Dynamics and Synchronization of Dual Phase

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Abstract This article presents dynamics of dual phase, which is connected with dual character of dions – hypothetical particles possessing both electrical and magnetic charges. For description of dual phase the set of equations is received from condition that Maxwell equations conserve their electrical character under dual transformations, what corresponds to effective electrical charge of dion. The mechanism of dual phase synchronization in different parts of space is revealed. It is shown, that solutions of Maxwell equations for spherical waves with zero orbital moment momentum correspond to monopole radiation of dion. Goldstone character of dual phase so as its geometrical character is shown. This article supplements previous investigations of G. Rainich[1] and J. Wheeler and C. Misner[2, 3]. Results of this article can explain many-years failures in searching magnetic charges (monopoles).

Keywords Dion, Magnetic Charge, Goldstone Mode

1. Introduction

Maxwell equations can be written in the form symmetric with respect to permutation of electrical and magnetic charges[4]:

\[
\begin{align*}
\text{rot} \mathbf{H} - \frac{\partial \mathbf{E}}{\partial t} &= \varepsilon \mathbf{J}, \quad \text{div} \mathbf{E} = \rho, \\
\text{rot} \mathbf{E} + \frac{\partial \mathbf{H}}{\partial t} &= -g \mathbf{J}, \quad \text{div} \mathbf{H} = g \rho
\end{align*}
\]

(1)

Here \( \mathbf{E} \) and \( \mathbf{H} \) – are electrical and magnetic fields, \( \rho \) and \( \mathbf{J} \) – number of particles’ and flow of charge densities, correspondingly(in systems of Heavyside-Lorenz units). We suppose that particles called “dions” which play role of the sources of fields have both electrical (\( e \)) and magnetic charge (\( g \)). It is known that with the help of dual transformation of fields:

\[
\begin{align*}
\mathbf{E} &= \cos \theta \cdot \mathbf{E} + \sin \theta \cdot \mathbf{H}, \\
\mathbf{H} &= -\sin \theta \cdot \mathbf{E} + \cos \theta \cdot \mathbf{H}
\end{align*}
\]

(2)

\( \cos \theta = \frac{e}{q}, \sin \theta = \frac{g}{q}, q = \sqrt{e^2 + g^2} \)

\( \theta \)- is a parameter of dual transformation (dual phase), one can eliminate magnetic charge from the (1), and they will look as follows[1]:

\[
\begin{align*}
\text{rot} \mathbf{H} - \frac{\partial \mathbf{E}}{\partial t} &= q \mathbf{J}, \quad \text{div} \mathbf{E} = \rho, \\
\text{rot} \mathbf{E} + \frac{\partial \mathbf{H}}{\partial t} &= 0, \quad \text{div} \mathbf{H} = 0
\end{align*}
\]

(3)

These equations describe electrodynamics of particles which have effective electrical charge \( q \). Thus, magnetic charge which is introduced in a such way became elusive[4]. Value of parameter \( \theta \) is considered as constant but undetermined one due to indeterminacy of values of \( e \) and \( g \). From this point of view charges, both electrical and magnetic take their denominations in the result of agreement and we are free change this one and name electrical charge as magnetic and vice versa, so as for fields. Such metaphysical way of thinking is inappropriate in contemporary physics.

On the other hand it is supposed in present article that some field is connected with phase \( \theta \), what imply definite restrictions on values of \( \theta \) in neighbouring points of space and time. These restrictions have a form of equations which can be received from the Lagrangian (or from the equations) of electromagnetic fields in the assumption that \( \theta \) – is independent dynamical variable. Some hypothetical mass-less particle can be connected with that field, which transfers interaction between dions[5], just like photons transfer interaction between electrically charged particles.

Analogous problems were concerned earlier by G. Rainich[1], J. Wheeler and C. Misner[2, 3] for charged particles interacting with their own gravitational field and by L. Witten[6], who studied problem of initial conditions for Maxwell-Einstein equations. Despite great significance of that works some problems stayed unsolved, such as problem of dual phase synchronization what has been stressed by R. Penrose3.

These reasons can be supplemented by arguments, which are analogous to ones that leads to the idea of dynamical

1 Analogously, equations of particles’ motion can be rewritten in the form, corresponding to particles with effective electric charge \( q \).
2 Analogously electric charge can be eluded in the same manner [4].
3 Unpublished result, for reference see [2].
nature of electrical charge and are formulated in terms of calibration fields, which are responsible for electromagnetic interaction\[4, 7].

2. Basic Equations

Field equations are usually derived from Lagrangian, which is connected with corresponding fields. In problems of dual electrodynamics this approach leads to well-known difficulties\[4]. Nonetheless, attempts like that are undertaken. For this one could take for example, a Lagrangian of type presented below for electromagnetic field, which is created by dually charged particles\[4]:

\[
A = -m \int ds + q \int d^4 x \cdot \mathbf{v}_\mu C_\mu - \frac{1}{2} \int d^4 x \cdot \mathbf{D}_{\mu \nu} \\
C_\mu = \cos \theta \cdot A_\mu + \sin \theta \cdot B_\mu \\
\Phi_{\mu \nu} = \cos \theta \cdot F_{\mu \nu} + \sin \theta \cdot \tilde{F}_{\mu \nu} = \hat{\mathbf{\partial}}_\mu C_\nu - \hat{\mathbf{\partial}}_\nu C_\mu
\]

\(A_\nu\) – is a vector potential and, \(B_\nu\) – pseudo-vector one; \(F_{\mu \nu}\) – is a tensor of electromagnetic field; sign \(\tilde{}\) (tilde) denotes tensor which is dual for \(F_{\mu \nu}\); \(m\) – mass of dions, \(v_\mu\) – their velocity. Taking variation of \(A\) on potentials one can receive Maxwell equations for photons:

\[
\hat{\mathbf{\partial}}_\nu \Phi_{\mu \nu} = q v_\mu
\]

(5)

If one takes a derivative of (5) and considers \(\theta\) as a function of co-ordinate and time he receives equations which describe dynamics of \(\theta\), which look as follows\[4]:

\[
\Phi_{\mu \nu, \theta} \cdot \hat{\mathbf{\partial}}_\nu, \theta = 0
\]

(6)

Comma means derivative on corresponding variable, \(\hat{\mathbf{\partial}}_\nu = \partial / \partial x_\nu\).

Equations (6) have nontrivial solutions if their determinant \(A = 0\). Explicit calculation gives us:

\[
\Delta = (\vec{E} \vec{H})^2 = \frac{H^2 - E^2}{2} \sin 2 \chi + \vec{E} \vec{H} \cos 2 \chi
\]

(7)

\[
\chi = \theta + \frac{\pi}{2}
\]

Thus, nontrivial dynamics of phase \(\theta\) takes place if both invariants of electromagnetic field are equal zero: \(H^2 - E^2 = 0\) and \(\vec{E} \vec{H} = 0\).

Let us consider some possible solutions of equations (6). Taking algebraic supplements for first two rows of \(\Phi_{\mu \nu, \theta} = \Phi_{\mu \nu}(\chi)\) one can receive two different systems of equations:

\[
\hat{\mathbf{\partial}}_i \chi = -\mathbf{H}_i, \\
\hat{\mathbf{\partial}}_i \chi = 0
\]

(8a)

\((i = x, y, z)\)

and

\[
\hat{\mathbf{\partial}}_i \chi = -\mathbf{E}_i, \\
\hat{\mathbf{\partial}}_i \chi = -\mathbf{E}_i
\]

(8b)

If one chooses another rows (or columns) he receives the same results, which differ only with denomination of co-ordinate axes. Combination of equations (8) and (3) gives equations for \(\chi\):

\[
\frac{\partial^2 \chi}{\partial x^2} - \frac{\partial^2 \chi}{\partial y^2} = 0
\]

(9)

\[
\Delta \chi = 0
\]

\[
\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}
\]

Co-ordinate axis \(x\) is standing out from others because it picks a direction of solution’s propagation as a wave with light speed \(c\). It is obvious that it is conditioned by external reasons and \(x\)-direction can be prescribed to any arbitrary axis.

One can receive analogous results directly from equations (3), if suppose their invariance under variations of \(\theta\) (or \(\chi\)).

If one substitutes in equations (1) expressions for the fields \(\mathbf{E}\) and \(\mathbf{H}\) from (2) he receives equations for dual phase under supposition that equations for \(\mathbf{E}\) and \(\mathbf{H}\) have the usual form (3):

\[
\nabla \theta \cdot \mathbf{E} = 0, \nabla \theta \cdot \mathbf{H} = 0,
\]

(10)

\[
\nabla \theta \times \mathbf{E} + \mathbf{H} \cdot \mathbf{\dot{\theta}} = 0,
\]

(10a)

\[
\nabla \theta \times \mathbf{H} + \mathbf{E} \cdot \mathbf{\dot{\theta}} = 0
\]

(10b)

From the first pair of equations (10) one can receive:

\[
\nabla \theta = -\alpha \cdot \mathbf{\dot{S}}, \nabla \cdot \mathbf{\dot{S}} = \mathbf{E} \times \mathbf{H} = \mathbf{E} \times \mathbf{\dot{H}}
\]

(11a)

\(\mathbf{S}\) – is vector of density of energy flux of electromagnetic field, \(\alpha\) – some co-efficient. Second pair of equations (10) leads to equation:

\[
\hat{\mathbf{\partial}}_i \theta = \frac{\alpha}{W} \cdot \dot{\mathbf{S}}, W = \frac{\mathbf{E}^2 + \mathbf{H}^2}{2} = \frac{\dot{\mathbf{E}}^2 + \dot{\mathbf{H}}^2}{2}
\]

(11b)

\(W\) – is density of energy of electromagnetic field. Substituting (11a) in (11b) one can receive equation:

\[
\hat{\mathbf{\partial}}_i \theta + \mathbf{\dot{V}} \cdot \nabla \theta = 0, \mathbf{\dot{V}} = \frac{\dot{\mathbf{S}}}{W}
\]

(12)

Here \(\mathbf{V}\) is a speed of electromagnetic energy propagation. If \(V = c\), what takes place in so-called wave zone (see below), equation (12) coincides with first equation (9), where \(c = 1\). Thus \(x\)-direction in (9) coincides with the direction of electromagnetic energy propagation. Second equation in (9) one can receive under supposition that \(\text{div} \mathbf{S} = 0\).

3. Solutions of the Equation for Dual Phase

a) Vacuum solution.

In this case \(\text{div} \mathbf{S} = 0\). Solutions of the equation (9) can be received if one imposes some boundary conditions on them. Let us treat dual phase’s behaviour at the vicinity of dion. It seems naturally to suppose that far from \(x\)-axis perturbations of phase \(\theta\) are small and electrodynamics’ equations

\[\text{put} \ c = 1 \ \text{in equations (4) and below.} \]

\[\text{Equations (10) are not equivalent to the equations (6), which are corresponding to the first pair of Maxwell equations. To make equivalence complete one must add to (6) equations } \Phi_{\mu \nu, \theta} = \theta, \text{what leads to the second pair [8].}\]
have usual form corresponding to zero value of dual phase, i.e. \( \theta = 0 \) asymptotically. Character feature of the equations (9) is that their solutions which are limited everywhere are absent. For instance, solution which satisfies abovementioned conditions looks as follows:

\[
\theta_0 (r) = bK_0(kr) \cdot \exp[\pm i(kx - \omega t)]
\]

\[
\rho = \sqrt{y^2 + z^2}, k = \frac{\omega}{c}
\]

where \( K_0 \) is modified Bessel’s function, \( k \) – wave number, \( \omega \) – frequency, \( c \) – speed of light in vacuum, \( b \) – is a constant which is found from the condition of normalizing:

\[
\int \theta_0^2 (r) \theta_0 (r) d\vec{r} = 2\pi \delta(k - k')
\]

what leads to \( b = k/\sqrt{\pi} \).

The function \( K_0(z) \) has irregularity near \( z = 0 \). This makes interpretation of the solution (13) problematic, what is connected with arising near \( x - \) axis regions where variation of \( \theta \) is of the order \( 2\pi \). Let us remind that the accuracy of phase \( \theta \) determination is equal to \( 2\pi \), too. Implementation of quantum theory in small distances doesn’t improve situation, because quantum-mechanical phase’s operator can not be determined[9].

b) Solution in electromagnetic fields of electrical dipole.

Electromagnetic fields of the elementary electrical dipole which is located at point \( r = 0 \) look as follows (in spherical co-ordinates)[10]:

\[
H_r = \frac{\sin \theta}{4\pi r} \left( \frac{\hat{p} + \hat{p}}{r} \right)
\]

\[
E_\theta = \frac{\sin \theta}{4\pi r r^2} \left( \frac{\hat{p} + \hat{p}}{r} \right)
\]

\[
E_r = \frac{\cos \theta}{4\pi r} \left( \frac{\hat{p} + \hat{p}}{r^2} \right)
\]

\( p(t) \) – is electric moment of dipole which is directed along polar axis \( z \); point means time derivation; \( c = 1 \). If one expresses values of \( S \) \& \( W \) from (15) and substitute them in equations (11) and (12) he can find their solutions. Explicit expression for nonzero components of \( S \) looks as follows (analogous expression for \( W \) is not of very importance):

\[
S_\theta = \left( \frac{\sin \theta}{4\pi r^2} \right) \left\{ \hat{p}^2 + \frac{1}{2r} \frac{d}{dt} \left( \hat{p}^2 + \left( \frac{\hat{p} + \hat{p}}{r} \right)^2 \right) \right\}
\]

\[
S_\rho = -\frac{\sin 2 \theta}{(4\pi r)^2} \frac{1}{2r} \frac{d}{dt} \left( \hat{p} + \frac{\hat{p}}{r} \right)^2
\]

In the wave zone \( ^7 (r >> \lambda, \lambda – \) wave length of radiation) one has:

\[
S_\theta = 0, S_\rho = W = \left( \frac{\sin \theta}{4\pi r^2} \right) \hat{p}^2
\]

Then equations (11a) and (12) look as follows:

\[
\frac{\partial \theta}{\partial t} + \alpha S_\rho = 0
\]

\[
\frac{\partial \theta}{\partial r} = 0
\]

\[
\frac{\partial \varphi}{\partial r} + c \frac{\partial \varphi}{\partial r} = 0
\]

In last equation light speed \( c \) is written explicitly. It is obvious that its solution has a form of travelling wave \( \theta(\rho - ct) \).

First two equations in (18) are equivalent to equation (11a). In order to treat their solutions one must know \( \alpha \). It can be done by two ways. First, if role of equation (11a) besides definition of space dependence of \( \theta \), consists in formulating initial conditions for equation (11b) too, then time derivative of its left side must equal zero. It can be shown that it is really true if \( \alpha = b/W \), where \( b \) does not depend on fields \( E \) and \( H \). Indeed, if one calculates abovementioned derivative he finds:

\[
\nabla \theta + (\alpha \hat{S}_\rho) = b \left[ \nabla (V^2) + \hat{V}_r \right] = 0
\]

because speed \( V = c \) in wave zone. Second, one can calculate \( \alpha \), using equation (11a) in case of propagation of plane electromagnetic wave between two ideally reflecting mirrors and taking into account that phase redundancy \( \Delta \theta \) is proportional to \( 2\pi \) along close trajectory.

Solving equation (18) for \( \alpha = b/W \), one can find explicit expression of phase \( \theta = -b(r - ct) \), what means that \( b = -k \), where \( k \) – is wave number. It is in agreement with well-known equation [8]: \( \omega_\rho + k_t = 0 \), where \( \omega = - \theta_i = ck, k = \theta_r \).

Let us formulate boundary conditions for equation (18). Remind that it isn’t problem for calculation, but thinking: dual phase value \( \theta = 0 \) near a system of electrically charged particles (elementary dipole in this case). So, whole picture of dual phase distribution for arbitrary \( 0 < r < \infty \) and \( t > 0 \) can be found from solution of (18) with that condition: \( \theta = 0 \) for \( r \leq r_0, 0 < t < \infty \), where \( r_0 \) is sufficiently small distance. Solution of (18) looks as follows [11]: \( \theta(\rho, t) \) for \( (r - r_0)/c < 1 < \infty \).

One can find from equations (11) and (18) that \( \text{rot grad } \theta = 0 \) due to \( \text{rot } S \neq 0 \). This means that topological structure of space near charged particles has no Euclid character.

Due to theorem of duality[12] one can assert that analogous result takes place at the vicinity of elementary magnetic dipole.

4. Electrodynamics of Dions.

In article [13] solutions of the Maxwell \(^9\) equations were received for radiation field of TM-type wave with zero orbital moment momentum \( l = 0 \):

---

\(^7\) Strictly speaking, equations (11) and (12) for phase \( \theta \) are valid in the wave zone only, where conditions \( H^2 - E^2 = 0 \) and \( \mathbf{EH} = 0 \) are satisfied.

\(^8\) This is quite analogous to electrodynamics where one pair of Maxwell equations which does not contain time derivatives is used for formulating initial conditions for second pair equations [14].

\(^9\) And of Maxwell-Einstein equation too because wave (19) does not create gravitational field [13].
$E_\rho = \frac{i \omega e}{cr} \text{ctg} \theta \cdot e^{i \rho (\frac{r}{c} - t)}$

$H_\rho = \frac{-i \omega m}{cr} \text{ctg} \theta \cdot e^{i \rho (\frac{r}{c} - t)}$ (19)

$E_\rho = \frac{e}{r^2} e^{i \rho (\frac{r}{c} - t)}$

$H_\rho = \frac{-m}{r^2} e^{i \rho (\frac{r}{c} - t)}$

where $e$ is constant. Let us prove that these fields describe radiation of dion placed at the origin of co-ordinates which phase varies linearly in time. For this consider together with (19) radiation field of TE-type wave using dual symmetry of Maxwell equation, i.e. substituting in (19) $E \to -H$ and $H \to E$, and so as $e \to m$:

$H_\rho = -\frac{i \omega m}{cr} \text{ctg} \theta \cdot e^{i \rho (\frac{r}{c} - t)}$

$E_\rho = \frac{i \omega e}{cr} \text{ctg} \theta \cdot e^{i \rho (\frac{r}{c} - t)}$ (20)

$H_\rho = \frac{m}{r^2} e^{i \rho (\frac{r}{c} - t)}$

$E_\rho = \frac{-i \omega m}{ct} \frac{r^2}{r^2 - \rho^2} e^{i \rho (\frac{r}{c} - t)}$

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REFERENCES