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Quasidistributions in Nonclassical Joint Gaussian Statistics

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Abstract We derive one-dimensional and two-dimensional wave quasidistributions for Gaussian processes including their nonclassical regimes.

Keywords Quantum measurement, Optical parametric processes, Nonclassical light, Quantum noise

1. Introduction

It is well known from the papers by R. J. Glauber [1, 2] published in 1963, founding the modern quantum optics, that optical processes having completely quantum behaviour, i.e. having no classical analogue, are described by means of quasidistributions, e.g. using the weighting function in the diagonal Glauber-Sudarshan representation of the density matrix [3]. Such quasidistributions have some properties of classical distribution functions, e.g. they are normalised, some other properties are violated because they are in general generalised functions (linear functionals); they can be more singular than the Dirac function and they can take on negative values. Reviews of quasidistributions used in quantum optics can be found in books [4, 5]. This reflects the physical fact that the quantum dynamics are much more rich than the classical dynamics and hence for some quantum effects it may happen that there is a debt of probability expressed by negative values of the classical tool of probability function used for description of a quantum system. Such an approach represents important point of view based on wave properties of quantum systems.

We start with the joint Gaussian generating function [6] to show how to derive one-dimensional and two-dimensional wave quasidistributions including nonclassical regimes, where quantum-noise components take on negative values and when standard Cauchy integrals may fail. The method is related to the so-called generalised superposition of coherent and quantum-noise fields [3]. The method can also be applied to treat a shifted Gaussian process with a stimulating coherent field [7]. In particular it is suitable for description of optical parametric processes [6, 7].

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2. Generating Function

The s-ordered generating function is obtained for the Gaussian process in the form [6]

$$G_s(\lambda_1,\lambda_2) = \left(1+\lambda_1 B_{1s}+\lambda_2 B_{2s}+\lambda_1\lambda_2 K_s\right)^{-M}, \quad (1)$$

where λ_1 and λ_2 are parameters of the generating function and the s-ordered noise functions are defined as $B_{1,2s} = B_{1,2} + (1-s)/2$ in terms of noise functions $B_{1,2}$; M is number of equally behaved modes (temporal, spatial and polarisation in the spirit of Mandel-Rice formula) [6]. The quality of the process is characterised by the determinant involved in a Fourier transform $K = B_1B_2 - |D_{1,2}|^2 < 0$ or $K = B_1^2 - |C_1|^2 < 0$ in quantum regimes related to quasidistributions, whereas in classical regimes $K \geq 0$; here $D_{1,2}$ and C_1 are correlation coefficients of the quantum noise between modes and self-interacting noise coefficient in the mode, respectively. The quantum-classical border is given by $K = 0$.

Determinants K are crucial for the judgement of classicality or nonclassicality of the field. Negative values of the determinant K mean that a given field cannot be described classically, which is, for instance, the case of the field composed of photon pairs. The quantities $B_{1,2} + K$ can be considered as characteristics of fictitious quantum noise present in the fields, giving declination from the purity of the process, i.e. from the diagonal form of the joint photon-number distribution $p(n_1, n_2)$ when photons are ideally paired and $K = -B_1 = -B_2$; in inpure cases physically they are positive. However, in some compound cases they can characterise partial virtual distributions taking on negative values giving finally the actual physical distribution after the convolution process of partial distributions [8, 9] leading to quantum oscillations of the physical distribution. Of course the sign of the determinant K is decisive for obtaining the wave joint distributions by means of Fourier transformation.

3. Joint Wave Quasidistributions

The s-ordered joint wave quasidistribution $P_s(W_1, W_2)$ is obtained by means of the double Fourier transformation as

$$P_s\left(W_1,W_2\right)=\frac{1}{4\pi^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\exp\left(-\mathrm{i}s_1W_1-\mathrm{i}s_2W_2\right)G_s\left(-\mathrm{i}s_1,-\mathrm{i}s_2\right)\mathrm{d}s_1\mathrm{d}s_2$$
$$=\frac{1}{\Gamma\left(M\right)}\left(\frac{K_s^2W_1W_2}{\left|D_{12}\right|^2}\right)^{(M-1)/2}\exp\left[-\frac{W_1B_{2s}+W_2B_{1s}}{K_s}\right]I_{M-1}\left(2\sqrt{\frac{\left|D_{12}\right|^2W_1W_2}{K_s^2}}\right), \tag{2}$$

under the assumption that the s-ordered determinant $K_s = B_{1s}B_{2s} - |D_{12}|^2 = K + (1 - s)(B_{1s} + B_{2s})/2 + (1 - s)^2/4$ is positive [6].

If the s-ordered determinant K_s is negative, the joint signal-idler quasidistribution P_s of integrated intensities exists as a generalised function that can take on negative values or can even have singularities stronger than the Dirac function. It can be obtained in the following regularised form [6], as we will discuss in greater detail in the following:

$$P_s\left(W_1,W_2\right)=\frac{\left(W_1W_2\right)^{(M-1)/2}}{\pi\Gamma\left(M\right)\left(-K_s\right)^{1/2}\left(B_{1s}B_{2s}\right)^{M/2}}\exp\left(-\frac{W_1}{2B_{1s}}-\frac{W_2}{2B_{2s}}\right)\mathrm{sinc}\left[A\left(\frac{B_{2s}}{B_{1s}}W_1-W_2\right)\right]; \tag{3}$$

here $\mathrm{sinc}(x) = \sin(x)/x$, $A = (-K_sB_{2s}/B_{1s})^{-1/2}$. Oscillating behaviour is typical for the quasidistribution P_s written in (3). Compared to our considerations in [6] the formula (3) is exact because we can show that the poles in the upper half complex plane give no contribution to the corresponding integral (see next section).

If $K_s = 0$ we have the quantum-classical border with the diagonal quasidistribution

$$P_s\left(W_1,W_2\right)=\frac{W_1^{(M-1)}}{\Gamma\left(M\right)B_{1s}^M}\exp\left(-\frac{W_1}{B_{1s}}\right)\delta\left(\frac{B_{2s}}{B_{1s}}W_1-W_2\right). \tag{4}$$

There exist threshold values s_{th} of the ordering parameter s for given values of quantum noise parameters $B_{1,2}$ and D_{12} determined by $K_s = 0$ (the corresponding joint s-ordered wave distribution (4) is diagonal):

$$s_{\mathrm{th}}=1+B_1+B_2-\sqrt{(B_1+B_2)^2-4K}, \tag{5}$$

$-1 \le s_{\mathrm{th}} \le 1$. Quasidistributions P_s for $s \le s_{\mathrm{th}}$ are ordinary functions with non-negative values whereas those for $s > s_{\mathrm{th}}$ are generalised functions with negative values and oscillations. Writing this expression in the form $1 + B_1 + B_2 - [(B_1 - B_2)^2 + 4|D_{12}|^2]^{1/2}$ we see that it is directly related to the principal squeeze parameter for equally behaved modes $\lambda = 1 + 2(B - |D_{12}|)$. In this sense the quasidistribution behaves in a quantum way for s between the threshold value s_{th} and 1 (Glauber-Sudarshan quasidistribution), for s equal or less than the threshold value, the vacuum fluctuations related to the s-ordered quasidistribution compensate nonclassicality in the field (expressed e.g. by the squeezing of vacuum fluctuations in the field) and the quasidistribution behaves classically.

Even if negative probabilities, which can be reconstructed from experimental data ([10] and references therein), represent only qualitative phenomenon reflecting debt of probabilities in richer quantum dynamics compared to classical dynamics and we cannot interpret them directly, they have direct consequences in the discrete region, i.e. in $p(n_1, n_2)$: In the quantum region $K < 0$ and $K + B \ge 0$, i.e. $K + B < B$; in the pure case we have $K = -B$ and $K + B=0$, which leads to the diagonal Mandel-Rice formula for $p(n_1, n_2)$ giving the oscillating sum-number distribution with zero odd values [6]; in general we have quantum oscillations and squeezed-form distribution up to the border between quantum and classical regions for $K = 0$ ($K + B = B$) which reflect negative probabilities. Then in the classical region $K \ge 0$, i.e. $K + B \ge B$, quasidistributions behave classically and forms of $p(n_1, n_2)$ change among that for $K = 0$ and the isotropic case $p(n_1, n_2) = p(n_1)\, p(n_2)$.

We can mention that quantum entanglement equivalent to the above quantum phenomena is obtained if $KK_{\mathcal{A}} < 0$, i.e. $K < 0$ because always $K_{\mathcal{A}} > 0$, where $K_{\mathcal{A}}$ is the corresponding determinant for antinormal operator ordering, i.e. B is substituted by $(B + 1)$ [8]. A number of other quantities can be derived from the basic joint distributions [8], such as conditional number distributions $p_{c,2}(n_2; n_1)$ which are sub-Poissonian in quantum regimes giving the corresponding Fano factor $F_{c,2} < 1$, sub-Poissonian difference-number distribution $p_-(n)$, sum- and difference-wave quasidistributions $P_{s\pm}(W)$ exhibiting classical and quantum behaviour, respectively; the principal squeezing $\lambda = 1 + B_1 + B_2 - 2\mathrm{Re}(\overline{D}_{12}) - |C_1 + C_2 + D_{12}|$ (it holds that $\lambda \ge s_{\mathrm{th}-}$) and sub-shot-noise behaviour $R = 1 + K/B < 1$ (the same modes) can also be obtained for a more general compound parametric process [8].

4. Forms of Quasidistributions

Now we can follow the way how to derive quasidistributions (2) and (3) in classical and quantum regions in greater detail explaining the mathematical basis. This is based on paper [11] involving some corrections. We can mention that in general parametric processes the noise quantities $B_{1,2}$ can also be negative as describing the difference between phase independent noise and phase dependent noise arising as a result of self-interaction in a mode [8]. So we distinguish this in our discussion.

In this sense we have eight possibilities with respect to signs of $B_{1,2s}$ and K_s.

The wave quasidistribution is determined by the Fourier transform of the generating function,

$$P_s\left(W_1,W_2\right)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\exp\left(-\mathrm{i}s_1W_1-\mathrm{i}s_2W_2\right)}{\left[1-\mathrm{i}s_1B_{1s}-\mathrm{i}s_2B_{2s}-s_1s_2K_s\right]^M}\mathrm{d}s_1\mathrm{d}s_2. \tag{6}$$

Writing the polynomial in the denominator in the form

$$\left(1-\mathrm{i}s_1B_{1s}-\mathrm{i}s_2B_{2s}-s_1s_2K_s\right)=-\left(\mathrm{i}B_{1s}+s_2K_s\right)\left[s_1+\frac{s_2\left|D_{12}\right|^2+\mathrm{i}\left(B_{1s}+s_2^2B_{2s}K_s\right)}{B_{1s}^2+s_2^2K_s^2}\right], \tag{7}$$

we see that when

1. $B_{1,2s}>0$, $K_s>0$, then the poles lie in the lower complex half-plane $\Pi^{(-)}$ and both the integrals with respect to variables s_1 and s_2 are performed as two Cauchy integrals, arriving to the I_M-distribution (2) which is regular and non-negative. Under these conditions the system behaves classically.

2. $B_{1,2s}>0$, $K_s<0$

This is the standard quantum case. To have the pole in $\Pi^{(-)}$ when integrating along s_1 to be able to use the Cauchy integral, it must be

$$B_{1s}-s_2^2B_{2s}(-K_s)>0, \tag{8}$$

i.e. s_2 is filtered out the interval $(-A,+A)$, $A=(B_{1s}/B_{2s}(-K_s))^{1/2}$ and we arrive at the integral

$$P_s\left(W_1,W_2\right)=\frac{W_1^{M-1}}{2\pi\Gamma(\mathrm{M})}\int_{-A}^{+A}\frac{\exp\left(-\mathrm{i}s_2W_2-\dfrac{1-\mathrm{i}s_2B_{2s}}{-\mathrm{i}s_2K_s+B_{1s}}W_1\right)}{\left(-\mathrm{i}s_2K_s+B_{1s}\right)^M}\mathrm{d}s_2. \tag{9}$$

Now we can show that the pole $s_2=-\mathrm{i}B_{1s}/K_s$ lying in the upper half-plane $\Pi^{(+)}$ in this case gives no contribution to the integral (9) which is analytic in the lower half-plane $\Pi^{(-)}$. If we integrate the integral (9) with respect to W_2 (it generally represents a generalised function) and use one factor from the denominator, we have

$$\frac{1}{-\mathrm{i}s_2\left(-\mathrm{i}s_2K_s+B_{1s}\right)}=\frac{1}{B_{1s}}\left(\frac{1}{-\mathrm{i}s_2}-\frac{K_s}{-\mathrm{i}s_2K_s+B_{1s}}\right). \tag{10}$$

The integral involving the second term is zero because taking it from $-\infty$ to $+\infty$ (the integral for the filtered values of s_2 is zero because this is related to the opposite inequality in (8) and the pole lies in the upper half-plane), the pole is at $s_2=-\mathrm{i}B_{1s}/K_s$ in $\Pi^{(+)}$ and performing the derivative with respect to W_2 we have the same integral with the factor in the denominator decreased by one; hence one factor $-\mathrm{i}s_2K_s+B_{1s}$ in the denominator is replaced by B_{1s}. Successively we replace all these factors by B_{1s} including the corresponding terms in the exponential function decomposing it into the series. We finally obtain exactly the above sinc-quasidistribution (3) after explicit symmetrisation of the expression (we change the order of integrations along s_1 and s_2 obtaining the same result changing only indices 1 and 2, multiply the results and take the square root).

3. $B_{1s}<0$, $B_{2s}>0$, $K_s>0$

To have the pole in s_1 in $\Pi^{(-)}$ we must have similarly to (8)

$$-\left(-B_{1s}\right)+s_2^2B_{2s}K_s>0, \tag{11}$$

which means that $|s_2| > -A_1$, $A_1 = (-B_{1s}/B_{2s}K_s)^{1/2}$, s_2 is not restricted and takes on all values on the real axis $(-\infty,+\infty)$. The integral in (6) with respect to s_1 is the Cauchy integral, however the pole in s_2 lies in $\Pi^{(+)}$ and the integral in s_2 is zero. The quasidistribution $P_s(W_1,W_2)$ has a point support and cannot directly be determined by the Cauchy theorem. However, when changing the order of integrations integrating first along s_2 and then along s_1, cases 3 and 4 are interchanged and both the quasidistributions are determined as in the following case 4.

4. $B_{1s} > 0$, $B_{2s} < 0$, $K_s > 0$

In this case we have

$$B_{1s} - s_2^2\left(-B_{2s}\right)K_s > 0, \tag{12}$$

$|s_2| < A_2$, $A_2 = (B_{1s}/(-B_{2s})K_s)^{1/2}$, the pole in s_2 is in $\Pi^{(-)}$, and consequently $P_s(W_1,W_2)$ is expressed as the above finite integral (9) in s_2, however the pole in s_2 cannot be neglected as in case 2.

5. $B_{1s} < 0$, $B_{2s} < 0$, $K_s > 0$

In this case

$$-\left(-B_{1s}\right) - s_2^2\left(-B_{2s}\right)K_s > 0, \tag{13}$$

which means that $s_2^2 < -A_3^2$, $A_3 = (-B_{1s}/(-B_{2s})K_s)^{1/2}$, which cannot be fulfilled on the real axis s_2; the quasidistribution could be found in the general complex plane $s_2' = -is_2$. The pole in s_2 lies in $\Pi^{(+)}$.

6. $B_{1s} < 0$, $B_{2s} > 0$, $K_s < 0$

We have

$$-\left(-B_{1s}\right) - s_2^2\left(B_{2s}\right)\left(-K_s\right) > 0, \tag{14}$$

giving $s_2^2 < -A_4^2$, $A_4 = (-B_{1s}/B_{2s}(-K_s))^{1/2}$, and the condition cannot again be fulfilled on the real axis s_2 as in case 5; the pole in s_2 is in $\Pi^{(-)}$.

7. $B_{1s} > 0$, $B_{2s} < 0$, $K_s < 0$

We have

$$B_{1s} + s_2^2\left(-B_{2s}\right)\left(-K_s\right) > 0, \tag{15}$$

and $s_2^2 > -A_5^2$, $A_5 = (B_{1s}/(-B_{2s})(-K_s))^{1/2}$ is fulfilled for all real s_2; the pole in s_2 lies in $\Pi^{(+)}$. Thus s_2 is not filtered and the Cauchy integral is zero as in case 3. However, from the symmetry, changing the order of integrations, we have case 6.

8. $B_{1s} < 0$, $B_{2s} < 0$, $K_s < 0$

This is physically more interesting case compared to cases 5–7 above. In this case the necessary condition is

$$-\left(-B_{1s}\right) + s_2^2\left(-B_{2s}\right)\left(-K_s\right) > 0, \tag{16}$$

with the pole in s_2 in $\Pi^{(-)}$. Thus $|s_2| > -A_6$, $A_6 = (-B_{1s}/(-B_{2s})(-K_s))^{1/2}$. The integration in s_2 is not restricted as in case 3 and $P_s(W_1,W_2)$ is of the form of I_{M-1}-distribution (2), however with respect to $K_s < 0$, it can change the sign in dependence on the number of modes M, thus being the quasidistribution taking on negative values in these cases.

The above arguments are also correct for the corresponding generating function (4) in [7] and the joint distribution (13) in [7] appropriate for the optical parametric process stimulated by a coherent light. The most general description can combine the results from [7] and [8] giving wave quasidistributions for a general optical parametric process stimulated by chaotic and squeezed light with initial coherent stimulating components. Roughly speaking, all these properties of the spontaneous process are only shifted along coherent components [12] as result of the fact that all these quantities are determined by normal moments obtained by means of derivatives at zeros of parameters of the generating function.

All the distributions discussed above were extensively illustrated in papers [6–9] (and references therein), therefore we prefer to illustrate here one-dimensional distributions discussed in the following which can be obtained as a special case of the two-dimensional problem only.

5. One-dimensional Process

As the illustration we can now give the simplest case of the single mode second subharmonic generation (which means that $\lambda_1 = \lambda$, $\lambda_2 = 0$, $B_1 = B$, $C_1 = C$, $B_2 = C_2 = D_{12} = \overline{D}_{12} = 0$ in general parametric process [8]).

i) Spontaneous process

In this case the normal generating function ($s = 1$) has the form [3]

$$G(-\mathrm{i}s) = \frac{1}{\left[1-\mathrm{i}s\left(B+|C|\right)\right]^{\frac{M}{2}}\left[1-\mathrm{i}s\left(B-|C|\right)\right]^{\frac{M}{2}}} \tag{17}$$

for the degenerate case (for M even when the cross-correlation coefficient D_{12} stands instead of self-correlation coefficient C we have the same generating function for the compound mode of the non-degenerate process, $B_1 = B_2 = B$). Clearly the second-order polynomial in the denominator of (1) can be written as $1 - 2\mathrm{i}Bs - Ks^2$ (s is now a Fourier variable, $\lambda = -\mathrm{i}s$; generalisation to the s-ordering is straightforward writing $B_s = B + (1 - s)/2$ instead of B), where the determinant $K = B^2 - |C|^2$ (in the non-degenerate case $K = B^2 - |D_{12}|^2$). If the field behaves nonclassically ($K < 0$), the mean number of quantum-noise photons $F = B + |C|$ is positive and the corresponding distribution is the Rayleigh gamma distribution [3], whereas $E = B - |C|$ is negative, leading to sub-Poisson behaviour, squeezing of vacuum fluctuations, quantum oscillations in photon-number distribution and wave quasidistributions, quantum entanglement, etc. Attempt to solve the problem of determining the quasidistribution in nonclassical region in one dimension was not successful earlier [13, 14]. Generalising the polynomial to the two dimensional form as $1 - \mathrm{i}s_1B - \mathrm{i}s_2B - s_1s_2K$ and performing the inverse Fourier transforms after s_1 and s_2 determining the Glauber-Sudarshan wave distribution $P_N(W_1,W_2)$ for $s = 1$ for simplicity, we obtain again filtering of values of s_2 for $K < 0$ to have the Cauchy integral after s_1 with the pole in $\Pi^{(-)}$ as above, giving the frequency filtering $|s_2| \le (-K)^{-1/2}$. Thus we have the partial integral

$$P_-(W) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\exp(-\mathrm{i}sW)}{(1-\mathrm{i}sE)^{M/2}}\,\mathrm{d}s. \tag{18}$$

Using the well-know identity $\int_0^\infty \exp(-\mathrm{i}sW)\,\mathrm{d}W = -\mathrm{i}P\frac{1}{s} + \pi\delta(s)$ ($P\frac{1}{s}$ means the principal value of the integral), we verify that $\int_0^\infty P_-(W)\,\mathrm{d}W = 1.$ Integrating and deriving this integral with respect to W as above and using successively the decomposition

$$\frac{1}{-\mathrm{i}s(1-\mathrm{i}sE)} = \frac{1}{-\mathrm{i}s} - \frac{E}{1-\mathrm{i}sE} \tag{19}$$

as above, we obtain successively after integration in $\Pi^{(-)}$ and taking only not filtered values of s:

$$P_-(W) = \frac{\sin(AW)}{\pi W}, \quad A = (-K)^{-1/2}. \tag{20}$$

The resulting quasidistribution is the convolution of the Rayleigh gamma distribution related to F [3],

$$P_+(W) = \frac{W^{M/2-1}}{\Gamma(M/2)F^{M/2}}\exp\left(-\frac{W}{F}\right), \tag{21}$$

and of the sinc-quasidistribution (20) taking on negative values,

$$P(W) = \int_0^W P_+(W')P_-(W-W')\,\mathrm{d}W'. \tag{22}$$

For the compound mode of the non-degenerate process $E = B - |D_{12}|$, $F = B + |D_{12}|$, $A = (-K)^{1/2}$, $B = B_1 = B_2$, $K = EF$.

ii) Stimulated process

In this case [3]

$$G(-\mathrm{i}s) = \frac{1}{(1-\mathrm{i}sE)^{M/2}} \exp\left(\frac{\mathrm{i}sA}{1-\mathrm{i}sE}\right) \tag{23}$$

where E is the above quantum noise component and $\mathcal{A}$ is a coherent component related to the initial field [3, 12]. Decomposing the exponential function and applying the above arguments, we arrive at the shifted sinc-quasidistribution

$$P_{-}(W) = \frac{\sin(A(W-\mathcal{A}))}{\pi(W-\mathcal{A})}. \tag{24}$$

The resulting quasidistribution is again the convolution of the regular I_{M-1}-distribution related to F [3],

$$P_{+}(W) = \frac{1}{F}\left(\frac{W}{\mathcal{B}}\right)^{(M/2-1)/2} \exp\left(-\frac{W+\mathcal{B}}{F}\right) I_{M/2-1}\left(2\frac{\sqrt{W\mathcal{B}}}{F}\right), \tag{25}$$

and of this sinc-quasidistribution (24) taking on negative values; here $\mathcal{B}$ is another partial coherent component related to the initial field [3, 12]. For $B = B_1 = B_2$ we have

$$(A,B) = \frac{1}{2}\left[|\xi_1|^2 + |\xi_2|^2 \pm \frac{1}{|D_{12}|}\left(D_{12}\xi_1^*\xi_2^* + \text{c.c.}\right)\right] \tag{26}$$

in the non-degenerate case and

$$(A,B) = \frac{1}{2}\left[|\xi|^2 \pm \frac{1}{2|C|}\left(C\xi^{*2} + \text{c.c.}\right)\right] \tag{26}$$

in the degenerate case.

6. Illustrations

The above discussion is based on results of paper [11] where two-dimensional illustrations are available. We will complete this here by adding one-dimensional quasidistributions for single and compound modes as illustrations.

We have chosen $|\xi|^2 = |\xi_1|^2 = |\xi_2|^2 = 1$ and the phase relations $\arg(C) - 2\arg(\xi) = \arg(D_{12}) - \arg(\xi_1) - \arg(\xi_2) = \pi/2$ for simplicity. In Figs. 1a, b we show the one-dimensional quasidistributions $P_{\pm}(W)$ and the resulting quasidistribution $P(W)$ for the compound mode of nondegenerate parametric process and $M = 2$ (one mode of photon pairs). The same for $M = 8$ (four modes of photon pairs) is illustrated in Figs. 2a, b. Analogous results for degenerate parametric process are given in Figs. 3a, b and 4a, b for $M = 1$ and $M = 4$, respectively. Negative values of the quasidistributions exhibit nonclassical behaviour of the process and one can see that the increasing number of degrees of freedom M (and also of the coherent component $\mathcal{B}$) leads to reducing quantum oscillations to negative values and nonclassical properties.

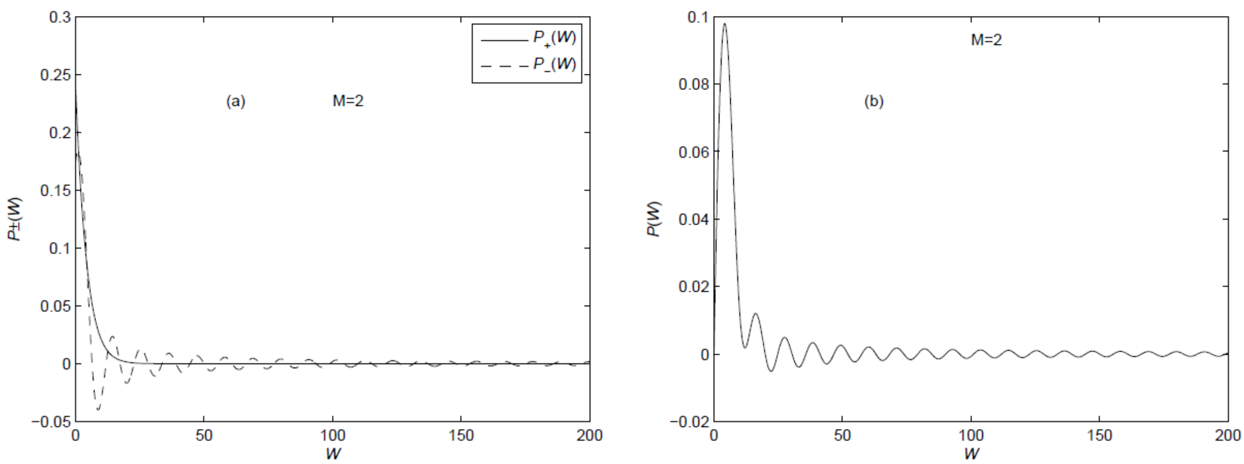

Figure 1. Partial one-dimensional wave quasidistributions $P_{\pm}(W)$ versus integrated intensity W (a) and the resulting quasidistribution $P(W)$ (b) for non-degenerate parametric process and one compound mode

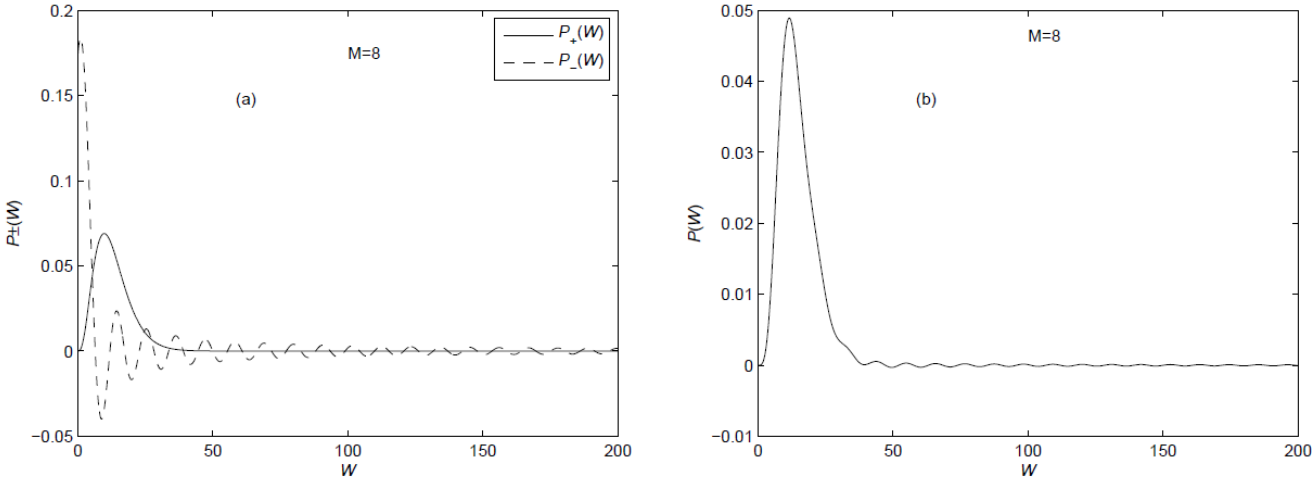

Figure 2. Partial one-dimensional wave quasidistributions $P_\pm(W)$ versus integrated intensity W (a) and the resulting quasidistribution $P(W)$ (b) for non-degenerate parametric process and four compound modes

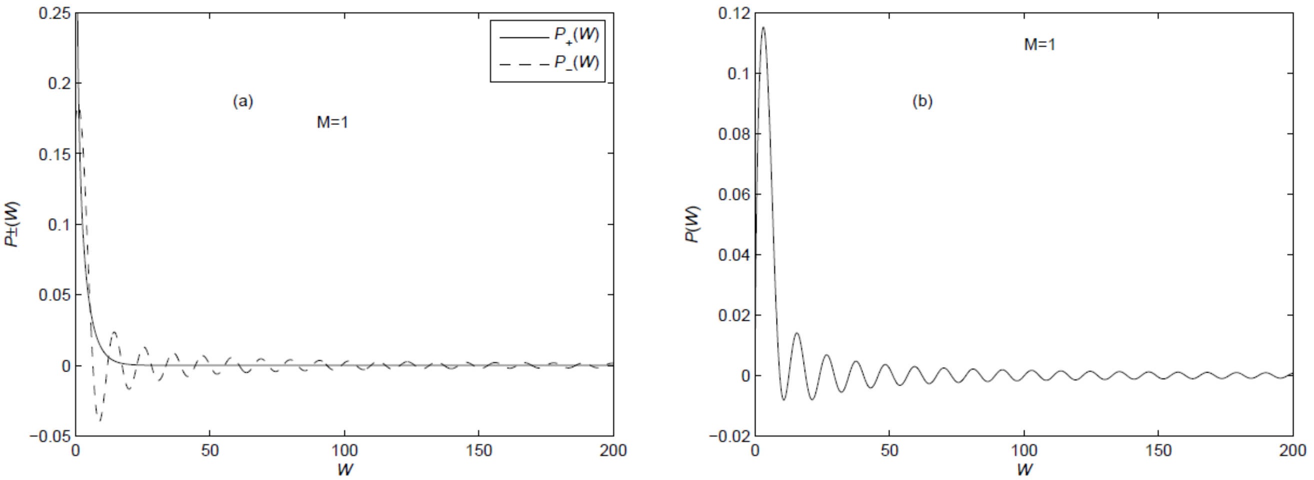

Figure 3. Partial one-dimensional wave quasidistributions $P_\pm(W)$ versus integrated intensity W (a) and the resulting quasidistribution $P(W)$ (b) for degenerate parametric process and one mode

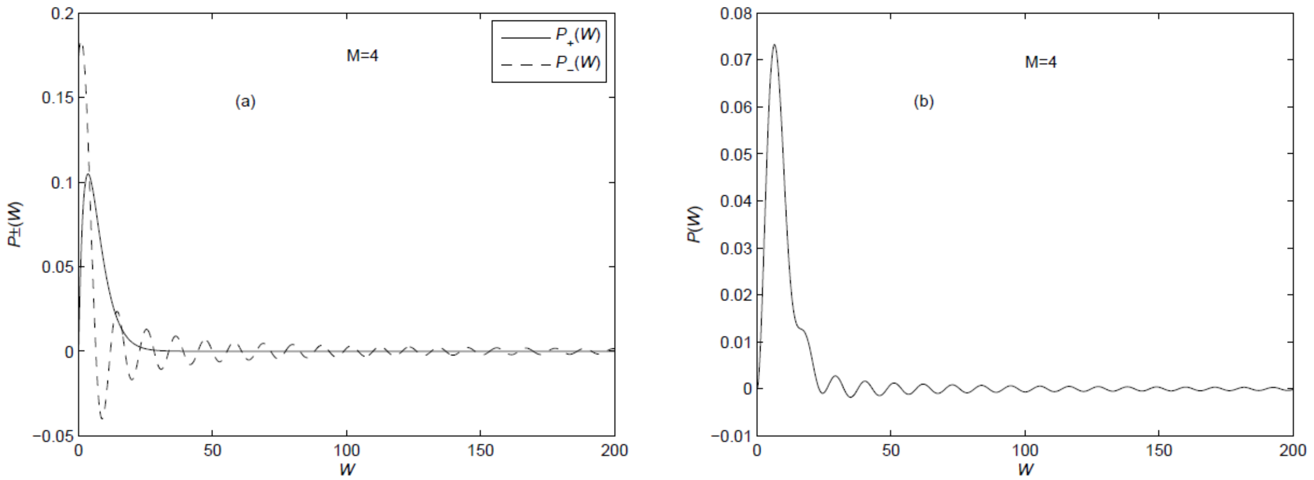

Figure 4. Partial one-dimensional wave quasidistributions $P_\pm(W)$ versus integrated intensities W (a) and the resulting quasidistribution $P(W)$ for degenerate parametric process and four modes

7. Conclusions

We have derived in greater detail and demonstrated classical and nonclassical behaviour of wave quasidistributions. We have demonstrated the important role of the sinc-quasidistributions in such a description of nonlinear optical processes in nonclassical regimes. The method described is directly suitable for quantum description of optical parametric processes, but it can also be used for more complex nonlinear optical processes, such as Raman scattering [15, 16].

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