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# Investment Decisions in a New Labor-Managed Industry

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**Abstract** This paper considers a continuous-time dynamic model of the strategic capacity investment of two labor-managed firms in a new industry and constructs a set of perfect equilibria of the model. The paper then finds that there are perfect equilibria where neither firm invests to its steady-state reaction curve.

**Keywords** Continuous-time dynamic model; Labor-managed firms; Strategic capacity investment

## 1. Introduction

The study of strategic interactions between firms with investments as the control variables has been examined by many economists.[1] Wenders[36], Spence[27], and Dixit[5] study the possibility of firms using excess capacity to strategic investments. This idea is extended to a two-stage model by Dixit[6], to a three-stage model by Ware[35], and a continuous-time dynamic model by Spence[28]. Spence examines strategic investment decisions for private firms in a new industry or market by using a continuous-time asymmetric dynamic model, namely, where leading and following firms exist. He shows that the equilibrium is for the leading firm to invest as quickly as possible to some capital level and then stops. His result is much like the equilibrium in a static Stackelberg game. Fudenberg and Tirole [8] establish the existence of a set of perfect equilibria by using Spence's dynamic model and suggest that the steady state of the game is usually on neither firm's steady-state reaction curve; that is, there are early-stopping equilibria where neither firm invests up to its steady-state reaction curve. Ohnishi[22] discusses the perfect equilibrium outcomes of a continuous-time mixed model of the strategic investment decisions of public and private firms in a new industry or market and shows that the equilibrium outcomes of the mixed market model differ from those of Fudenberg and Tirole's pure market model; that is, there are no early-stopping equilibria where neither firm invests to its steady-state reaction curve. In addition, Ohnishi[23] discusses the perfect equilibria of a continuous-time model of the strategic investment decisions of labor-managed income-per-worker-maximizing and profit-maximizing firms and shows that there exists a particular equilibrium in which neither firm invests to its steady-state reaction curve.

We examine a continuous-time dynamic game of the strategic investment decisions of labor-managed income-per-worker-maximizing firms in a new industry or market. The pioneering work on a theoretical model of a labor-managed firm was conducted by Ward[34]. Since then, many economists have studied the behaviors of labor- managed firms.[2] For example, Laffont and Moreaux[17] examine the welfare properties of free entry Cournot equilibria in labor-managed economies and show that Cournot equilibria are efficient provided that the market is sufficiently large.[3] Zhang[37] applies a Dixit[6]-Bulow et al.[2] framework of entry deterrence to a labor-managed industry and show that a labor-managed incumbent firm has a greater incentive to hold excess capacity to deter entry than a corresponding profit-maximizing incumbent firm. Okuguchi [25] examines two models of duopoly with product differentiation and with only labor-managed firms, in one of which two firms' strategies are outputs (labor-managed Cournot duopoly) and prices become strategic variables in the other (labor-managed Bertrand duopoly). He shows that reaction functions are upward-sloping under general conditions in both labor-managed Bertrand and Cournot duopolies with product differentiation.[4] Lambertini and Rossini[19] analyze the behavior of labor-managed firms in a two-stage Cournot duopoly model with capital strategic interaction, and show that labor-managed firms choose their capital commitments according to the level of interest rate, unlike what usually happens when only profit-maximizing firms operate in the market. Lambertini[18] examines a spatial differentiation duopoly model and shows that if both firms are labor- managed, there exists a (symmetric) subgame perfect equilibrium in pure strategies with firms located at the first and third quartiles, if and only if the setup cost is low

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[1] See Tirole [32], Gilbert [12], and Fudenberg and Tirole [10] for excellent surveys on strategic capacity investment.

[2] See Ireland and Law [15], Stephan [30], or Bonin and Putterman [1] for excellent surveys of theoretical work on labor-managed firms.

[3] For free entry models, see also Hill and Waterson [14] and Kamshad [16].

[4] See also Delbono and Rossini [4], Futagami and Okamura [11], and Lambertini and Rossini [19].

enough. Cellini and Lambertini[3] take a differential game approach to study the dynamic behavior of labor-managed firms, in the presence of price stickiness. They show that, provided the membership of labor-managed firms is given, the steady state equilibrium allocation reached by an oligopoly populated by labor-managed firms is the same as in an oligopoly populated by profit-maximizing firms, and the result holds under both the open-loop information structure and the memoryless closed-loop information structure. Drago and Turnbull[7] provide a model of work effort and wage incentives in the worker-owned or labor-managed firm and show that if employee-owners can establish binding effort matching agreements, purely collective incentives are optimal. There are many further excellent studies.

The purpose of this paper is to characterize the perfect equilibrium of a continuous-time dynamic model where labor-managed firms compete for capital investment in a new industry.

The paper is organized as follows. In Section 2, we present the elements of the continuous-time model with labor- managed firms. Section 3 shows the equilibrium of the model, and the conclusion appears in Section 4.

# 2. The Model

Let us consider a market with two labor-managed firms, firm 1 and firm 2. For the remainder of this paper, when *i* and *j* are used to refer to firms in an expression, they should be understood to refer to 1 and 2 with $i \neq j$. Time *t* is continuous, and the horizon is infinite.

Firm *i*'s income per worker at time *t* is given by

$$\omega_i(k_i, k_j, a_i) = \frac{p(K)k_i - m_i k_i - a_i}{l_i(k_i)} \tag{1}$$

where $k_i$ denotes firm *i*'s current capital stock, $p(K)$ price as a function of capital stock $(K = k_1 + k_2)$, $m_i$ firm *i*'s maintenance cost per unit of capital, $a_i$ firm *i*'s rate of investment in its own capital, and $l_i(k_i)$ the quantity of labor utilized. We assume that $p' < 0$ and $p'' < 0$. In addition, we assume that $l_i' > 0$ and $l_i'' > 0$.

The constant cost of one unit of investment is one, capital stocks cannot decrease, and each firm has a constant upper bound on the amount of its capital investment at every time *t*. Hence, $dk_i / dt = a_i \in [0, \overline{a}_i]$. At time zero, each firm enters the market with $k_i(0) \geq 0$ and can start investing.

Firm *i* maximizes the net present value of income per worker, given by

$$\Omega_i = \int_0^\infty \omega_i(k_i(t), k_j(t), a_i(t)) e^{-\delta t} dt \tag{2}$$

where $\delta \geq 0$ is the common rate of interest.

Firm *i*'s steady-state reaction function $R_i(k_j)$ is defined as the locus of points that give the final optimal level of capital $k_i$ for each value of the final level of capital $k_j$. Under our assumptions, the steady-state reaction functions are upward sloping.[5] We examine the perfect equilibrium of this game. The perfect equilibrium is a strategy combination that induces a Nash equilibrium for the subgame starting from every possible initial state in the state space.

# 3. Equilibrium

In this section, we demonstrate the equilibrium of the continuous-time model described in the previous section. We discuss each firm's actual investment by using Figure 1. Here, $R_i$ is firm *i*'s steady-state reaction curve, and $I_i$ is those points at which firm *i*'s iso-income-per-worker curve is tangent to the industry growth path (the path along which both firms are investing as rapidly as they can). Each firm can start investing at time zero. Each firm will invest as quickly as possible, given the constraints. The industry continues to grow along the industry growth path, as long as neither firm has stopped investing. Each firm will stop inventing at a point that it finds optimal.

The main result of this study is described by the following proposition, which states that there are early-stopping equilibria in the continuous-time labor-managed market model.

**Proposition 1.** The intersections of $I_1$ and $R_2$, $I_1$ and $I_2$, and $R_1$ and $I_2$ are denoted by $A$, $B$, and $C$, respectively. $E$ is the lines formed by the kinked line $ABC$ and the reaction curves ($R_1$ to the northeast of $C$ and $R_2$ to the northeast of $A$). $E$ is depicted in Figure 1. One can construct perfect equilibrium strategies such that the equilibrium path stops on $E$.

Proof. We begin by dividing the state space into six regions as depicted in Figure 1: Region I is the set below $R_2$ to the right of $R_1$; Region II is the set above $R_2$ to the left of $R_1$; Region III is the shaded part (including the kinked line $NABCN$); Region IV is the set not above $R_2$ to the left of $R_1$ (including $R_1$ to the southwest of $C$ and excluding Region III); Region V is the intersection of the set not above $R_1$ and the set not below $R_2$; and Region VI is the rest of the figure.

First, we consider starting at an arbitrary point in Region I. Since $\omega_1(k_1, k_2, a_1)$ is assumed to be concave in $k_1$, firm 1 wishes to be as close to $R_1$ as possible. Firm 1's income per worker will decrease if firm 1 invests whether firm 2 invests or not. Capital stocks cannot decreases. Hence, firm 1 never invests in this region. Since $\omega_2(k_1, k_2, a_2)$ is assumed to be concave in $k_2$, firm 2 wishes to be as close to $R_2$ as possible. Given $k_1$, an increase in $k_2$ increases firm 2's income per worker. Therefore, the best firm 2 can do is to invest. Hence, since firm 2 unilaterally continues to invest, the state will reach from Region I to either Region III, Region IV, Region V, or Region VI. Since the arguments for both firms in Region II are exactly symmetric, they are omitted.

Second, we show the strategies in Region IV. Since $\omega_i(k_i, k_j, a_i)$ is assumed to be concave in $k_i$, firm *i* wishes to be as close to $R_i$ as possible. Given $k_j$, an increase in $k_i$ increases firm *i*'s income per worker. Given $k_i$, an increase in $k_j$ decreases firm *i*'s income per worker. Firm *i*'s proposed strategy in Region IV is to invest as quickly as

[5] For the reaction functions of labor-manage firms, see Stewart [31], Delbono and Rossini [4], Lambertini and Rossini [19], and Ohnishi [23, 24].

possible, so it can deviate only by investing less quickly. However, given firm *j*'s strategy, such deviation can never lead to a steady state preferred by firm *i*. Firm *i* cannot gain by deviating. Hence, given firm *j*'s strategy, the best firm *i* can do is to invest as quickly as possible. Since firm *i* continues to invest, the state will reach from Region IV to Region III.

Third, we show the strategies in Region III. Given $k_j$, an increase in $k_i$ decreases firm *j*'s income per worker. Furthermore, if each firm continues to invest, then its income per worker decreases. Firm *i*'s income per worker decreases more when firm *j* unilaterally invests than when both firms invest. Therefore, the best firm *i* can do is to invest if firm *j* invests. If firm *j* invests, since firm *j*'s income per worker decreases, firm *j* does not invest. Hence, neither firm has an incentive to invest. Since the arguments for both firms in Region VI are exactly symmetric, they are omitted.

Fourth, we show the strategies in Region V. Each firm wishes to be as close to its own reaction curve as possible. Given $k_j$, an increase in $k_i$ decreases each firm's income per worker. Firm *i*'s income per worker will decrease if firm *i* invests whether firm *j* invests or not. Hence, neither firm has an incentive to invest.

Consequently, each firm's optimization problem at any point of these regions, given the other firm's strategy, induces a Nash strategy at any point of these regions. Thus, the strategies are in perfect equilibrium, and the proposition follows. Q.E.D.

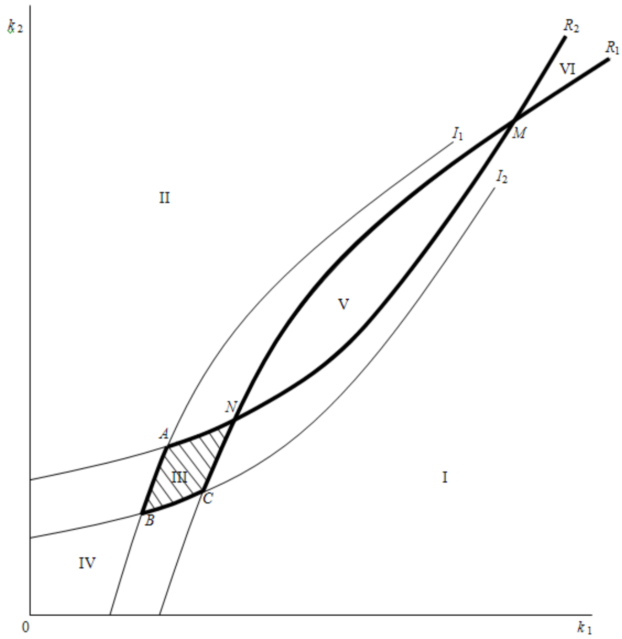


**Figure 1**. $E$ is the bold line

# 4. Conclusions

We have constructed a set of perfect equilibria of a continuous-time dynamic game where two labor-managed income-per-worker-maximizing firms compete for capital investment in a new industry or market. Fudenberg and Tirole[8] suggest that if two profit-maximizing firms compete in a continuous-time dynamic game, there are early-stopping equilibria where each firm does not invest to its steady-state reaction curve. We have found that there are early-stopping equilibria in the continuous-time labor- managed market model.

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