The Variable Rest Mass Interpretation of Gravitation

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Abstract Einstein’s theory of gravitation is presently interpreted assuming the general invariance of the rest mass of objects. This assumption is shown to be untenable by a straightforward argument proving that rest masses are reduced in a gravitational field. Indeed, it is argued that a change in the gravitational potential energy of a body resides in the body itself as a change in rest mass. Several important consequences follow regarding the very nature of the gravitational interaction – most notably that particles of zero rest mass, such as photons and gravitons, are not directly affected by a gravitational field, although they do respond to the altered geometry associated with the field. In cosmology, the variable rest mass concept enables a reconciliation of the observed red shift with the requirement of momentum conservation, which is demanded by the presumed large-scale homogeneity of the universe. These ideas and other developments show that the variable rest mass concept forms the basis for what may fairly be considered to be an important contribution to our understanding of gravitational physics.

Keywords Gravitation, Variable Rest Mass, Black Holes, Red Shift, Geodesics, Photons, Gravitons, Cosmology, Noether’s Theorem

1. Introduction

Part I deals with the gravitational field of stars and black holes. A simple thought experiment is presented proving that rest masses are reduced in a time-independent gravitational field. The strong equivalence principle together with simple quantum theory shows that the reduction of rest masses causes not only a slowing of clock rates, but also an increase in the size of all objects, including measuring rods. The currently accepted (proper) metric thus underestimates both time and space intervals. An alternate metric correcting these deficiencies is introduced, revealing the true structure of spacetime in the Schwarzschild gravitational field. In particular, light is shown to move with constant momentum along the spatial geodesics of the new metric, indicating that gravity does not couple to the electromagnetic field.

Part II deals with cosmology. Arguments presented in Part I establish the fact that rest masses are reduced in a gravitational field. This suggests that the concept of variable rest mass may play a significant role in cosmology. Indeed, it turns out that the concept of evolving rest mass represents the only way in which the cosmological red shift can be consistent with the conservation of momentum, which is required by the presumed homogeneity of the universe.

It is concluded that the universe is not expanding, but rather that rest masses have been, and are, increasing as a simple exponential function of world time. Thus the red shift is not caused by the expansion of space itself; the accepted understanding, but instead results from the fact that the frequency of any particular quantum transition, being proportional to rest mass, was, in past eras, reduced as compared with the present value for the same transition.

Finally, it must be understood that a new theory of gravitation is not being proposed; everything is grounded in Einstein’s original theory. This paper simply develops an interpretation (which is actually inherent in Einstein’s theory) that enables a true understanding of the theory.

Part I: Stars & Black Holes

2. Rest Mass and Gravitational Potential Energy

The concept of rest mass reduction in a gravitational field is intuitive and easily understood: it is the literal embodiment of gravitational potential energy. When a mass is raised against gravity, the increase in potential energy, \( \Delta W \), is stored in the body itself as an increase in rest mass: \( \Delta m = \Delta W/c^2 \). But this relativistic concept dates only from the beginning of the twentieth century (Poincaré and Einstein). Maxwell and others, lacking this key connection between mass and energy, could only postulate that the energy must reside in the gravitational field. Awkwardly, this required that the gravitational field had to have a negative energy density. Maxwell refused to accept the idea, but the concept has persisted, and over time has become part
of the canon of physics, in spite of the fact that it was introduced ad hoc long ago and has no theoretical or experimental support. The very simple thought experiment described below will prove that rest masses are reduced in gravitational field, in conformity with the concept that the gravitational potential energy of a body resides in the body itself.

3. A Positronium thought Experiment

Consider the annihilation of a ground-state para-positronium system deep in a time-independent gravitational field. In a time-independent field, energy is conserved, implying that photons move upward with constant energy. Nevertheless, because of the gravitational red shift, the total energy of the photons emitted from the annihilation event will be measured by distant observers unaffected by the field to be reduced as compared with the rest mass energy of such a system measured locally by these observers. The unavoidable conclusion is that the rest mass of the positronium system was reduced by the action of the gravitational field.

4. The Strong Equivalence Principle

As formulated by R. H. Dicke[1; 4], the strong equivalence principle (SEP) states:

In a freely falling, non-rotating laboratory, the local laws of physics take on some standard form, including a standard numerical content, independent of the position of the laboratory in space and time.

This principle is routinely misunderstood as implying that nothing can change, whereas it only requires that any such changes occur in concert so as to be undetectable to observers in the laboratory. But clearly, the SEP does require that all physical quantities of the same dimensionality must vary, if they do in fact vary, in precisely the same manner.

We shall distinguish between true values and measured values: Thus regarding the red shift in a time-independent gravitational field, local observers, with their varying frequency standards, measure different values for the frequency of an ascending light ray, the frequency of which must be a true constant. In the conventional scheme ‘true’ values will be marked with a hat (^), while ‘true’ values in the Variable Rest Mass (VRM) scheme will be marked with an asterisk.

Thus the true characteristic time intervals associated with every physical object of non-zero rest mass must bear a constant ratio to the Rydberg period, \( P_R = h^3 / m_e c^4 \).

Likewise, the true length (breadth & height!) of every physical object of non-zero rest mass must bear a constant ratio to the Bohr radius, \( a_0 = h^2 / m_e c^2 \).

Thus the positronium thought experiment, together with the SEP, implies the every rest mass, including the electron rest mass, \( m_e \), will be reduced in a gravitational field.

5. Implications of Rest Mass Reduction

Rest mass reduction, by increasing the Rydberg period, is thus identified as the cause of the gravitational red shift, which is thereby finally understood. The concomitant increase in the Bohr radius implies a heretofore unsuspected phenomenon, a gravitational size dilation effect. Importantly, the length of any measuring rod is increased, implying that currently accepted geometry, which may be called proper geometry, is incorrect.

6. Regarding the Speed of Light

Note that according to the SEP, the speed of light should bear a constant ratio to the speed defined as \( a_0 / P_R = c^2 / h \); in other words, \( (c^2 / h) / \bar{c} \) must be a constant. But this is just the fine structure ‘constant’, which is observed to be truly constant, unaffected by gravity. Thus the presently accepted idea that the true speed of light is reduced in a gravitational field must be rejected.

7. The Variable Rest Mass Metric

Here it is appropriate to introduce a new metric, which corrects the proper metric for the two effects resulting from rest mass reduction. We restrict our attention to the Schwarzschild field. The proper metric may be written:

\[
\begin{align*}
d\tilde{s}^2 &= f^2c^2dt^2 - f^{-2}dr^2 - r^2[d\theta^2 + \sin^2 \theta d\phi^2] \\
f &= \sqrt{1 - r_s / r}, \quad r_s = 2GM / c^2
\end{align*}
\]

Since proper time intervals and distance intervals are underestimated by the same factor, \( f \), the correction is effected by simply multiplying by \( f^{-2} \): The VRM metric is thus

\[
ds^2 = c^2 dt^2 - \frac{1}{f^2} dr^2 - f^{-2} r^2 [d\theta^2 + \sin^2 \theta d\phi^2]
\]

Note that this does not represent a solution to the field equations: it simply introduces a system for the measurement of time and distance that is not influenced by the field. Time is measured using signals from a remote clock (the ‘clock at infinity’), while distances are measured by electromagnetic echo ranging calculated using the same time system.

Note that minimizing the distance between two points in space using the VRM spatial metric

\[
d\tilde{s} = f^{-4} dx^2 + f^{-2} r^2 [d\theta^2 + \sin^2 \theta d\phi^2]
\]

is identical to setting \( d\tilde{s} = 0 \) in the proper space-time metric and minimizing the world time for light to traverse the interval between the same two points, Thus light rays move along the spatial geodesics of the VRM metric.
8. Spatial Geodesics Defined
Kinematically: Optical Geometry

Abramowicz et al.[2] proved that in a static gravitational field a body will not experience velocity-dependent forces (centrifugal and Coriolis forces) if it is constrained to move along a path that a light ray might follow. By analogy to Newtonian mechanics, in which such forces vanish for bodies moving in straight lines, the authors defined a new geometry by identifying light rays as the geodesics (the nearest thing to a straight line in non-Euclidean geometry) of the new geometry. This new geometry they named, appropriately, ‘Optical Geometry’. It is identical to the VRM geometry that results from correcting for the elongation of measuring rods.

9. Gravity Does Not Couple to Photons

Proponents of the usual interpretation insist that the deflection of light rays indicates that photons are subject to gravity, but the arguments given above prove that photons are just following the true geodesics of the non-Euclidean geometry that gravity induces. Thus photons are not responding to the force of gravity per se (as proved by the fact that their momentum remains unchanged) but are only responding to the non-Euclidean geometry inherent in the gravitational field. It must be noted however that we are here referring to free radiation. Captive radiation, as for instance in the body of a star, exhibits an equivalent rest mass and does interact with the gravitational field.[3; 34,100]

Thus the gravitational field does not couple to the free electromagnetic field. This is a fact of inestimable importance in that it indicates that the basic, non-dynamical component of a gravitational field, if it is to be quantized, must be quantized with a spin-zero scalar field. This follows since, uniquely, spin-zero scalar fields couple only to the trace of the energy-momentum tensor of matter fields, and the trace of the energy-momentum tensor of the electromagnetic field vanishes identically.

It is noteworthy that when the straightforward procedure for quantizing a field is applied to the gravitational field, a mixture of spin-zero and spin-two gravitons emerge. Physicists, believing incorrectly that gravity couples with electromagnetic fields, have resorted to a rather contrived mathematical artifice to eliminate the spin-zero graviton.[4; 35-38].

10. Proper Metric Retained

It must be emphasized that it is not being suggested that the proper metric should be discarded. Certainly the proper metric correctly describes the dynamical behavior of non-zero rest mass matter, which the VRM metric does not describe. In fact, rest mass reduction is confirmed by an exact integral of motion[5; 292] derived in the proper metric of the Schwarzschild field, namely,

$$mc^2 f / \sqrt{1-v^2/c^2} = \text{const.}$$

The factor $f = \sqrt{1-r_s/r}$ is exactly the factor by which rest masses are reduced: $m^* = fm$.

The integral is thus identified as expressing the conservation of energy – rest mass energy being exchanged for kinetic energy and vice versa. This is consistent with the view that gravitational potential energy resides in the body itself as a change in rest mass. Finally, this, together with the fact that gravity does not couple to photons, indicates that gravity couples only to bodies of non-zero rest mass.

11. Black Hole Structure

Before considering how the true geometry of a black hole is revealed in the VRM interpretation, a review of Schwarzschild static black hole structure according to the mainstream interpretation seems appropriate. The basic structure is a singularity hidden behind a surface called the event horizon, from which the escape velocity is equal to the speed of light. This surface is very special in that light sent from a finite proper distance directly toward the event horizon never reaches that surface. Proponents of the accepted interpretation ‘explain’ that because of ‘the slowing of the flow of time itself’, the speed of light goes to zero as light approaches the event horizon, i.e., $\ddot{c} \to 0$ as $r \to r_s$.

Another oddity is that light does not follow the geodesics (shortest paths) of the proper geometry. Again, proponents ‘explain’ that light’s speed is higher along a path outside of the geodesic path, where there is less ‘slowing of the flow of time itself’. But, as previously noted, the claim that light’s speed is reduced in a gravitational field (for whatever reason) is untenable if the Strong Equivalence Principle is accepted.

According to the proper metric, every sphere centered on the black hole is convex when viewed from the outside, except for the event horizon itself, which, in the proper metric, appears to have zero curvature. Nevertheless, for any location inside the locus of photon orbits at $\frac{3}{2} r_s$, centrifugal force acts inwardly! Clearly, this phenomenon, the Abramowicz Effect[6], cannot be accounted for in the context of the proper geometry of the conventional interpretation.

The puzzling phenomena that appear in the conventional interpretation are easily understood in terms of the VRM interpretation. First of all, the geometry is very different. In the VRM metric, the area of a centered sphere is equal to

$$4\pi r^2 f^{-2} = 4\pi r^2 (1-r_s/r)^{-1},$$

and differentiating with respect to $r$, one has

$$d/dr [r^2 (1-r_s/r)^{-1}] = (2r-3r_s)(1-r_s/r)^{-2}.$$
of a wormhole-like structure connecting our familiar universe with another infinite three-space, which may be called ‘innerspace.’ In the VRM metric, the stenosphere has zero curvature, which explains why centrifugal force vanishes there. This vanishing, in turn, confirms the fact that photons do not feel the force of gravity, since on the stenosphere there is no centrifugal force to counter an imagined force of gravity.

Note that inside the stenosphere, the surface of a centered sphere, viewed from the ‘outside,’ will be concave rather than convex. Thus, regarding the Abramowicz Effect, the correct geometry shows that the direction of centrifugal force obeys the usual pattern inside the stenosphere: the force is directed from the concave side to the convex side of the circle on which a body is constrained to move.

Regarding the other puzzles, it is obvious that light cannot reach the event horizon since that surface is infinitely distant in ‘innerspace’ (the seeming finite proper distance to that surface is an artifact of the limitless elongation of measuring rods as $r \rightarrow r_3$).

That proper geodesics lie inside VRM geodesics is easily understood; proper geodesics ‘cheat’ by taking advantage of the elongation of measuring rods implied by the gravitational size dilation effect. Another significant feature of the VRM interpretation is the non-existence of the baleful singularity that seemingly lurks behind the event horizon in the usual interpretation.

12. Conclusions for Part I

The variable rest mass concept as it applies to the gravitational field of stars and black holes provides many valuable insights, most notably, that gravity couples only to bodies of non-zero rest mass. It follows as a corollary that only such bodies act as sources of gravitational fields. Thus the hypothesized graviton, being long range and therefore massless, will not act as a gravitational source. The quantum theory of gravity should therefore be quite different from, and much simpler than, the non-linear nightmare that one would expect on the basis of the usual assumption that all forms of energy act as sources of gravitational fields. Also, since light is not directly affected by gravity, the non-dynamic part of the gravitational field must be quantized with a zero-spin scalar graviton. Viewed optimistically, it would seem that one may hope that the VRM interpretation of gravity as it applies to stars and black holes will in time be accepted.

Part II: Cosmology

13. The Standard Interpretation of the Cosmological Red Shift

Hubble’s discovery of the systematic cosmological red shift immediately suggested that galaxies were flying through space away from us and from one another – the further, the faster. This in turn suggested that the universe grew from an incredibly hot and dense condition (a singularity!) billions of years ago. Later, solutions of the field equations revealed that the galaxies were not actually moving through space, but rather that space itself was expanding. The conventional understanding holds that the wavelength of light is continually stretched in flight by the expansion of space. Thus the redshift, $(z + 1)$, shown by a galaxy is held to be proportional to the ratio of the scale of the present universe to that of the universe at the time of emission.

Neither astronomers nor cosmologists seem to be concerned with the fact that this explanation of the cosmological red shift violates a very deep principle of mathematical physics, namely, Noether’s theorem. Observation shows that, on a sufficiently large scale, the universe is homogeneous and isotropic to a very high degree, and virtually every cosmological model assumes this at the outset. But according to Noether’s theorem, homogeneity implies that momentum must be conserved. Perhaps no one even considered this problem since, as every observation appears to have demonstrated, momentum seems to be manifestly not conserved.

14. The Variable Rest Mass Interpretation of the Cosmological Red Shift

The microwave photons of the cosmical background radiation field that we detect today were born in 3000$^\circ$K hydrogen-helium plasma at ‘recombination’ time, when the plasma first became transparent. Their wavelength has seemingly increased by a factor of about 1000 – how is it possible that their momentum has not changed? The only possibility is that over the aeons, measuring instruments have changed, and are changing, decreasing their characteristic wavelengths – for example, diffraction gratings have shrunk and are shrinking. And the only way this might occur is if all rest masses have been and are increasing in proportion to $a(t) = A(t)/\bar{A}$, in which $A(t)$ is the function that is presently interpreted as representing the increasing scale of the universe, and $\bar{A}$ is its present value.

To prove that momentum conservation requires rest masses to increase in proportion to $a(t)$, one need only consider the motion of a test mass through space. In this case, there is an easy integral of motion for the Robertson-Walker metric:

$$a(t)\beta / \sqrt{1 - \beta^2} = \text{const.}[3; 77].$$

Since the momentum is just $m^* c \beta / \sqrt{1 - \beta^2}$, it is clear that momentum conservation requires that the true rest mass, $m^*$, must be proportional to $a(t)$, that is, $m^* = ma(t)$, where $m$ is the proper (constant) rest mass.

A much more elaborate calculation is required to solve the field equations for cosmology under the assumption that
momentum is conserved[3; 71-76]. The surprising result is that \( a(t) \) turns out to be a simple exponential function of world time. But upon thoughtful reflection, it is obvious (in hindsight) that such must be the case, as will now be shown.

15. Momentum Conservation, of Itself, Implies that Rest Masses Increase as an Exponential Function of Time

Consider a source of light and two observers lying on a straight line at distances \( d_1 \) and \( d_2 > d_1 \) from the source. Generally, \( \lambda_{obs} = \lambda_{std} a (t_{obs}) / a(t_{emit}) \)

Let \( \lambda_1 \) & \( \lambda_2 \) be the wavelengths observed at \( d_1 \) & \( d_2 \).

In the case considered, one may write \( \lambda_2 = \lambda_{std} a(t_2) = \lambda_{std} a(t_1 + (d_2 - d_1) / c) \).

Momentum conservation implies that wavelengths of light remain constant in traveling between any two points. If that is so, the spectra measured at \( d_2 \) will differ from that observed at \( d_1 \) only as a result of the changes in the instruments at \( d_1 \) caused by the increase in rest masses during the time interval \( (d_2 - d_1) / c \). And of course, the same is true for the time interval \( t_2 = d_1 / c \). Thus the function on the right hand side above must be representable as the product of some function of \( d_1 / c \) and the same function of \( (d_2 - d_1) / c \).

Thus

\[
a[d_1 / c + (d_2 - d_1) / c] = f(d_1 / c) \cdot f[(d_2 - d_1) / c]
\]

But when \( d_2 = d_1 \), we must have \( f = a \) thus the function \( a \) satisfies the functional equation

\[
a(x + y) = a(x) \cdot a(y).
\]

Only the exponential function satisfies this equation. Thus we conclude that

\[
a(t) \approx \exp[\omega(t - \tilde{t})].
\]

16. The Mass Evolution Function

Let the true rest mass, \( m^* \) to be equal to the nominal proper value, \( m \), at the present time, \( \tilde{t} : \)

\[
m^* = m a(t) = m \exp[\omega(t - \tilde{t})]
\]

Regarding the evolution of proper time,

\[
dT = (m^* / m) dt = \exp[\omega(t - \tilde{t})] dt,
\]

\[
T = \int_{\tilde{t}}^{t} \exp[\omega(t' - \tilde{t})] dt' = \omega^{-1} \exp[\omega(t - \tilde{t})]
\]

\[
= \omega^{-1} a(t) \quad \Rightarrow \quad a = \omega T
\]

Thus rest mass evolves as a simple linear function of proper time.

The present proper age of the universe is just the present proper time

\[
\tilde{T} = \int_{\tilde{t}}^{t} \exp[\omega(t' - \tilde{t})] dt' = \omega^{-1}
\]

Regarding the cosmic background radiation emitted at 'recombination' time, \( T_{rec} \), we know that

\[
(\lambda_{obs} / \lambda_{std})_{rc} = (m / m_{rc}^*) \approx 10^3, \text{ and hence } \tilde{T} - T_{rc} = \omega^{-1} \ln(10^3) \approx 6.908 \omega^{-1}
\]

The proper time interval from the ‘big bang’ to ‘recombination’ is

\[
T_{rec} = \int_{\tilde{t}}^{t} \exp[\omega(t' - \tilde{t})] dt' = \omega^{-1} \exp[-\omega(\tilde{T} - T_{rc})]
\]

\[
= \omega^{-1} \exp[-\ln(10^3)] = 10^{-3} \omega^{-1}
\]

The proper time interval between the ‘recombination’ event and the present is

\[
\tilde{T} - T_{rc} = (1 - 10^{-3}) \omega^{-1} = 0.999 \omega^{-1}
\]

Finally, the frequency parameter, \( \omega \), is easily identified as the Hubble constant, \( H_0 \); Thus, for nearby galaxies, we may write

\[
\lambda_{obs} / \lambda_{std} \approx 1 + \nu / c = 1 + H_0 D / c = 1 + H_0 [c (\tilde{t} - t)] / c = 1 + H_0 (\tilde{t} - t)
\]

But

\[
\lambda_{obs} / \lambda_{std} = 1 / a(t) = \exp[\omega(\tilde{t} - t)]
\]

\[
\approx 1 + \omega(\tilde{t} - t), \text{ for nearby galaxies, and so } \omega = H_0.
\]

17. The Fundamental Role of the Higgs Field

The recent (July 2012) success of the Large Hadron Collider experiments strongly supports the existence of a Higgs field. According to the standard model, the Higgs field gives mass to all those fields that will eventually have rest mass, including the Higgs itself. If this is so, it suggests a way to understand the cosmological exponential increase in rest masses predicted by the VRM cosmology. If the Higgs field acts upon itself, it is very likely that it is not time-independent. It is also reasonable to assume that the Higgs field momentum is conserved, as required by the presumed homogeneity of the universe. Then, by the same argument developed above regarding the cosmological evolution of rest mass, the Higgs field, \( H^0 \), must evolve as an exponential function of world time:

\[
H^0(t) = \tilde{H}^0 \exp[H_0(t - \tilde{t})]
\]

in which \( \tilde{H}^0 \) is the present strength of the Higgs field, and \( H_0 \) is Hubble’s constant. Thus the function \( a(t) \), which is assumed to affect all fields, may be identified as

\[
a(t) = H^0(t) / \tilde{H}^0 = \exp[H_0(t - \tilde{t})]
\]

Because the Higgs field is a spin-zero field, it will not interact directly with the electromagnetic field: but since it possess rest mass, inhomogeneities in the Higgs field will
produce gravitational fields, which, by altering the geometry of space, will affect the trajectory of light rays. In this way, it could be considered a candidate for the role of ‘dark matter.’

18. Other Rest Mass Evolution Cosmologies

The theory of Hoyle and Narlikar evolved out of their quasi-steady state cosmology, which postulated a continuous creation of matter. After several modifications, Hoyle and Narlikar developed a theory in which the particle number is constant, but in which particle masses may vary with position in spacetime. It is a conformally invariant theory of gravity featuring variable particle rest masses[7,8]. The theory is based upon an action-at-a-distance interaction between masses, and is Machian in that all particles serve as sources for a scalar ‘mass function’, the local value of which, in turn, determines a particle’s mass. The Hoyle-Narlikar cosmologies bear an eerie similarity to the VRM cosmology presented here: the cosmic red shift is attributed to the epochal increase of the mass function, rather than to an expansion of space.

Jayant Narlikar and Halton Arp[8,9], elaborated on the Hoyle-Narlikar theory, proposing that the mass function may exhibit departures from a smooth decrease with time and distance, exhibiting regions in which rest masses are very much smaller than the values typical for sources at same distance. Matter in such regions would exhibit anomalously high red shifts. Arp had compiled a catalog of peculiar galaxies, and later found that many quasars, point-like sources exhibiting very large red shifts, were often associated with these peculiar galaxies, which themselves did not possess large red shifts. The red shifts of these quasars have been interpreted by mainstream astronomers as indicating great remoteness, and consequently these objects are believed to radiate at unaccountably high power levels. Arp disputes the idea that red shifts are always correlated with distance, proposing that newly created matter appears initially with very low rest mass, and consequently has an anomalously high red shift. Narlikar and Arp suggested that surfaces may exist on which the mass function takes on a zero value. Near such a region, they argue, quantum fluctuation will be sufficient to cause the production of new particles, resulting in a sort of mini-big bang, as it were.

The assumed homogeneity of the universe is really a simplifying assumption, valid only on the largest scales beyond that of galaxy superclusters. On smaller scales, in some regions of space, there may exist at any particular time, \( t \), very large deviations from the value of the rest mass evolution function \( a(t) \), even to the extent suggested by Narlikar and Arp.

19. Conclusions for Part II

The evolving rest mass interpretation of the cosmic red shift has not been widely accepted, to say the least. The idea that the universe is not expanding but has evolved from a state (infinitely long ago in terms of world time) in which all fields were massless is very far from mainstream thinking and will be extremely hard for the physics community to accept. Nevertheless, it is the only cosmology that is consistent with Noether’s theorem. Also, this cosmology does have several attractive features – there is no singularity, smoothness and flatness are guaranteed by the very slow development (with respect to world time, \( t \) ) of rest mass, and temperatures (as assessed in the VRM reference frame) are for all times past only modestly higher than present values[3, 78]. Importantly, it should be easy to make a decision regarding the acceptance of the VRM cosmology since the theory makes the very definite prediction that the mass evolution function must be a simple exponential function of world time or, equivalently, a linear function of proper time, \( \tau \).

REFERENCES