a connection with weaker schools by increasing tax incentives it offers.

We need the following preliminary definitions to state our main results:

Definition 2.3. We will use $\boldsymbol{G_n}$ to denote the network where the relation matrix V has entries specified as

$$v_{ij} = 1 \text{ for } 1 \le i \le m, 1 \le j \le n$$

Definition 2.4. We will use $\boldsymbol{G_h}$ $(1 \le h \le n-1)$ to denote the network where the relation matrix V have entries specified as

$$v_{ij} = 1 \text{ for } 1 \le i \le m, 1 \le j \le h$$

$$v_{ij} = 0 \text{ for } 1 \le i \le m,\ h+1 \le j \le n$$

Definition 2.5. We will use $\boldsymbol{G_0}$ to denote the network where the relation matrix V have entries specified as

$$v_{ij} = 0 \text{ for } 1 \le i \le m,\ 1 \le j \le n$$

Thus we have the following main results based on the game theoretical approach:

Theorem 2.4. (Pairwise stability)

1. If $r > (c-t)/p_n$, then network G_n is pairwise stable.

2. If $\frac{c-t}{p_{h-1}} < r < \frac{c-t}{p_h}$ for $2 \le h \le n$, then network G_{h-1} is pairwise stable.

3. If $r < (c-t)/p_1$, then network G_0 is pairwise stable.

Theorem 2.5. (Efficiency)

1. If G_h $(0 \le h \le n)$ is pairwise stable, then severing a link from G_h always results in a smaller efficiency of the entire network.

2. If $a > d/p_n$, then the complete network G_n has the maximum efficiency among all networks that could be possibly formed.

Remark 1. A direct implication of this result is that a pairwise stable network does not have maximum efficiency unless it is a complete one. This is in line with the phenomenon of "tension between stability and efficiency" often observed on a social network. In Section 4, we gives a detailed analysis of such “tension” phenomenon under a further assumption that the capacity for a university’s research resource is not unlimited. This is a natural assumption. It implies a university cannot accept unlimited projects from member companies in the network because this will drastically reduce the university’s performance on each project.

There are essentially two ways to reduce the lower bound in Theorem 2.5: decreasing d or increasing p_n. The first criterion requires that the cost for member schools to participate is not too big, while the second criterion demands a higher success rate from all member schools. If either of these two criterion is applicable, then it is more likely that the maximum efficiency is obtained when a complete network is formed.

3. Proof of Main Results

3.1. Proof of Theorem 2.4

First of all, we claim that the member companies have to carefully determine whether a link shall be added or served in the network in order to maximize their utility functions but the member universities always want to serve more links with companies.

Under Assumption 2.3, the utility function of school j is $u_j(g) = n_j(g) \cdot (c-d)$. Since $c-d > 0$, (this is because c>t+d>d by last bullet of Assumption 2.3), a school always want to build relations to increasing its utility with as many companies as possible. On the other hand, the utility of a company does not always increase if new relations to a school are added. A company will benefit from connecting to more member schools only if the return r is large enough. This could be seen by analyzing the utility function of a company:

1. If company i has a relation with each school, then

$$u^i(g_n) = \left(\sum_{j=1}^{n} p_j\right) \cdot r + n(t-c)$$

2. Suppose school k is not connected with company i. But all the other schools are connected with company i. Then,

$$u^i(g_{n-1}) = \left(\sum_{j=1, j\neq k}^{n} p_j\right) \cdot r + (n-1)(t-c)$$

There could be a total of n different utilities for a company if it is connected with n-1 schools. In this circumstance, the maximum utility for a company is obtained if the unconnected school is the weakest school, school n (recalling that $p_1 > p_2 > \cdots > p_n$):

$$u^i_{max}(g_{n-1}) = \left(\sum_{j=1}^{n-1} p_j\right) \cdot r + (n-1)(t-c)$$

3. Similarly, let $u^i_{max}(g_{n-k})$ be the maximum utility for a company if it is connected to n-k schools. Then, in general,

$$u^i_{max}(g_{n-k}) = \left(\sum_{j=1}^{n-k} p_j\right) \cdot r + (n-1)(t-c)$$

for $0 \le k \le n$.

Let's prove the first part of the main theorem. If $r > (c-t)/p_n$, then $p_n \cdot r + t - c > 0$, which leads to $p_h \cdot r + t - c > 0$ for all $0 \le h \le n$, since $p_n < p_{n-1} < \cdots p_1$. But

$$p_h \cdot r + t - c = u^i_{max}(g_h) - u^i_{max}(g_{h-1})$$

for all $1 \le h \le n$, therefore,

$$u^i_{max}(g_n) > u^i_{max}(g_{n-1}) > \cdots u^i_{max}(g_0)$$

This implies that as long as $r > (c-t)/p_n$, the overall maximum utility of a company is obtained when it is connected to all schools. So the complete network in this case should be pairwise stable.

In general, if $(c-t)/p_{h-1} < r < (c-t)/p_h$, $2 \le h \le n$, then

$$p_j \cdot r + t - c > 0 \text{ for } 1 \le j \le h-1$$

while

$$p_j \cdot r + t - c < 0 \text{ for } h \le j \le n$$

Therefore,

$$u^i_{max}(g_{h-1}) > u^i_{max}(g_h) > \cdots > u^i_{max}(g_n)$$

and

$$u^i_{max}(g_{h-1}) > u^i_{max}(g_{h-2}) > \cdots > u^i_{max}(g_0)$$

Thus, the overall maximum utility of a company is obtained when it is connected to the strongest h-1 schools but disconnected with the rest of the schools. Therefore, if $(c-t)/p_{h-1} <$ r $< (c-t)/p_h$, $2 \le h \le n$, a network with the weakest n-h+1 schools excluded is pairwise stable.

3.2. Proof of Theorem 2.5

In this section, we study the efficiency (total utility) of the network. The efficiency function v(g) for a network G could be calculated as

$$v(g) = (r+a)\vec{1} \cdot V \cdot P - \vec{1} \cdot D$$

Under Assumption 2.3, the efficiency function for the pairwise stable networks G_h $(0 \le h \le n)$ is given by

$$v(g_h) = m\left(\sum_{j=1}^{h} p_j\right) \cdot (r+a) - (mh)d$$

- Our first observation is that severing any link in a pairwise stable network G_h ($0 \le h \le n$) will result in smaller efficiency.

If a link between company i (for any $1 \le i \le m$) and school j (for $1 \le j \le h$) (note schools h+1, ..., n are not connected to any company in the pairwise stable network G_h) is severed, then efficiency of the network is reduced by an amount of $p_j(r+a) - d$:

$$v(g_h) - v(g_h - ij) = p_j(r+a) - d$$

for $1 \le i \le m$, $1 \le j \le h$.

On the other hand, by Theorem 2.4, G_h is pairwise stable if $(c-t)/p_{h-1} <$ r $< (c-t)/p_h$, $2 \le h \le n$, which yields

$$p_h \cdot r > c - t > d$$

Thus, $p_h \cdot r - d > 0$. But p_h is the smallest for all p_j's where $1 \le j \le h$, hence

$$p_j \cdot r - d > 0$$

This in turn implies that when G_h is pairwise stable, regardless of the value the return constant for the government a (which we assume is positive) might take,

$$v(g_h) - v(g_h - ij) > 0$$

for $1 \le i \le m$, $1 \le j \le h$.

Therefore, severing a link from a pairwise stable network G_h will reduce the efficiency.

- The interesting question to ask now is to find a network which has the maximum efficiency.

To answer this question, we have to examine

$$v(g + ij) - v(g) = p_j(r+a) - d$$

where ij is not a link in network G.

Obviously, if a is large enough, $a > d/p_n$, then for all $1 \le j \le n$, $p_j a - d > p_n a - d > 0$, thus,

$$v(g + ij) - v(g) > 0$$

for $1 \le i \le m$, $h+1 \le j \le n$.

Hence, adding a link to any network G will increase the efficiency if a has a lower bound d/p_n.

4. Optimization of Companies' Utility Function-Another Perspective

Since companies are leading to form the network in this CUG model, a more detailed study of the company utility function is necessary to establish a more in-depth investigation of a member company's incentive and behavior when allocating their resources to the member schools in the research center network.

Based on the analysis in Section 2 and taking consideration of the restrictions of universities' research capacity, an a slightly modified model is constructed to study the resource allocation of the companies to universities. In order to maximize the utility of a company, it is not always true that companies only choose to collaborate with the best university.

4.1. Analysis and Setting

Assumption 4.1: Links are formed between companies and schools only. No links are formed between different companies or different schools.

This assumption is identical with Assumption 2.1.

Assumption 4.2: The return for company i from successfully completed projects done by school j is modeled by $rp_j c_{ij}$, where r is a constant uniform for all companies and all schools.

Here in this section, the definition of return is slightly different from Section 2. The "return" in Assumption 4.2 includes both the profit from the successful projects by universities and the revenue from government's tax incentive policy for funding the non-profit institutions, as we assume the government return is proportional to the company's investment to universities. We make such modification because our focus in this section is on what happens if research projects from member companies are overflowing the research capacity of a university.

Assumption 4.3: Different universities belong to different tiers. We use a concept "research capacity" to describe the fact. One measurement of the research capacity given by Section 2 is the "success rate", we keep using it here. Furthermore, we assume that the research capacity of each university is not unlimited, and represent the research capacity of university j by R_j. This R_j is described by the standard upper funding limit that university j can accept. This premise is reasonable. Though universities want to receive as much funding as possible, their amount of workforce (the faculty) are limited, which means the amount of projects they can work on at one time are limited. We use R_j, the standard upper funding limit to describe the standard amount of projects university j can hold at one time while maintaining its maximum research "success rate" p_{j0}. And without loss of generality, we assume

$$p_{10} > p_{20} > \cdots > p_{n0} \quad (1)$$

On the other hand, we assume companies are rational enough in evaluating the budget of the proposed project, which only depends on the size and the difficulty of the project. The total amount of funds that university j receives is F_j. Now we define the following function:

$$s_j = \begin{cases} 0, & \text{if } F_j \le R_j \\ F_j - R_j, & \text{if } F_j > R_j \end{cases} \quad (2)$$

s_j describes the excessive research loads university j has received. $s_j > 0$ is quite natural as we assume universities want to receive as much funding as possible. They will still try to get funds if the projects are "a little bit overflow". The sacrifice will be lowering the success rate p_{j0} to p_j (The university will tries to avoid this situation as it will harm their research reputation). p_j is the success rate when university j has funds F_j and $0 < p_j < p_{j0}$.

We assume the decrease of p_j is proportional to the size of s_j. Thus

$$\frac{\partial p_j}{\partial s_j} = -\tau_j s_j \quad (3)$$

where τ_j is positive constant and is another indication of the research capacity of university j. A smaller τ_j indicates a better stability of university j when encountering overflowing research loads. Without loss of generality, we assume

$$\tau_1 < \tau_2 < \cdots < \tau_n \quad (4)$$

From (3), we get the following representation for p_j:

$$p_j = p_{j0} - \frac{\tau}{2} s_j^2 = p_{j0} - \frac{\tau}{2}\left(F_j - R_j\right)^2 \quad (5)$$

4.2. Modified Company Utility Function

The return of project c_{ij} is $rc_{ij}p_j$, thus the utility function for company i is the sum of returns of all projects with universities.

$$u_i = \sum_{j=1}^{n}\left\{-c_{ij} + rc_{ij}\left[p_{j0} - \frac{\tau}{2} s_j^2\right]\right\} \quad (6)$$

The above utility function is subjected to the following constraints:

- Budget constraint:

$$\sum_{j=1}^{n} c_{ij} \le C_i$$

- Success rate: $0 < p_j < p_{j0}$ this means that the university will be rational enough that they will stop to accept any projects if s_j is big enough.

Assumption 4.4: $rp_{j0} > 1$

Remark 2. This assumption is natural, since companies expect positive return, which means that their investment brings them profit, not deficit.

Without loss of generality, we also assume that companies want to maximize their utilities by cooperating with universities that have better research reputations with better success rate. They will always consider providing funds to universities in the following priority order $(1,2,\cdots,n)$. If a better university can not accept their projects (we simply assume that the only reason is that the university has enough projects for its research resource), they will consider the next until one university is available.

4.3. Optimization of the Utility Function

1. Case 1: $s_1 = 0$. This condition means that the top university still has enough research resource and is open to companies' project. This implies

$$s_1 = s_2 = \cdots = s_n \quad (7)$$

by Remark 2. Then the representation for u_i is simplified as

$$u_i = \sum_{j=1}^{n}\left(-c_{ij} + rc_{ij}p_{j0}\right) \quad (8)$$

This is very close to the utility function provided in Section 2. And the analysis of the companies' incentive and behavior of distributing their budget strictly follows the version in Section 2 and Section 3. The game theory results apply to this case. That is companies will always consider cooperating with the best school in the network as it will give them maximum investment returns.

2. Case 2: $s_1 > 0$. This condition means that at least one of the university is "standardly full", though it is possible that it still available and open to new projects. In this scenario, we are interested in the following optimization problem: How should companies distribute their budgets among universities in the network of the research center to get the maximum return. Converting this problem into mathematical language, it is to find the vector $(c_{i1}, c_{i2}, \cdots c_{im})$ that maximizes the utility function u_i. Our method for this problem is based on the maximum theorem of calculus, i.e. the derivatives of the utility function u_i should be zero.

Now we suppose that university 1 to l $(1 \le l \le n)$ is standardly full in its research capacity but still open to new projects. The excess of research capacity is $(s_1, \cdots s_l, 0, 0, \cdots 0)$. When company i make a cooperation with universities in the network by allocation its resource $(c_{i1}, c_{i2}, \cdots c_{in})$, the utility function becomes

$$u_i = \sum_{j=1}^{l}\left\{-c_{ij} + rc_{ij}\left[p_{j0} - \frac{\tau}{2}(s_j + c_{ij})^2\right]\right\} + \sum_{j=l+1}^{n}\left\{-c_{ij} + rc_{ij}p_{j0}\right\} \quad (9)$$

First derivative: If $1 \le k \le l$,

$$\frac{\partial u_i}{\partial c_{ik}} = \frac{\partial u_{ik}}{\partial c_{ik}} + \sum_{j \ne k}\frac{\partial u_{ij}}{\partial c_{ik}}$$

$$= -1 + r\left[p_{k0} - \frac{\tau_k}{2}(s_k + c_{ik})^2\right] - \tau_k r c_{ik}(s_k + c_{ik})$$

$$= -1 - \frac{\tau_k r}{2}(s_k + c_{ik})^2 + r\left[p_{k0} - \tau_k c_{ik}(s_k + c_{ik})\right] \quad (10)$$

If $l + 1 \le k \le n$,

$$\frac{\partial u_i}{\partial c_{ik}} = \frac{\partial u_{ik}}{\partial c_{ik}} = -1 + rp_0 \quad (11)$$

Second derivative: If $1 \le k \le l$,

$$\frac{\partial^2 u_i}{\partial^2 c_{ik}} = -r\tau_k c_{ik} - 2r\tau_k(s_k + c_{ik}) < 0 \quad (12)$$

The above inequality is because we assume r, τ_k, c_{ik} positive and s_k nonnegative for all k.

If $l + 1 \le k \le n$,

$$\frac{\partial^2 u_i}{\partial^2 c_{ik}} = 0 \quad (13)$$

The second derivative of u_i shows that u_i is concave down. Thus, a maximum can be obtained and the solution of this optimization problem is the solution of the following system of equations:

$$\begin{cases} \frac{\partial u_i}{\partial c_{i1}} = 0 \\ \cdots \\ \cdots \\ \frac{\partial u_i}{\partial c_{in}} = 0 \end{cases} \quad (14)$$

which is equivalent to the system of equations:

$$\begin{cases} -1 - \frac{\tau_1 r}{2}(s_1 + c_{i1})^2 + r[p_{10} - \tau_1 c_{i1}(s_1 + c_{i1})] = 0 \\ \cdots\cdots\cdots \\ -1 - \frac{\tau_l r}{2}(s_l + c_{il})^2 + r[p_{l0} - \tau_l c_{il}(s_l + c_{il})] = 0 \end{cases} \quad (15)$$

In the above equation system, we ignore the part

$$(\frac{\partial u_i}{\partial c_{i,l+1}}, \cdots \frac{\partial u_i}{\partial c_{i,n}})$$

This is because for $l + 1 \leq k \leq n$, equation (11) together with Assumption 4.4 indicate that

$$\frac{\partial u_i}{\partial c_{ik}} = -1 + rp_0 > 0 \text{ for } l + 1 \leq k \leq n \quad (16)$$

This implied the equation

$$\frac{\partial u_i}{\partial c_{ik}} = 0 \text{ for } l + 1 \leq k \leq n \quad (17)$$

will not have a solution.

However, (16) shows that the return u_i will increase as long as the funding for university k $(l + 1 \leq k \leq n)$, c_{ik}, increases. Therefore, the company's strategy to receive the maximum return is to fund the first *l* universities according to the solution of equation system (15). If their budget is more than $\sum_{k=1}^{l} c_{ik}$, then the rest should all go to $c_{i,l+1}$, since university *l+1* has the best success rate among the next *n-l* universities.

Now we need to prove the existence of the solution of the system of equations (15). Therefore we can show that the maximum does exist and it can be obtained. Notice that each equation in (15) is a second order polynomial and the equations are independent to each other. So the existence is guaranteed by the quadratic formula.

Simplifying *k*-th equation in (15), we get that

$$\frac{3r\tau_k}{2} c_{ik}^2 + 2r\tau_k c_{ik} - rp_{k0} + 1 + \frac{r\tau_k}{s_k^2} = 0 \quad (18)$$

The discriminant $\Delta = b^2 - 4ac$ of the quadratic polynomial is

$$\Delta = (2r\tau_k s_k)^2 - 4 \cdot \frac{3r\tau_k}{2}(-rp_{k0} + 1 + \frac{r\tau_k}{2} s_k^2)$$
$$= r^2\tau_k^2 s_k^2 + 6r\tau_k(rp_{k0} - 1) > 0 \quad (19)$$

and the axis of symmetry is

$$-\frac{b}{2a} = -\frac{2r\tau_k s_k}{3r\tau_k} = -\frac{2s_k}{3} < 0 \quad (20)$$

(19) and (20) show that (18) has two roots and at least one root is negative. If the other root of equation (18) is positive, it guarantees the existence of solutions for the system of equations (15). This requires the following inequality.

$$-rp_{k0} + 1 + \frac{r\tau_k}{2} s_k^2 < 0 \quad (21)$$

Keeping in mind that s_k is positive and solving the above inequality, we get that

$$s_k < \sqrt{\frac{2rp_{k0} - 2}{r\tau_k}} \quad (22)$$

This is a sufficient condition to guarantee that (15) has a solution under Assumption 4.1-4.4

$$(c_{i1}, \cdots, c_{il}, c_{i,l+1}, 0 \cdots, 0)$$

which provides an optimum plan of resource allocation for company *i* to maximize its utility.

Since equation (18) has only two roots and one of them is negative, the postive root c_{ik} is also the unique solution to equation (18). This implies (15) has a uique solution under the condition (22).

So we have proved the following theorem:

Theorem 4.1: Suppose

1. Company *i* have some budgets to fund some projects with universities.
2. University 1 to *l* have excessive research load while university *l+1* to *n* have less research load than their standard research capacity. $(1 \leq l \leq n)$.
3. Companies allocate their funds among the universities in the network in order to maximize their utilities.

Then company *i* will find a unique maximum return u_i if the excessive research load s_k, $1 \leq k \leq l$ satisfies

$$s_k^2 < \frac{2rp_{k0} - 2}{r\tau_k}, \ 1 \leq k \leq l \quad (23)$$

Remark 3: From (23), we can get the following inequality:

$$\frac{1}{2}\tau_k s_k^2 < p_{k0} - \frac{1}{r} \quad (24)$$

Compare the left hand side of the above inequality and equation (5), we get

$$p_{k0} - p_k < p_{k0} - \frac{1}{r} \quad (25)$$

which yields,

$$rp_k > 1 \quad (26)$$

This result indicates a necessary and sufficient condition to guarantee that the utility function for company *i* will have a maximum solution when, taking the universities' research capacity into consideration, all university can still give a positive return if they are funded by company *i* for some project.

Furthermore, we have the following theorem regarding companies' choosing the right project partner:

Theorem 4.2: Under the same assumptions in Theorem 4.1 and the following condition

$$c_{ik} s_k \geq \frac{p_{k0}}{\tau_k} \text{ for some k, } 1 \leq k \leq l \quad (27)$$

companies should stop funding new projects with university k but consider the next university: university k+1.

Remark 4: Theorem 4.2 gives a necessary condition of giving up cooperating with university *k* for projects. It shows that when companies consider cooperating with universities, they should not only consider the reputations, but also their research capacity. Nevertheless, the research capacity of a university is closely related to its reputation. But once a university has enough projects, the company should take this into their considerations; qualitatively check whether their investment gives them positive revenue return. (27) can be converted to

$$c_{ik} \geq \frac{p_{k0}}{\tau_k s_k} \tag{28}$$

For the right hand side of (28), the numerator can be interpreted as the maximum change in the sucess rate of university k, the denominator is the "changing rate" of success rate of university k. Thus, (28) means company i can make a quick estimate of the ratio of maximum sucess rate change and change rate for university k, and if they make to have a profit, their project investment should not exceed the value of this ratio.

Proof: From (10) we see that if

$$r(p_{k0} - \tau_k c_{ik}) \leq 0 \tag{29}$$

(10) will be negative permanently, which means u_i will decrease as long as c_{ik} increases. Practically, the company's revenue will decrease if they continue to give university k money, and they definitely want to avoid this situation.

On the other hand, (29) is equivalent to

$$p_{k0} \leq \tau_k c_{ik} s_k \tag{30}$$

From here we get the condition (27).

5. Summary and Open Problems

In this paper, we studied a cooperative model representing the relation network in a consortium among commercial companies, universities and local government. By game theoretical approach, we have shown that under the simplified assumptions that links could be formed only between a company and a university and that returns from a completed project to a member company and local government are depending on the university research strength, a pairwise stable network could be formed between the strongest universities (strongest in terms of the success rate of completing a project) and member companies. The higher the return to a company, the more complete the network is. Unfortunately, the total utility of the entire network might not be maximized over such pairwise stable network unless it is a complete one, which, in return, demands high return.

A further investigation on the impact of an institution's capacity of research resources on the link formations between member companies and member universities is presented in Section 4. This supplements the results of game theoretical approach in Section 2 and 3 by showing that both research strength and research capacity are important factors for a university's performance on a project. Taking the university's research capacity into account, a network consisting only the links from member companies to the few top research universities on one hand could be pairwise stable, but on the other hand, might not be strategically wise in getting the overall optimal return among member companies. This result reinforces the aforementioned claim in Section 2 and Section 3 that a stable network composed of only the links from member companies to the strongest member universities could not yield the maximized overall utility of the network, or simply, there is tension between stability and efficiency in this CUG network. One solution to such an issue might be strengthening the role of government agency in coordinating the cooperation between member companies and member universities to achieve to better return yields for companies. Thus they have more incentives to form a complete network.

Some open questions that are worth to be investigated include: can a complete pairwise stable network be obtained without high return if cooperation among universities is permitted in the network, or/and if the return to the company is a function of a university's research capabilities? More advanced tools need to be employed if the assumptions are loosened.

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