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The Problem of Meeting of N of Fuzzy Objects

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Abstract In the given article we consider a problem of a meeting of fuzzy linear objects and we receive a necessary condition of an optimality.

Keywords Fuzzy Differential Inclusion, Control System, Optimal Control, Meeting Problem

1. Introduction

The first research of the differential equations with set-valued right-hand side has been fulfilled by A. Marchaud[1,2] and S.C. Zaremba[3]. In the early sixties, T. Wazewski[4,5], A.F. Filippov[6] had been obtained fundamental results about existence and properties of solutions of the differential equations with set-valued right-hand side (differential inclusions). Connection deriving between differential inclusions and optimum control problems was one of the most important outcomes of these papers. These outcomes became impulse for development of the theory of differential inclusions[7-9].

Considering of the differential inclusions required to study properties of set-valued maps, i.e. an elaboration the whole tool of mathematical analysis for set-valued maps[7,10,11].

In work[12] annotate of an R-solution for differential inclusion is introduced as an absolutely continuous set-valued maps. Various problems for the R-solution theory were considered in[8,13]. The basic idea for a development of an equation for R-solutions (integral funnels) is contained in[14].

In the eighties the last century the control theory in the conditions of uncertainty began to be formed. The control differential equations with set of initial conditions[15-17], control set differential equations[18-21] and the control differential inclusions[21-32] are used in the given theory for exposition of dynamic processes.

In recent years, the fuzzy set theory introduced by Zadeh[33] has emerged as an interesting and fascinating branch of pure and applied sciences. The applications of fuzzy set theory can be found in many branches of regional, physical, mathematical, differential equations, andengineering sciences. Recently there have been new advancesin the theory of fuzzy differential equations[34-47] and inclusions[48-53] as well as in the theory of control fuzzy differential equations[54-57] and inclusions[57-59].

In this article we consider a problem of a meeting of fuzzy linear objects and we receive a necessary condition of an optimality.

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2. The Fundamental Definitions and Designations

Let $comp(R^n)(conv(R^n))$ be a set of all nonempty (convex) compact subsets from the space R^n,

$$h(A,B)=\min_{r\ge 0}\{S_r(A)\supset B,\, S_r(B)\supset A\}$$

be Hausdorff distance between sets A and B, $S_r(A)$ is r -neighborhood of set A.

Let E^n be the set of all $u:R^n\to[0,1]$ such that u satisfies the following conditions:

- u is normal, that is, there exists an $x_0\in R^n$ such that $u(x_0)=1$;
- u is fuzzy convex, that is,

$$u(\lambda x+(1-\lambda)y)\ge \min\{u(x),u(y)\}$$

for any $x,y\in R^n$ and $0\le\lambda\le 1$;
- u is upper semicontinuous,
- $[u]^0=cl\{x\in R^n:u(x)>0\}$ is compact.

If $u\in E^n$, then u is called a fuzzy number, and E^n is said to be a fuzzy number space. For $0<\alpha\le 1$, denote

$$[u]^\alpha=\{x\in R^n:u(x)\ge\alpha\}.$$

Then from 1)-4), it follows that the α -level set $[u]^\alpha\in conv(R^n)$ for all $0\le\alpha\le 1$.

Let θ be the fuzzy mapping defined by $\theta(x)=0$ if $x\ne 0$ and $\theta(0)=1$.

Define $D:E^n\times E^n\to[0,\infty)$ by the relation

$$D(u,v) = \sup_{0\le\alpha\le1} h\left(\left[u\right]^{\alpha},\left[v\right]^{\alpha}\right),$$

where h is the Hausdorff metric defined in $comp(R^n)$. Then D is a metric in E^n. Further we know that[60]:

1. $\left(E^n, D\right)$ is a complete metric space,
2. $D\left(u+w, v+w\right) = D\left(u,v\right)$ for all $u,v,w\in E^n$,
3. $D\left(\lambda u, \lambda v\right) = \left|\lambda\right| D\left(u,v\right)$ for all $u,v\in E^n$ and $\lambda\in R$.

Definition 1.[36] A mapping $F:[0,T]\to E^n$ is measurable if for all $\alpha\in[0,1]$ the set-valued map $F_\alpha:[0,T]\to conv\left(R^n\right)$ defined by $F_\alpha(t) = \left[F(t)\right]^\alpha$ is Lebesgue measurable.

Definition 2.[36] A mapping $F:[0,T]\to E^n$ is said to be integrably bounded if there is an integrable function $h(t)$ such that $\|x(t)\|\le h(t)$ for every $x(t)\in F_0(t)$.

Definition 3.[36] The integral of a fuzzy mapping $F:\left[0,T\right]\to E^n$ is defined levelwise by $\left[\int_0^T F(t)dt\right]^\alpha = \int_0^T F_\alpha(t)dt$. The set $\int_0^T F_\alpha\left(t\right)dt$ of all $\int_0^T f(t)dt$ such that $f:[0,T]\to R^n$ is a measurable selection for $F_\alpha:[0,T]\to conv\left(R^n\right)$ for all $\alpha\in\left[0,1\right]$.

Definition 4.[36] A measurable and integrably bounded mapping $F:\left[0,T\right]\to E^n$ is said to be integrable over $\left[0,T\right]$ if $\int_0^T F(t)dt\in E^n$.

Note that if $F:\left[0,T\right]\to E^n$ is measurable and integrably bounded, then F is integrable. Further if $F:\left[0,T\right]\to E^n$ is continuous, then it is integrable.

Now we consider following control differential equations with the fuzzy parameter

$$\dot{x} = f\left(t,x,w,v\right),\ x(0) = x_0, \tag{1}$$

where $\dot{x}$ means dx/dt; $t\in R_+$ is the time; $x\in R^n$ is the state; $w\in R^m$ is the control; $v\in V\in E^k$ is the fuzzy parameter; $f:R_+\times R^n\times R^m\times R^k\to R^n$.

Let $W:R_+\to conv(R^m)$ be the measurable set-valued map.

Definition 5. The set LW of all measurable single-valued branches of the set-valued map $W(\cdot)$ is the set of the admissible controls.

Further we consider following control fuzzy differential inclusions

$$\dot{x}\in F(t,x,w),\quad x(0) = x_0, \tag{2}$$

where $F:R_+\times R^n\times R^m\to E^n$ is the fuzzy map such that $F\left(t,x,w\right)\equiv f\left(t,x,w,V\right)$.

Obviously, the control fuzzy differential inclusion (2) turns into the ordinary fuzzy differential inclusion

$$\dot{x}\in\Phi\left(t,x\right),\ x(0) = x_0, \tag{3}$$

if the control $\tilde{w}(\cdot)\in LW$ is fixed and $\Phi\left(t,x\right)\equiv F\left(t,x,\tilde{w}(t)\right)$.

If right-hand side of the fuzzy differential inclusion (3) satisfies some conditions (for example look[12]) then the fuzzy differential inclusions (3) has the fuzzy R-solution.

Let $X(t)$ denotes the fuzzy R-solution of the differential inclusion (3), then $X(t,w)$ denotes the fuzzy R-solution of the control differential inclusion (2) for the fixed $w(\cdot)\in LW$.

Definition 6. The set $Y(T) = \left\{X\left(T,w\right): w\left(\cdot\right)\in LW\right\}$ be called the attainable set of the fuzzy system (2).

3. The Some Properties of the Fuzzy R-solution and Time-optimal Problem

3.1. The Some Properties of the Fuzzy R-solution

Further in the given paper, we consider following control linear fuzzy differential inclusions

$$\dot{x}\in A(t)x + G(t,w),\ x(0) = x_0, \tag{4}$$

where $A(t)$ is $\left(n\times n\right)$ -dimensional matrix-valued function; $G:R_+\times R^m\to E^n$ is the fuzzy map.

In this section, we consider the some properties of the fuzzy R-solution of the control fuzzy differential inclusion (4).

Let the following condition is true.

Condition A:

1) $A(\cdot)$ is measurable on $\left[0,T\right]$;

2) The norm $\left\|A(t)\right\|$ of the matrix $A(t)$ is integrable on $\left[0,T\right]$;

3) The set-valued map $W:\left[t_0,T\right]\to conv\left(R^m\right)$ is measurable on $\left[0,T\right]$;

4) The fuzzy map $G:\left[0,T\right]\times R^m\to E^n$ satisfies the conditions

a) measurable in t;

b) continuous in w;

1) There exist $v(\cdot)\in L_2\left[0,T\right]$ and $l(\cdot)\in L_2\left[0,T\right]$ such that

$$h(W(t),0)\le v\left(t\right),\ D(G(t,w),\theta)\le l\left(t\right)$$

almost everywhere on $\left[0,T\right]$ and all $w\in W\left(t\right)$.

2) The set $Q(t) = \left\{G\left(t,w(t)\right): w(\cdot)\in LW\right\}$ is compact and convex for almost every $t\in\left[0,T\right]$, i.e. $Q(t)\in conv(E^n)$.

Theorem 1[61]. Let the condition A is true.

Then for every $w(\cdot)\in LW$ there exists the fuzzy R-solution $X\left(\cdot,w\right)$ such that

1) the fuzzy map $X\left(\cdot,w\right)$ has form

$$X\left(t,w\right) = \Phi\left(t\right)x_0 + \Phi\left(t\right)\int_0^t \Phi^{-1}(s)G(s,w(s))ds,$$

where $t\in\left[0,T\right]$; $\Phi\left(t\right)$ is Cauchy matrix of the differential equation $\dot{x} = A(t)x$;

2) $X(t,w)\in E^n$ for every $t\in\left[0,T\right]$;

3) the fuzzy map $X\left(\cdot,w\right)$ is the absolutely continuous fuzzy map on $\left[0,T\right]$.

Theorem 2[61]. Let the condition A is true.

Then the attainable set $Y(T)$ is compact and convex.

Remark. Properties of space $comp(E^n)$ have been considered in work[62].

3.2. Time-optimal Problem

Consider the following time-optimal problem: it is necessary to find the minimal time T and the control $w^*(\cdot)\in LW$ such that the fuzzy R-solution of system (4) satisfy of the condition:

$$X(T,w^*)\cap S_k\neq\varnothing, \quad (5)$$

where $S_k\in E^n$ is the fuzzy terminal set.

Theorem 3[61]. (necessary optimal condition for the time-optimal problem (4),(5)). Let the condition A is true and the pair $(T,w^*(\cdot))$ is optimality of the control problem (4),(5).

Then there exists the vector-function $\psi(\cdot)$, which is the solution of the system

$$\dot\psi=-A^T(t)\psi,\ \psi(T)\in S_1(0)$$

such that

1) $C([G(t,w^*)]^1,\psi(t))=\max_{w\in W(t)} C([G(t,w)]^1,\psi(t))$

almost everywhere on $[0,T]$;

2) $C\left(\left[X(T,w^*)\right]^1,\psi(T)\right)=-C\left(\left[S_k\right]^1,-\psi(T)\right)$,

where $C(P,\psi)=\max_{p\in P}(p_1\psi_1+\ldots+p_n\psi_n)$, $\psi\in R^n$, $P\in conv(R^n)$.

4. The Problem of Meeting of N of Fuzzy Objects

Further, we consider N linear control differential inclusions with fuzzy parameters

$$\dot x^i\in A^i(t)x^i+G^i(t,w^i),\ x^i(0)=x_0^i,\ i=\overline{1,N}, \quad (6)$$

where $x^i\in R^n$; $t\in R_+$; $A^i(t):R_+\to R^{n\times n}$ is a matrix $n\times n$; $G^i(t,w^i):R_+\times R^{k_i}\to E^n$ is a fuzzy map; $w^i\in W^i\subset R^{k_i}$ is a control parameter; $x_0^i\in R^n$.

Consider the following optimal control problem (***problem A***): it is necessary to find the minimal time T^* and controls $w_*^i(\cdot)\in LW^i$, $i=\overline{1,N}$ such that the fuzzy R-solutions of system (6) satisfy of the condition:

$$\bigcap_{i=1}^N X^i(T^*,w_*^i)\neq\varnothing. \quad (7)$$

Definition 6. The collection $(T^*,w_*^1(\cdot),...,w_*^N(\cdot))$ is said to be optimality if

$$\bigcap_{i=1}^N X^i(T^*,w_*^i)\neq\varnothing \text{ and } \bigcap_{i=1}^N X^i(\tau,w^i)=\varnothing$$

for every $0\le\tau<T^*$ and all $w^i(\cdot)\in LW^i$, $i=\overline{1,N}$.

Further we will reduce necessary conditions of an optimality of collection $(T^*,w_*^1(\cdot),...,w_*^N(\cdot))$ for meeting problem.

Theorem 4. Let the following conditions hold for every $i\in\{1,...,N\}$:

1) $A^i(\cdot)$ is measurable on $[0,T^*]$;

2) The norm $\|A^i(t)\|$ of the matrix $A^i(t)$ is integrable on $[0,T^*]$;

3) The set-valued map $W^i:[t_0,T]\to conv(R^{k_i})$ is measurable on $[0,T^*]$;

4) The fuzzy map $G^i:[0,T^*]\times R^{k_i}\to E^n$ satisfies the conditions

a) measurable in t;

b) continuous in w^i;

5) There exist $v^i(\cdot)\in L_2[0,T^*]$ and $l^i(\cdot)\in L_2[0,T^*]$ such that

$$h(W^i(t),0)\le v^i(t),\ D(G^i(t,w^i),\theta)\le l^i(t)$$

almost everywhere on $[0,T^*]$ and all $w^i\in W^i(t)$.

6) The set $Q^i(t)=\{G^i(t,w^i(t)):w^i(\cdot)\in LW^i\}$ is compact and convex for almost every $t\in[0,T^*]$, i.e. $Q^i(t)\in conv(E^n)$ and the pair $(T^*,w_*^1(\cdot),...,w_*^N(\cdot))$ is optimality for the problem (6),(7).

Then there exist $j\in\{1,...,N\}$ and solution $\psi^j(\cdot)$ of the differential equation $\dot\psi^j=-(A^j(t))^T\psi^j$, $\|\psi^j(T^*)\|=1$ such that

1) $C([G^j(t,w_*^j)]^1,\psi^j(t))=\max_{w^j\in W^j(t)} C([G^j(t,w^j)]^1,\psi^j(t))$

almost everywhere on $[0,T^*]$;

2) $C\left(\left[X^j(T^*,w_*^j)\right]^1,\psi^j(T^*)\right)=-C\left(\left[\bigcap_{i=1}^N X^i(T^*,w_*^i)\right]^1,-\psi^j(T^*)\right)$.

Proof. We associate with the control fuzzy system (6) the following control fuzzy system

$$\dot x\in A(t)x+G(t,w),\ x(0)=x_0, \quad (8)$$

where $x=(x^1,...,x^N)$, $x^i\in R^n$, $i=\overline{1,N}$, $w=(w^1,...,w^N)$, $w^i\in R^{k_i}$, $i=\overline{1,N}$,

$$A(t)=\begin{pmatrix}A^1(t)&0&\dots&0\\0&A^2(t)&\dots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\dots&A^N(t)\end{pmatrix},$$

$$G(t,w)=\begin{pmatrix}G^1(t,C^1w)\\G^2(t,C^2w)\\\vdots\\G^N(t,C^Nw)\end{pmatrix},$$

$$C^1=\begin{pmatrix}I^1&0&\dots&0\\0&0&\dots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\dots&0\end{pmatrix},\ \dots,\ C^N=\begin{pmatrix}0&\dots&0&0\\\vdots&\ddots&0&0\\0&\dots&0&0\\0&\dots&0&I^N\end{pmatrix},$$

I^i is a unit matix $(k_i\times k_i)$, $W=\prod_{i=1}^N W^i$, $x_0=(x_0^1,...,x_0^N)^T$.

Under the conditions of theorem, we have conditions

1) $A(\cdot)$ is measurable on $\left[0,T^*\right]$;

2) The norm $\|A(t)\|$ of the matrix $A(t)$ is integrable on $\left[0,T^*\right]$;

3) The set-valued map $W:\left[t_0,T^*\right]\rightarrow conv\left(R^{k_1\times\ldots\times k_N}\right)$ is measurable on $\left[0,T^*\right]$;

4) The fuzzy map $G:\left[0,T\right]\times R^{k_1\times\ldots\times k_N}\rightarrow E^{Nn}$ satisfies the conditions

a) measurable in t;

b) continuous in w;

5) There exist $v(t)=\sup\limits_{i=\overline{1,N}} v^i(t)$ and $l(t)=\sup\limits_{i=\overline{1,N}} l^i(t)$ such that

$$h(W(t),0)\le v(t),\ D(G(t,w),\theta)\le l(t)$$

almost everywhere on $\left[0,T^*\right]$ and all $w\in W(t)$.

The set $Q(t)=\left\{G(t,w(t)):w(\cdot)\in LW\right\}$ is compact and convex for almost every $t\in\left[0,T^*\right]$, i.e. $Q(t)\in conv(E^{Nn})$.

Let fuzzy set $S_K\in E^{Nn}$ such that $\left[S_K\right]^\alpha=\left\{x\in R^{Nn}\mid x^1=\ldots=x^N,\ x^i\in R^n, i=\overline{1,N}\right\}$ for all $\alpha\in\left[0,1\right]$.

Now we consider the following optimal control problem (***problem B***): it is necessary to find the minimal time T^* and the control $w^*\in LW$ such that the fuzzy R-solution of system (8) satisfies of the condition $X\left(T^*,w^*\right)\cap S_K\neq\varnothing$.

Using the results of[32], we know that the ***problem A*** and the ***problem B*** is the equivalent, i.e. the collection $\left(T^*,w_*^1(\cdot),\ldots,w_*^N(\cdot)\right)$ is optimality for ***problem A*** if and only if the pair $\left(T^*,w^*(\cdot)\right)$ is optimality for ***problem B***, where $w^*(\cdot)=\left(w_*^1(\cdot),\ldots,w_*^N(\cdot)\right)$.

Hence and by the theorem 3, it follows that there exists solution $\psi(\cdot)$ of the differential equation $\dot\psi=-A^T(t)\psi,\ \psi\left(T^*\right)\in S_1(0)$ such that

1) $C([G(t,w^*)]^1,\psi(t))=\max\limits_{w\in W(t)} C(\left[G(t,w)\right]^1,\psi(t))$

almost everywhere on $\left[0,T\right]$;

2) $C\left(\left[X(T,w^*)\right]^1,\psi(T)\right)=-C\left(\left[S_k\right]^1,-\psi(T)\right)$.

From here theorem statements follow. The theorem is proved.

5. Conclusions

It is obviously possible to consider other problem of a $\alpha-$meeting: it is necessary to find the minimal time T_α^* and the controls $w_\alpha^i(\cdot)\in LW^i$, $i=\overline{1,N}$ such that the fuzzy R-solutions of system (6) satisfy of the condition:

$$\bigcap_{i=1}^N\left[X^i\left(T_\alpha,w_\alpha^i\right)\right]^\alpha\neq\varnothing.$$

In this case, necessary conditions will look like: there exist $j\in\{1,\ldots,N\}$ and solution $\psi^j(\cdot)$ of the differential equation $\dot\psi^j=-\left(A^j(t)\right)^T\psi^j$, $\left\|\psi^j\left(T_\alpha^*\right)\right\|=1$ such that

1) $C([G^j(t,w_\alpha^j)]^\alpha,\psi^j(t))=\max\limits_{w^j\in W^j(t)} C(\left[G^j(t,w^j)\right]^\alpha,\psi^j(t))$

almost everywhere on $\left[0,T_\alpha^*\right]$;

2) $C\left(\left[X^j(T_\alpha^*,w_\alpha^j)\right]^\alpha,\psi^j(T_\alpha^*)\right)=-C\left(\left[\bigcap_{i=1}^N X^i\left(T_\alpha^*,w_\alpha^i\right)\right]^\alpha,-\psi^j(T_\alpha^*)\right).$

Finally, we note that $T_0^*\le T_\alpha^*\le T_1^*=T^*$ as

$$\left[X^i\left(t,w^i\right)\right]^0\subset\left[X^i\left(t,w^i\right)\right]^\alpha\subset\left[X^i\left(t,w^i\right)\right]^1$$

for every $t\ge0$ and $w^i(\cdot)\in LW^i$, $i=\overline{1,N}$.

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