

Pairwise Concurrence Dynamics in a Five Qubit Model

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Abstract In this paper, we study the entanglement in a five qubit model. For this purpose, we consider two non-interacting systems A and B which are correlated together initially. A is a two atoms Tavis-Cummings system and B is a Jaynes-Cummings System. By choosing a convenient initial condition which shows the initially correlation, our total system (A plus B) become a system which consist of five qubits. There are ten pairwise concurrences which we must calculate. We show that initially non-entangled qubits become entangled by passing time, the process which may be interpreted as entanglement transfer. We also show that our model in special cases become two-qubit and three-qubit models and analyze entanglement in these special cases.

Keywords Entanglement, Tavis-Cummings model, Jaynes-Cummings model, Concurrence, Qubit

1. Introduction

Entanglement is a fascinating aspect of quantum mechanics and the most bizzare features that can not be found in the classical world. It plays fundamental role in both quantum computation (QC) and quantum information processing (QIP)[1-4]. Recently, the time evolution of entanglement, such the behaviour of qubits in quantum computing, is studied in several context[5-9].

The purpose of this paper is to examine a system which consists of two non-interacting subsystems A and B. A is a two-atoms Tavis-Cummings system[10-19] and B is a Jaynes-Cummings system[20]. By choosing a convenient initial condition which provide the correlation between these two systems, this system will behave like a system which their elements (atoms plus cavities) are qubits and our system becomes a five qubit model. We study the entanglement in this system by calculating the reduced density matrice and hence the concurrence among each two qubits.

The Hamiltonian for our system is written as ($\hbar = 1$)

$$\begin{aligned} H &= H_A + H_B \\ &= \omega_{TC} \left(a^+ a + \frac{1}{2} \sigma_3^{(1A)} + \frac{1}{2} \sigma_3^{(2A)} \right) \\ &+ \lambda_{TC} \left[(\sigma_-^{(1A)} + \sigma_-^{(2A)}) a^+ + (\sigma_+^{(1A)} + \sigma_+^{(2A)}) a \right] \\ &+ \omega_{JC} \sigma_3^{(B)} + \lambda_{JC} (b^+ \sigma_-^B + b^- \sigma_+^B) + \omega_{JC} b^+ b \end{aligned} \quad (1)$$

Clearly there will be no interaction between subsystems A and B.

Now we must choose a convenient initial state to make sure two systems (A and B) are correlated together. We suppose that one atom from A (A_1) is entangled with the

Jaynes-Cummings atom B . Therefore, the initial state has two parts:

I) The initially Entangled atoms:

As we said before we suppose that one atom from the Tavis-Cummings system is entangled with the Jaynes-Cummings atom. We use a partially entangled atomic pure state that is a combination of the bell states:

$$|\Psi_{A,B}(0)\rangle = \cos \alpha |e_{1A} g_B\rangle + \sin \alpha |g_{1A} e_B\rangle \quad (2)$$

(II) The initially non entangled atom and cavities:

For our purpose, we also suppose that the non-entangled atom A_2 is in its ground state $|g_{2A}\rangle$ and both cavities which we denoted a and b are in their vaccum states $|0_a\rangle$ and $|0_b\rangle$. Thus the initial state for the total system is given by

$$\begin{aligned} |\Psi(0)\rangle &= |\Psi_{A,B}(0)\rangle \otimes |g_{2A}\rangle \otimes |0_a\rangle \otimes |0_b\rangle \\ &= (\cos \alpha |e_{1A} g_B\rangle + \sin \alpha |g_{1A} e_B\rangle) \\ &\otimes |g_{2A}\rangle \otimes |0_a\rangle \otimes |0_b\rangle \\ &= \cos \alpha |e_1 g_2; 0\rangle_A \otimes |g; 0\rangle_B + \sin \alpha |g_1 g_2; 0\rangle_A \otimes |e; 0\rangle_B \end{aligned} \quad (3)$$

Under these assumptions, there is never more than one photon in each cavity; therefore, the cavities a and b both behave like two-level systems and the total system will behave like a system which consists of five qubits.

In (1), the terms of Tavis-Cummings subsystem are $|e_1 g_2; 0\rangle$ and $|g_1 g_2; 0\rangle$. The state $|e_1 g_2; 0\rangle$ is coupled to the states $|g_1 g_2; 1\rangle$ and $|g_1 e_2; 0\rangle$, but the state $|g_1 g_2; 0\rangle$ is coupled only to itself.

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If we write the Hamiltonian for subsystem A in these states, we obtain

$$H^{(A)} = \begin{pmatrix} 0 & \lambda_{TC} & \lambda_{TC} & 0 \\ \lambda_{TC} & 0 & 0 & 0 \\ \lambda_{TC} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega \end{pmatrix} \quad (4)$$

The eigenvalues of these Hamiltonian are

$$E_1 = 0$$

$$E_2 = -\omega$$

$$E_3 = \lambda\sqrt{2}$$

$$E_4 = -\lambda\sqrt{2}$$

and the correlated eigenstates are

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{\sqrt{2}}(|e_1g_2;0\rangle - |g_1e_2;0\rangle) \\ |\psi_2\rangle &= |g_1g_2;0\rangle \\ |\psi_3\rangle &= \frac{1}{\sqrt{2}}|g_1g_2;1\rangle + \frac{1}{2}|e_1g_2;0\rangle + \frac{1}{2}|g_1e_2;0\rangle \\ |\psi_4\rangle &= \frac{1}{\sqrt{2}}|g_1g_2;1\rangle - \frac{1}{2}|e_1g_2;0\rangle - \frac{1}{2}|g_1e_2;0\rangle \end{aligned} \quad (6)$$

For using in next section:

$$\begin{aligned} |g_1g_2;1\rangle &= \frac{1}{\sqrt{2}}|\psi_3\rangle + \frac{1}{\sqrt{2}}|\psi_4\rangle \\ |e_1g_2;0\rangle &= \frac{1}{\sqrt{2}}|\psi_1\rangle + \frac{1}{2}|\psi_3\rangle - \frac{1}{2}|\psi_4\rangle \\ |g_1e_2;0\rangle &= -\frac{1}{\sqrt{2}}|\psi_1\rangle + \frac{1}{2}|\psi_3\rangle - \frac{1}{2}|\psi_4\rangle \\ |g_1g_2;0\rangle &= |\psi_2\rangle \end{aligned} \quad (7)$$

From ref[6] for Jaynes-Cummings system:

$$\begin{aligned} |\phi_0\rangle &= |g;0\rangle \\ |\phi_1^+\rangle &= c_0|e;0\rangle + s_0|g;1\rangle \\ |\phi_1^-\rangle &= -s_0|e;0\rangle + c_0|g;1\rangle \end{aligned} \quad (8)$$

where $\lambda^\pm = \omega_{JC} \pm \lambda_{JC}$ are eigenvalues for these eigenstates.

To use in section 2 :

$$\begin{aligned} |e;0\rangle &= c_0|\phi_1^+\rangle - s_0|\phi_1^-\rangle \\ |g;1\rangle &= s_0|\phi_1^+\rangle + c_0|\phi_1^-\rangle \\ |g;0\rangle &= |\phi_0\rangle \end{aligned} \quad (9)$$

for the resonance case (The case which will be used in this paper)

$$c_0 = s_0 = \frac{1}{\sqrt{2}} \quad (10)$$

Now, we are ready to study the entanglement in our model.

2. Pairwise Concurrences between each Two Qubits

We begin with relation (2), to obtain the time-evolution of entanglement, we first rewrite the initial state by using the eigenstates (6) and (8)

$$\begin{aligned} |\Psi(0)\rangle &= (\cos\alpha|e_{1A}g_B\rangle + \sin\alpha|g_{1A}e_B\rangle) \\ &\otimes |g_{2A}\rangle \otimes |0_a\rangle \otimes |0_b\rangle \\ &= \cos\alpha|e_1g_2;0\rangle_A \otimes |g;0\rangle_B + \sin\alpha|g_1g_2;0\rangle_A \otimes |e;0\rangle_B \end{aligned} \quad (5)$$

then

$$\begin{aligned} |\Psi(0)\rangle &= \cos\alpha \left[\frac{1}{\sqrt{2}}|\psi_1\rangle_A + \frac{1}{2}|\psi_3\rangle_A - \frac{1}{2}|\psi_4\rangle_A \right] \otimes |\phi_0\rangle_B \\ &+ \sin\alpha |\psi_2\rangle_A \otimes \left[\frac{1}{\sqrt{2}}|\phi_1^+\rangle_B - \frac{1}{\sqrt{2}}|\phi_1^-\rangle_B \right] \end{aligned} \quad (11)$$

The time-evolution of these eigenstates are specified as

$$\begin{aligned} |\psi_i(t)\rangle &= e^{-iE_it} |\psi_i\rangle \\ |\phi_1^\pm(t)\rangle &= e^{-i\lambda_\pm t} |\phi_1^\pm\rangle \end{aligned} \quad (12)$$

Then

$$\begin{aligned} |\Psi(t)\rangle &= \cos\alpha \left[\frac{1}{\sqrt{2}}|\psi_1(t)\rangle_A + \frac{1}{2}|\psi_3(t)\rangle_A - \frac{1}{2}|\psi_4(t)\rangle_A \right] \\ &+ \sin\alpha |\psi_2(t)\rangle_A \otimes \left[\frac{1}{\sqrt{2}}|\phi_1^+(t)\rangle_B - \frac{1}{\sqrt{2}}|\phi_1^-(t)\rangle_B \right] \end{aligned} \quad (13)$$

We transfer the time-dependent of these states to new coefficients and write the $|\Psi(t)\rangle$ as

$$\begin{aligned} |\Psi(t)\rangle &= \cos\alpha [a_1(t)|\psi_1\rangle_A + a_3(t)|\psi_3\rangle_A + a_4(t)|\psi_4\rangle_A] \\ &\otimes |\phi_0\rangle_B \\ &+ \sin\alpha |\psi_2\rangle_A \otimes [c(t)|\phi_1^+\rangle_B + s(t)|\phi_1^-\rangle_B] \end{aligned} \quad (14)$$

where

$$\begin{aligned} a_1(t) &= \frac{1}{\sqrt{2}} e^{-i(E_1 + \frac{1}{2}\omega_{JC})t} \\ a_3(t) &= \frac{1}{2} e^{-i(E_3 + \frac{1}{2}\omega_{JC})t} \\ a_4(t) &= -\frac{1}{2} e^{-i(E_4 + \frac{1}{2}\omega_{JC})t} \\ c(t) &= \frac{1}{\sqrt{2}} e^{-i(\lambda^+ + \omega_{TC})t} \\ s(t) &= -\frac{1}{\sqrt{2}} e^{-i(\lambda^- + \omega_{TC})t} \end{aligned} \quad (15)$$

are the time-dependent coefficients. Obviously, these

time-dependent coefficient satisfy the normalization condition.

In order to take traces over individual atoms or cavities, we need to revert to the bare bases and this leads to

$$\begin{aligned} |\Psi(t)\rangle = & \alpha_1(t)|e_1g_2;0\rangle_A \otimes |g;0\rangle_B + \alpha_2(t)|g_1e_2;0\rangle_A \otimes |g;0\rangle_B \\ & + \alpha_3(t)|g_1g_2;0\rangle_A \otimes |g;0\rangle_B + \alpha_4(t)|g_1g_2;0\rangle_A \otimes |e;0\rangle_B \\ & + \alpha_5(t)|g_1g_2;0\rangle_A \otimes |g;1\rangle_B \end{aligned} \quad (16)$$

The time-dependent coefficients are

$$\begin{aligned} \alpha_1(t) &= \left(\frac{1}{\sqrt{2}}a_1(t) + \frac{1}{2}a_3(t) - \frac{1}{2}a_4(t) \right) \cos \alpha \\ \alpha_2(t) &= \left(-\frac{1}{\sqrt{2}}a_1(t) + \frac{1}{2}a_3(t) - \frac{1}{2}a_4(t) \right) \cos \alpha \\ \alpha_3(t) &= \left(\frac{1}{\sqrt{2}}a_3(t) + \frac{1}{\sqrt{2}}a_4(t) \right) \cos \alpha \\ \alpha_4(t) &= \left(\frac{1}{\sqrt{2}}c(t) - \frac{1}{\sqrt{2}}s(t) \right) \sin \alpha \\ \alpha_5(t) &= \left(\frac{1}{\sqrt{2}}c(t) + \frac{1}{\sqrt{2}}s(t) \right) \sin \alpha \end{aligned} \quad (17)$$

Now, we are going to calculate the pairwise concurrences among qubits.

We first focus on C^{A_1B} , for this purpose, firstly, we must calculate the reduced matrix ρ^{A_1B} . The projections that are needed are

$$\begin{aligned} {}_A\langle g_2;0| \otimes {}_B\langle 0| \Psi(t) \rangle &= \alpha_1(t)|e_{1A}g_B\rangle + \alpha_4(t)|g_{1A}e_B\rangle \\ {}_A\langle g_2;0| \otimes {}_B\langle 1| \Psi(t) \rangle &= \alpha_5(t)|g_{1A}g_B\rangle \\ {}_A\langle g_2;1| \otimes {}_B\langle 1| \Psi(t) \rangle &= 0 \\ {}_A\langle g_2;1| \otimes {}_B\langle 0| \Psi(t) \rangle &= \alpha_3(t)|g_{1A}g_B\rangle \\ {}_A\langle e_2;0| \otimes {}_B\langle 0| \Psi(t) \rangle &= \alpha_2(t)|g_{1A}g_B\rangle \\ {}_A\langle e_2;0| \otimes {}_B\langle 1| \Psi(t) \rangle &= 0 \\ {}_A\langle e_2;1| \otimes {}_B\langle 0| \Psi(t) \rangle &= 0 \\ {}_A\langle e_2;1| \otimes {}_B\langle 1| \Psi(t) \rangle &= 0 \end{aligned} \quad (18)$$

From these projections, we see that A_1B mixed state has the form

$$\rho^{A_1B} = \begin{pmatrix} \nu & 0 & 0 & 0 \\ 0 & w & z & 0 \\ 0 & z^* & x & 0 \\ 0 & 0 & 0 & y \end{pmatrix} \quad (19)$$

For which the concurrence has the stated form[23]

$$C^{A_1B} = 2 \max \{0, |z| - \sqrt{wy}\} \quad (20)$$

where

$$z = \alpha_1(t)\alpha_4^*(t)$$

$$\nu = 0 \quad (21)$$

$$y = |\alpha_2(t)|^2 + |\alpha_3(t)|^2 + |\alpha_5(t)|^2$$

Because $\nu y = 0$ then

$$C^{A_1B} = 2|\alpha_1(t)\alpha_4^*(t)| \quad (22)$$

Figures 1 and 2 show the concurrence for A_1B mixed state. In this graphs $\omega_{TC} = \lambda_{TC} = \omega_{JC} = \lambda_{JC} = 1$

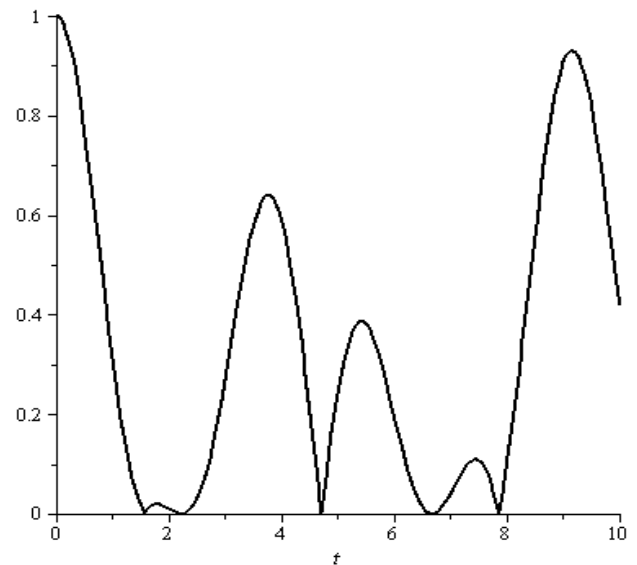


Figure 1. plot of A_1B concurrence for $\alpha = \frac{\pi}{4}$

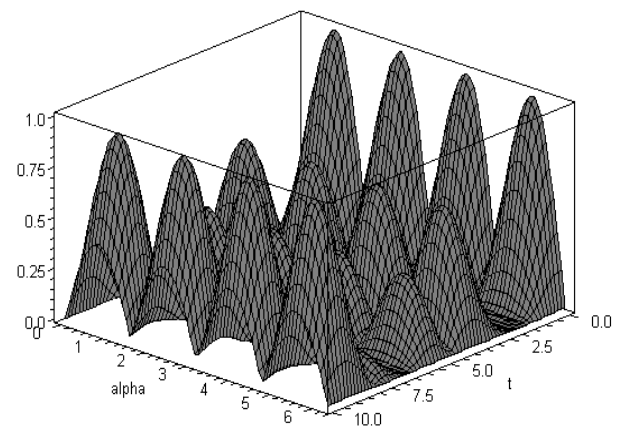


Figure 2. Surface plot of A_1B concurrence

As we expected from the initial state (3) for the case ($\alpha = \frac{\pi}{4}$) the concurrence at $t=0$ is equals to 1 that we can see in figure 1.

For concurrence calculation C^{A_2b} , like C^{A_1B} , we first must calculate ρ^{A_2b} . The non-zero projection are

$$\begin{aligned}
{}_A \langle e_1; 0 | \otimes {}_B \langle g | \Psi(t) &= \alpha_1(t) | g_{2A}; 0_b \rangle \\
{}_A \langle g_1; 0 | \otimes {}_B \langle e | \Psi(t) &= \alpha_4(t) | g_{2A}; 0_b \rangle \\
{}_A \langle g_1; 0 | \otimes {}_B \langle g | \Psi(t) &= \alpha_2(t) | e_{2A}; 0_b \rangle + \alpha_5(t) | g_{2A}; 1_b \rangle \\
{}_A \langle g_1; 1 | \otimes {}_B \langle g | \Psi(t) &= \alpha_3(t) | g_{2A}; 0_b \rangle
\end{aligned} \quad (23)$$

Again the decreasing density matrix has the form (19) with

$$\begin{aligned}
z &= \alpha_2(t) \alpha_5^*(t) \\
v &= 0
\end{aligned} \quad (24)$$

then

$$C^{A_2b} = 2 |\alpha_2(t) \alpha_5^*(t)| \quad (25)$$

is the concurrence between the initially non-entangled atom A_2 and the cavity of subsystem B. (Figures 3-4)

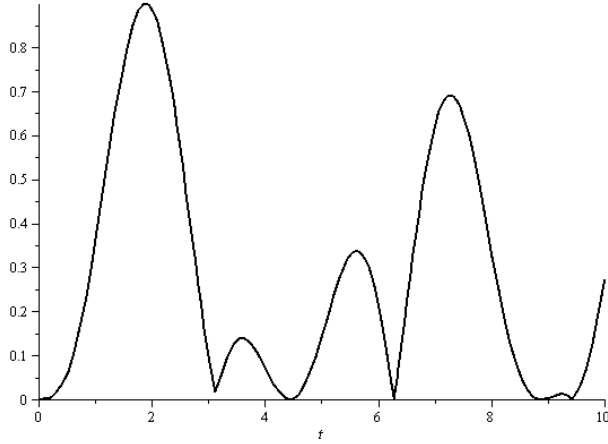


Figure 3. The plot of $C^{A_2b}(t)$ for $\alpha = \frac{\pi}{4}$

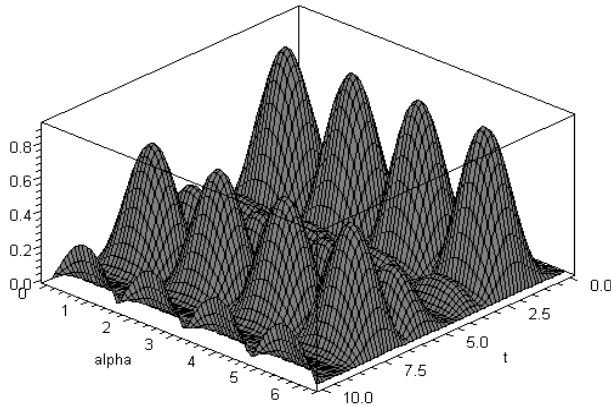


Figure 4. The surface plot of $C^{A_2b}(t, \alpha)$

For obtaining concurrence between two cavities:

$$\begin{aligned}
{}_A \langle e_1 g_2 | \otimes {}_B \langle g | \Psi(t) &= \alpha_1(t) | 0_a, 0_b \rangle \\
{}_A \langle g_1 e_2 | \otimes {}_B \langle g | \Psi(t) &= \alpha_2(t) | 0_a, 0_b \rangle \\
{}_A \langle g_1 g_2 | \otimes {}_B \langle e | \Psi(t) &= \alpha_4(t) | 0_a, 0_b \rangle \\
{}_A \langle g_1 g_2 | \otimes {}_B \langle g | \Psi(t) &= \alpha_3(t) | 1_a, 0_b \rangle + \alpha_5(t) | 0_a, 1_b \rangle
\end{aligned} \quad (26)$$

and the concurrence between two cavities becomes (Figures 5-6)

$$C^{ab} = 2 |\alpha_3(t) \alpha_5^*(t)| \quad (27)$$

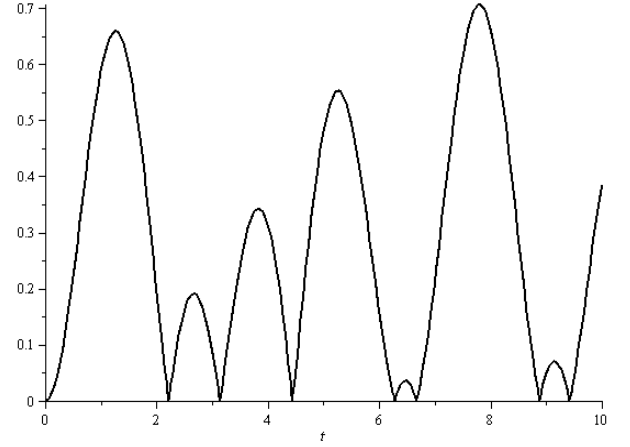


Figure 5. The plot of $C^{ab}(t)$ for $\alpha = \frac{\pi}{4}$

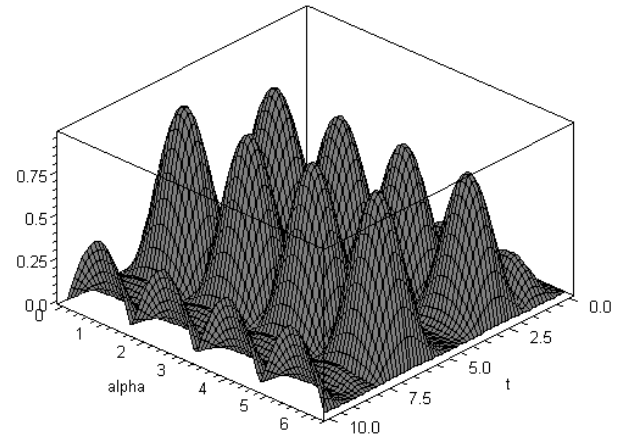


Figure 6. The surface plot of $C^{ab}(t, \alpha)$

Other remote concurrences between two subsystems are

$$\begin{aligned}
C^{A_1b}(t) &= 2 |\alpha_1(t) \alpha_5^*(t)| \\
C^{A_2B}(t) &= 2 |\alpha_2(t) \alpha_4^*(t)| \\
C^{aB}(t) &= 2 |\alpha_3(t) \alpha_4^*(t)|
\end{aligned} \quad (28)$$

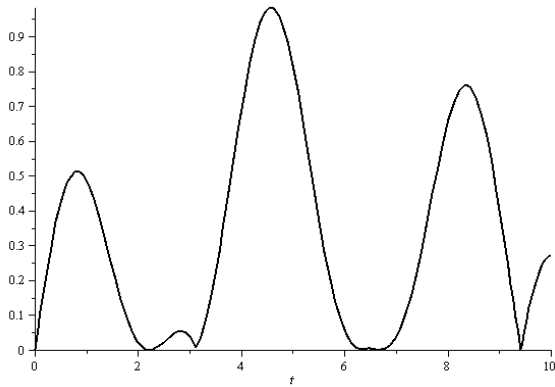


Figure 7. The plot of $C^{A_1 b}(t)$ for $\alpha = \frac{\pi}{4}$

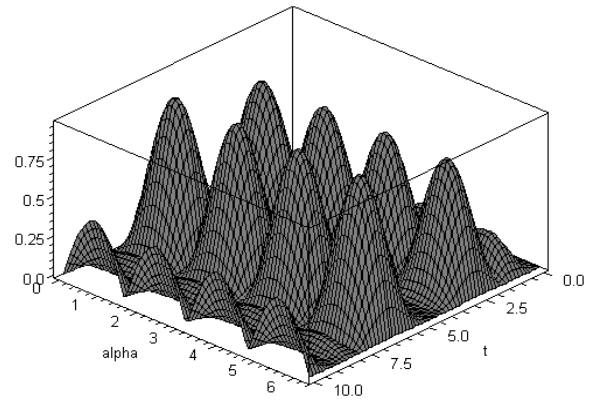


Figure 10. The surface plot of $C^{A_2 B}(t, \alpha)$

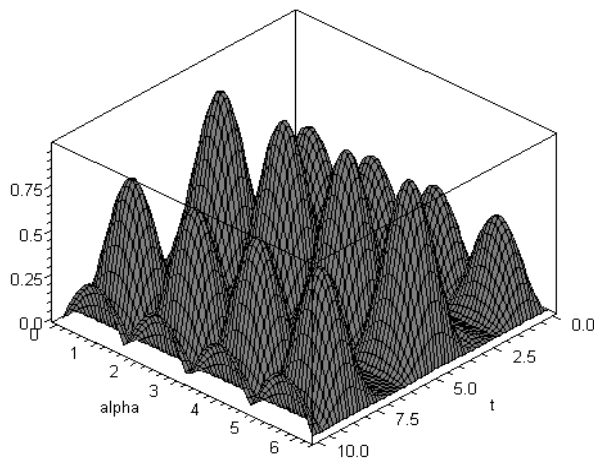


Figure 8. The surface plot of $C^{A_1 b}(t, \alpha)$

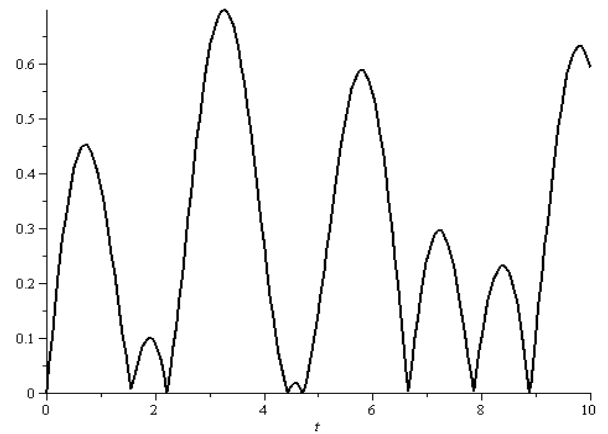


Figure 11. The plot of $C^{aB}(t)$

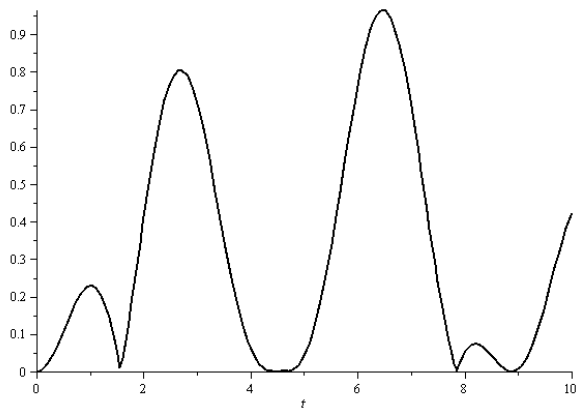


Figure 9. The plot of $C^{A_2 B}(t)$ for $\alpha = \frac{\pi}{4}$

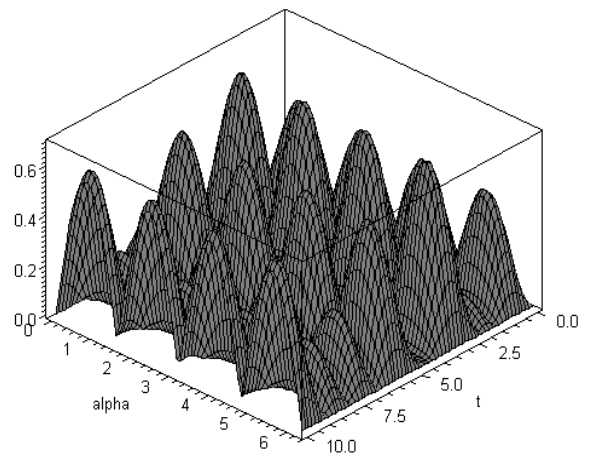


Figure 12. The surface plot of $C^{aB}(t, \alpha)$

Similarly, we obtain

$$\begin{aligned} C^{A_1 A_2}(t) &= 2 \left| \alpha_1(t) \alpha_2^*(t) \right|, \quad C^{A_1 A_a}(t) = 2 \left| \alpha_1(t) \alpha_3^*(t) \right| \\ C^{A_2 A_a}(t) &= 2 \left| \alpha_2(t) \alpha_3^*(t) \right|, \quad C^{B B_b}(t) = 2 \left| \alpha_4(t) \alpha_5^*(t) \right| \end{aligned} \quad (29)$$

Figures (1-12) show that, the local atom-cavity coupling not only cause the creation of entanglement between an atom and its cavities (or between two atoms in Tavis-Cummings system), but also generate non-local entanglement between cavities (Figure 5 and 6) or between initially non-entangled atom A_2 with Jaynes-Cummings cavity (Figure 3 and 4) if the atom-atom subsystem (A_1 and B) is initially prepared in an entangled state (2). We see also the creation of non-local entanglement between one atom and remote cavity mode (Figures 7-8 and 11-12). In next section we analyze the entanglement between each atom with its cavity for each individual systems Jaynes-Cummings and Tavis-Cummings as two special cases in our work.

3. Special Cases

In this section, we want to discuss about two special cases. In each of these cases, the number of qubits decreases and qubits are not entangled initially. As time goes, these qubits be entangled with each other for the interaction between them.

I) The case $\omega_{TC} = \lambda_{TC} = 0$

In this case, we only have a Jaynes-Cummings system, so our model becomes a two qubit model. If $\alpha = 0$ then

$$|\Psi(0)\rangle = |g; 0\rangle_B \quad (30)$$

Because this state couples only with itself, there is no Entanglement.

If $\alpha = \frac{\pi}{2}$ then

$$|\Psi(0)\rangle = |e; 0\rangle_B \quad (31)$$

And We can calculate the concurrence by helping (28). (Figure 13)

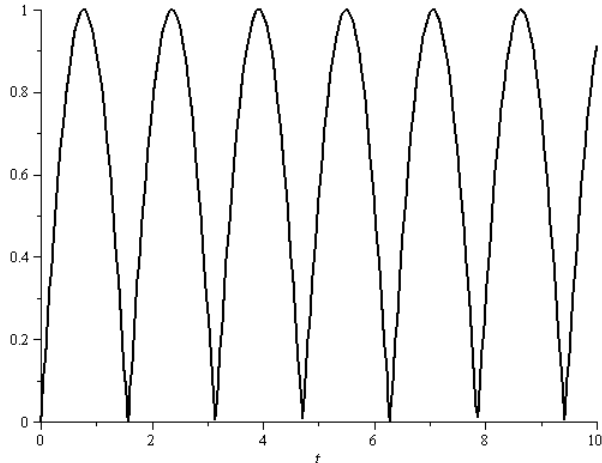


Figure 13. Plot of $C^{A_2 a}(t)$ for initial state $|\Psi(0)\rangle = |e_1 g_2; 0\rangle_A$

II) The case $\omega_{JC} = \lambda_{JC} = 0$

Now, we have a three qubit model. for $\alpha = 0$

$$|\Psi(0)\rangle = |e_1 g_2; 0\rangle_A \quad (32)$$

And the concurrences for this case is calculated helping (29).

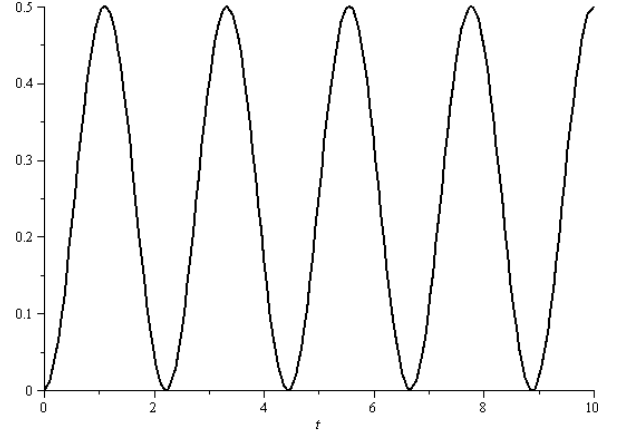


Figure 14. Plot of $C^{A_1 A_2}(t)$ for initial state $|\Psi(0)\rangle = |e_1 g_2; 0\rangle_A$

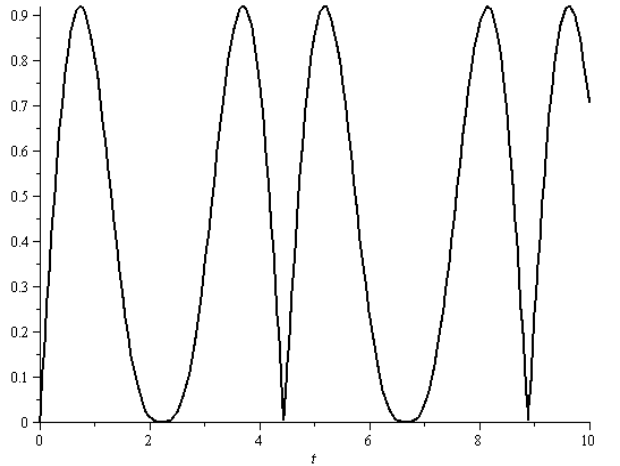


Figure 15. Plot of $C^{A_1 a}(t)$ for initial state $|\Psi(0)\rangle = |e_1 g_2; 0\rangle_A$

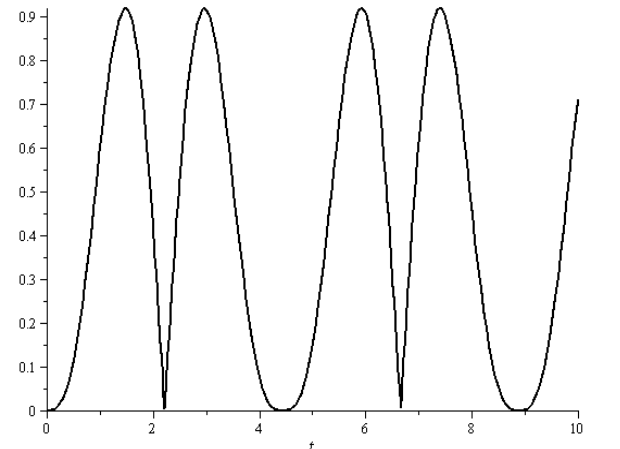


Figure 16. Plot of $C^{A_2 a}(t)$ for initial state $|\Psi(0)\rangle = |e_1 g_2; 0\rangle_A$

$$\text{For } \alpha = \frac{\pi}{2}, |\Psi(0)\rangle = |g_1 g_2; 0\rangle_A \quad (33)$$

Because this state couples only with itself, there is no Entanglement in this condition.

Figures 13-16 show that in both Jaynes-Cummings and Tavis-Cummings for interaction which there are in these systems the atom-cavity coupling cause the entanglement among qubits in each system separately.

4. Conclusions

We have examined Entanglement between two subsystems A and B. A is a two-atoms Tavis-Cummings system and B is a Jaynes-Cummings system. In our model, a distinctive feature is that there are no interaction between these two subsystems, but one atom from A is entangled with the Jaynes-Cummings atom B the case that makes sure us subsystems A and B are physically correlated. We show that initially non-entangled atoms or cavities become entangled with each other by passing time. The process which will be interpreted as entanglement transfer. The figures show the creation of entanglement among non-located qubits. It is a surprising point in quantum computing which shows a fundamental distinction between quantum and classical world.

If $A_1 \rightarrow 1, A_2 \rightarrow 2, a \rightarrow 3, B \rightarrow 4, b \rightarrow 5$

then in our notation

$$\rho^{ij}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |\alpha_i(t)|^2 & \alpha_i(t)\alpha_j^*(t) & 0 \\ 0 & \alpha_i^*(t)\alpha_j(t) & |\alpha_j(t)|^2 & 0 \\ 0 & 0 & 0 & 1 - |\alpha_i(t)|^2 - |\alpha_j(t)|^2 \end{pmatrix} \quad (34)$$

And we can write pairwise concurrences as the compact form

$$C^{ij}(t) = 2|\alpha_i(t)\alpha_j^*(t)| \quad (35)$$

Because $\nu_j = 0$ for all matrices, there is no negative term in (34). For this reason there are no ESD (Entanglement Sudden death) between qubits for this case of initial state which be used in our work.

The important point, which must be noted, is that the initial state has essential role for building the five qubit model. If we use another initial state, firstly we must make sure that the cavities are behaved like two stated systems; furthermore, the most important point is that to make sure that initial state must provides correlation between subsystems A and B. For example, if in the initial state (2)

the non-entangled atom was in its excited state $|e_2\rangle_A$ the total system will not behave like a five qubit system.

However, there are other initial states that if we are used then they will behave like a five qubit model. For example we can test

$$|\Psi(0)\rangle = (\cos\alpha|e_{1A}e_B\rangle + \sin\alpha|g_{1A}g_B\rangle) \otimes |g_2\rangle_A \otimes |0_a\rangle \otimes |0_b\rangle \quad (36)$$

and solve the problem with the method which we have used.

REFERENCES

- [1] McMahon, D.M., *Quantum computing explained*. 2008: Wiley-IEEE Computer Society Pr.
- [2] Hill, S. and W.K. Wootters, *Entanglement of a pair of quantum bits*. Physical Review Letters, 1997. 78(26): p. 5022-5025.
- [3] Latorre, J. and A. Riera, *A short review on entanglement in quantum spin systems*. Journal of Physics A: Mathematical and Theoretical, 2009. 42: p. 504002.
- [4] Amico, L., et al., *Entanglement in many-body systems*. Reviews of Modern Physics, 2008. 80(2): p. 517.
- [5] Chen, Q.H., et al., *Entanglement dynamics of two independent Jaynes-Cummings atoms without the rotating-wave approximation*. Physical Review A, 2010. 82(5): p. 052306.
- [6] Yönaç, M., T. Yu, and J. Eberly, *Pairwise concurrence dynamics: a four-qubit model*. Journal of Physics B: Atomic, Molecular and Optical Physics, 2007. 40: p. S45.
- [7] Ghoreishi, S.A., M. Sarbishaei, and K. Javidan, *Entanglement between Two Tavis-Cummings Systems with $N=2$* . International Journal of Theoretical and Mathematical Physics, 2012. 2(6): p. 187-195.
- [8] Yönaç, M., T. Yu, and J. Eberly, *Sudden death of entanglement of two Jaynes-Cummings atoms*. Journal of Physics B: Atomic, Molecular and Optical Physics, 2006. 39: p. S621.
- [9] Zhang, G. and X. Xie, *Entanglement between two atoms in a damping Jaynes-Cummings model*. Eur. Phys. J. D, 2010. 60: p. 423-427.
- [10] Tavis, M. and F.W. Cummings, *Exact solution for an N -molecule-radiation-field Hamiltonian*. Physical Review, 1968. 170: p. 379-384.
- [11] Tessier, T., et al., *Entanglement sharing in the two-atom Tavis-Cummings model*. Physical Review A, 2003. 68(6): p. 062316.
- [12] Fujii, K., et al., *Explicit Form of Solution of Two Atoms Tavis-Cummings Model*. Arxiv preprint quant-ph/0403008, 2004.
- [13] Fujii, K., et al., *Quantum Diagonalization Method in the Tavis-Cummings Model*. Arxiv preprint quant-ph/0410003, 2004.
- [14] Reslen, J. and L. Quiroga, *ENTANGLEMENT IN THE DICKE MODEL*. REVISTA COLOMBIANA DE FÍSICA, 2004. 36(2).
- [15] Zhang, G. and Z. Chen, *The entanglement character between atoms in the non-degenerate two photons Tavis-Cummings model*. Optics communications, 2007. 275(1): p. 274-277.

- [16] Lian, J., Y. Zhang, and J.Q. Liang, *Macroscopic Quantum States and Quantum Phase Transition in Dicke Models of Arbitrary Atom-Number*. Arxiv preprint arXiv:1110.3862, 2011.
- [17] Garraway, B.M., *The Dicke model in quantum optics: Dicke model revisited*. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 2011. 369(1939): p. 1137-1155.
- [18] Keeling J., *Quantum corrections to the semiclassical collective dynamics in the Tavis-Cummings model*. Physical Review A, 2009. 79(5): p. 053825.
- [19] López, C., et al., *Entanglement properties in the inhomogeneous Tavis-Cummings model*. Arxiv preprint quant-ph/0611045, 2006.
- [20] Gerry, C.C., P.L. Knight, and M. Beck, *Introductory Quantum Optics*. American Journal of Physics, 2005. 73: p. 1197.
- [21] Wootters, W.K., *Entanglement of formation of an arbitrary state of two qubits*. Physical Review Letters, 1998. 80(10): p. 2245-2248.
- [22] O'Connor, K.M. and W.K. Wootters, *Entangled rings*. Physical Review A, 2001. 63(5): p. 052302.