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Estimation of Population Mean Using Co-Efficient of Variation and Median of an Auxiliary Variable

J. Subramani [*], G. Kumarapandiyan

Department of Statistics, Ramanujan School of Mathematical Sciences, Pondicherry University, R V Nagar, Kalapet, 605014, Puducherry

Abstract The present paper deals with a class of modified ratio estimators for estimation of population mean of the study variable using the linear combination of the known values of the Co-efficient of Variation and the Median of the auxiliary variable. The biases and the mean squared errors of the proposed estimators are derived and are compared with that of existing modified ratio estimators. Further we have also derived the conditions for which the proposed estimators perform better than the existing modified ratio estimators. The performances of the proposed estimators are also assessed with that of the existing estimators for certain natural populations. From the numerical study it is observed that the proposed modified ratio estimators perform better than the existing modified ratio estimators.

Keywords Mean Squared Error, Modified Ratio Estimators, Natural Populations, Simple Random Sampling

1. Introduction

The simplest estimator of population mean is the sample mean obtained by using simple random sampling without replacement, when there is no additional information on the auxiliary variable available. Sometimes in sample surveys, along with the study variable Y , information on auxiliary variable X , correlated with Y , is also collected. This information on auxiliary variable X, may be utilized to obtain a more efficient estimator of the population mean. Ratio method of estimation is an attempt in this direction. This method of estimation may be used when (i) X represents the same character as Y, but measured at some previous date when a complete count of the population was made and (ii) the character X is cheaply, quickly and easily available. Consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ of N distinct and identifiable units. Let Y is a study variable with value Y_i measured on $U_i, i = 1,2,3,\dots,N$ giving a vector $Y = \{Y_1, Y_2, \dots, Y_N\}$ and let X is an auxiliary variable which is readily available. The problem is to estimate the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ with some desirable properties on the basis of a random sample selected from the population U using auxiliary information.

When the population parameters of the auxiliary variable X such as Population Mean, Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness, Median are known, a number of estimators such as ratio, product and linear regression estimators and their modifications are proposed in the literature. Before discussing further about the modified ratio estimators and the proposed modified ratio estimators the notations to be used in this paper are described below:

- $N-$ Population size
- $n-$ Sample size
- $f = n/N$, Sampling fraction
- $Y-$ Study variable
- $X-$ Auxiliary variable
- $\bar{X}, \bar{Y}$ $-$ Population means
- $\bar{x}, \bar{y}$ $-$ Sample means
- S_X, S_y $-$ Population standard deviations
- C_X, C_y $-$ Co-efficient of variations
- $\rho-$ Co-efficient of correlation
- $\beta_1 = \frac{N\sum_{i=1}^{N}(X_i-\bar{X})^3}{(N-1)(N-2)S^3}$, Co-efficient of skewness of the auxiliary variable
- $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$, Co-efficient of kurtosis of the auxiliary variable
- M_d $-$Median of the auxiliary variable
- $B(.)$ $-$ Bias of the estimator
- $MSE(.)$ $-$ Mean squared error of the estimator
- $\hat{\bar{Y}}_i\,(\hat{\bar{Y}}_{pi})$ $-$ Existing (proposed) modified ratio estimator of $\bar{Y}$

The Ratio estimator for estimating the population mean $\bar{Y}$ of the study variable Y is defined as

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\,\bar{X}$$

$$\text{where } \hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x} \text{ is the estimate of } R = \frac{\bar{Y}}{\bar{X}} = \frac{Y}{X} \qquad (1)$$

List of modified ratio estimators together with their biases, mean squared errors and constants available in the literature are classified into two classes namely Class 1, Class 2 and

* Corresponding author:
drjsubramani@yahoo.co.in (J. Subramani)
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are given respectively in Table 1 and Table 2 respectively.

It is to be noted that "the existing modified ratio estimators" means the list of modified ratio estimators to be considered in this paper unless otherwise stated. It does not mean to the entire list of modified ratio estimators available in the literature. For a more detailed discussion on the ratio estimator and its modifications one may refer to Cochran[1], Kadilar and Cingi[2,3], Koyuncu and Kadilar[4], Murthy[5], Prasad[6], Rao[7], Singh and Tailor[9, 11], Singh et.al[10], Sisodia and Dwivedi[12], Subramani and Kumarapandiyan [13,14,15,16,17], Upadhyaya and Singh[18], Yan and Tian[19] and the references cited there in.

The modified ratio estimators given in Table 1 and Table 2 are biased but have minimum mean squared errors compared to the classical ratio estimator. The list of estimators given in Table 1 and Table 2 uses the known values of the parameters like $\bar{X}$, C_x, β_1, β_2, ρ, M_d and their linear combinations. However, it seems, no attempt is made to use the linear combination of known values of the Co-efficient of variation and Median of the auxiliary variable to improve the ratio estimator. The points discussed above have motivated us to introduce modified ratio estimators using the linear combination of the known values of Co-efficient of variation and Median of the auxiliary variable. It is observed that the proposed estimators perform better than the existing modified ratio estimators listed in Table 1 and Table 2.

Table 1. Existing modified ratio estimators (Class 1) with their biases, mean squared errors and their constants

Estimator	Bias - B(.)	Mean squared error MSE(.)	Constant θ_i
$\hat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X} + C_x}{\bar{x} + C_x}\right]$ Sisodia and Dwivedi[12]	$\frac{(1-f)}{n}\bar{Y}(\theta_1^2 C_x^2 - \theta_1 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho)$	$\theta_1 = \frac{\bar{X}}{\bar{X} + C_x}$
$\hat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X} + \beta_2}{\bar{x} + \beta_2}\right]$ Singh et.al[10]	$\frac{(1-f)}{n}\bar{Y}(\theta_2^2 C_x^2 - \theta_2 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho)$	$\theta_2 = \frac{\bar{X}}{\bar{X} + \beta_2}$
$\hat{\bar{Y}}_3 = \bar{y}\left[\frac{\bar{X} + \beta_1}{\bar{x} + \beta_1}\right]$ Yan and Tian[19]	$\frac{(1-f)}{n}\bar{Y}(\theta_3^2 C_x^2 - \theta_3 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_x C_y \rho)$	$\theta_3 = \frac{\bar{X}}{\bar{X} + \beta_1}$
$\hat{\bar{Y}}_4 = \bar{y}\left[\frac{\bar{X} + \rho}{\bar{x} + \rho}\right]$ Singh and Tailor[9]	$\frac{(1-f)}{n}\bar{Y}(\theta_4^2 C_x^2 - \theta_4 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_x C_y \rho)$	$\theta_4 = \frac{\bar{X}}{\bar{X} + \rho}$
$\hat{\bar{Y}}_5 = \bar{y}\left[\frac{\bar{X} C_x + \beta_2}{\bar{x} C_x + \beta_2}\right]$ Upadhyaya and Singh[18]	$\frac{(1-f)}{n}\bar{Y}(\theta_5^2 C_x^2 - \theta_5 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_x C_y \rho)$	$\theta_5 = \frac{\bar{X} C_x}{\bar{X} C_x + \beta_2}$
$\hat{\bar{Y}}_6 = \bar{y}\left[\frac{\bar{X}\beta_2 + C_x}{\bar{x}\beta_2 + C_x}\right]$ Upadhyaya and Singh[18]	$\frac{(1-f)}{n}\bar{Y}(\theta_6^2 C_x^2 - \theta_6 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_6^2 C_x^2 - 2\theta_6 C_x C_y \rho)$	$\theta_6 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2 + C_x}$
$\hat{\bar{Y}}_7 = \bar{y}\left[\frac{\bar{X}\beta_1 + \beta_2}{\bar{x}\beta_1 + \beta_2}\right]$ Yan and Tian[19]	$\frac{(1-f)}{n}\bar{Y}(\theta_7^2 C_x^2 - \theta_7 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_7^2 C_x^2 - 2\theta_7 C_x C_y \rho)$	$\theta_7 = \frac{\bar{X}\beta_1}{\bar{X}\beta_1 + \beta_2}$
$\hat{\bar{Y}}_8 = \bar{y}\left[\frac{\bar{X} C_x + \beta_1}{\bar{x} C_x + \beta_1}\right]$ Yan and Tian[19]	$\frac{(1-f)}{n}\bar{Y}(\theta_8^2 C_x^2 - \theta_8 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_8^2 C_x^2 - 2\theta_8 C_x C_y \rho)$	$\theta_8 = \frac{\bar{X} C_x}{\bar{X} C_x + \beta_1}$
$\hat{\bar{Y}}_9 = \bar{y}\left[\frac{\bar{X} + M_d}{\bar{x} + M_d}\right]$ Subramani and Kumarapandiyan[15]	$\frac{(1-f)}{n}\bar{Y}(\theta_9^2 C_x^2 - \theta_9 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_9^2 C_x^2 - 2\theta_9 C_x C_y \rho)$	$\theta_9 = \frac{\bar{X}}{\bar{X} + M_d}$

Table 2. Existing modified ratio estimators (Class 2) with their biases, mean squared errors and their constants

Estimator	Bias-B(.)	Mean squared error MSE(.)	Constant R_i
$\hat{\bar{Y}}_1 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X}$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_1^2$	$\frac{(1-f)}{n}\left(R_1^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_1=\frac{\bar{Y}}{\bar{X}}$
$\hat{\bar{Y}}_2 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_2^2$	$\frac{(1-f)}{n}\left(R_2^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_2=\frac{\bar{Y}}{\bar{X}+C_x}$
$\hat{\bar{Y}}_3 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_3^2$	$\frac{(1-f)}{n}\left(R_3^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_3=\frac{\bar{Y}}{\bar{X}+\beta_2}$
$\hat{\bar{Y}}_4 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_4^2$	$\frac{(1-f)}{n}\left(R_4^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_4=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x}$
$\hat{\bar{Y}}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_5^2$	$\frac{(1-f)}{n}\left(R_5^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_5=\frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}$
$\hat{\bar{Y}}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_1)}(\bar{X}+\beta_1)$ Yan and Tian[19]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_6^2$	$\frac{(1-f)}{n}\left(R_6^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_6=\frac{\bar{Y}}{\bar{X}+\beta_1}$
$\hat{\bar{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_7^2$	$\frac{(1-f)}{n}\left(R_7^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_7=\frac{\bar{Y}}{\bar{X}+\rho}$
$\hat{\bar{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_8^2$	$\frac{(1-f)}{n}\left(R_8^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_8=\frac{\bar{Y}C_x}{\bar{X}C_x+\rho}$
$\hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_9^2$	$\frac{(1-f)}{n}\left(R_9^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_9=\frac{\bar{Y}\rho}{\bar{X}\rho+C_x}$
$\hat{\bar{Y}}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2$	$\frac{(1-f)}{n}\left(R_{10}^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_{10}=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho}$
$\hat{\bar{Y}}_{11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2$	$\frac{(1-f)}{n}\left(R_{11}^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_{11}=\frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$

2. Proposed Modified Ratio Estimators

In this section, we have suggested a class of modified ratio estimators using the linear combination of Co-efficient of variation and Median of the auxiliary variable. The proposed modified ratio estimators for estimating the population mean $\bar{Y}$ together with the first degree of approximation, the biases and mean squared errors and the constants are given below:

Table 3. Proposed modified ratio estimators with their biases, mean squared errors and their constants

Estimator	Bias B(.)	Mean squared errors MSE(.)	Constants θ_i or R_i
$\hat{\bar{Y}}_{p1}=\bar{y}\left[\frac{\bar{X}C_x+M_d}{\bar{x}C_x+M_d}\right]$	$\frac{(1-f)}{n}\bar{Y}(\theta_{p1}^2C_x^2-\theta_{p1}C_xC_y\rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2+\theta_{p1}^2C_x^2-2\theta_{p1}C_xC_y\rho)$	$\theta_{p1}=\frac{\bar{X}C_x}{\bar{X}C_x+M_d}$
$\hat{\bar{Y}}_{p2}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+M_d)}(\bar{X}+M_d)$	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p2}^2$	$\frac{(1-f)}{n}\left(R_{p2}^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_{p2}=\frac{\bar{Y}}{\bar{X}+M_d}$
$\hat{\bar{Y}}_{p3}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+M_d)}(\bar{X}C_x+M_d)$	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p3}^2$	$\frac{(1-f)}{n}\left(R_{p3}^2S_x^2+S_y^2(1-\rho^2)\right)$	$R_{p3}=\frac{\bar{Y}C_x}{\bar{X}C_x+M_d}$

3. Efficiency Comparison

For want of space; for the sake of convenience to the readers and for the ease of comparisons, the modified ratio estimators given in Table 1, Table 2 are represented into two classes as given below:

Class 1: The biases, the mean squared errors and the constants of the modified ratio type estimators $\hat{\bar{Y}}_1$ to $\hat{\bar{Y}}_9$ listed in the Table 1 are represented in a single class (say, Class 1), which will be very much useful for comparing with that of proposed modified ratio estimators and are given below:

$$B\left(\hat{\bar{Y}}_i\right)=\frac{(1-f)}{n}\bar{Y}\left(\theta_i^2C_x^2-\theta_iC_xC_y\rho\right)$$
$$MSE\left(\hat{\bar{Y}}_i\right)=\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_i^2C_x^2-2\theta_iC_xC_y\rho\right);\quad i=1,2,3,\ldots,9 \quad (2)$$

where $\theta_1=\frac{\bar{X}}{\bar{X}+C_x}, \theta_2=\frac{\bar{X}}{\bar{X}+\beta_2}, \theta_3=\frac{\bar{X}}{\bar{X}+\beta_1}, \theta_4=\frac{\bar{X}}{\bar{X}+\rho},$
$\theta_5=\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2}, \theta_6=\frac{\bar{X}\beta_2}{\bar{X}\beta_2+C_x}, \theta_7=\frac{\bar{X}\beta_1}{\bar{X}\beta_1+\beta_2},$
$\theta_8=\frac{\bar{X}C_x}{\bar{X}C_x+\beta_1}$ and $\theta_9=\frac{\bar{X}}{\bar{X}+M_d}$

Class 2: The biases, the mean squared errors and the constants of the 11 modified ratio estimators $\hat{\bar{Y}}_1$ to $\hat{\bar{Y}}_{11}$ listed in the Table 2 are represented in a single class (say, Class 2), which will be very much useful for comparing with that of proposed modified ratio estimators and are given below:

$$B\left(\hat{\bar{Y}}_j\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_j^2$$
$$MSE\left(\hat{\bar{Y}}_j\right)=\frac{(1-f)}{n}\left(R_j^2S_x^2+S_y^2(1-\rho^2)\right);\quad j=1,2,3,\ldots,11 \quad (3)$$

where $R_1=\frac{\bar{Y}}{\bar{X}}, R_2=\frac{\bar{Y}}{\bar{X}+C_x}, R_3=\frac{\bar{Y}}{\bar{X}+\beta_2},$
$R_4=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x}, R_5=\frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}, R_6=\frac{\bar{Y}}{\bar{X}+\beta_1}, R_7=\frac{\bar{Y}}{\bar{X}+\rho},$
$R_8=\frac{\bar{Y}C_x}{\bar{X}C_x+\rho}, R_9=\frac{\bar{Y}\rho}{\bar{X}\rho+C_x}, R_{10}=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho}$ and $R_{11}=\frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$

As derived earlier in section 2, the biases, the mean squared errors and the constants of the proposed modified ratio estimators are given below:

$$B\left(\hat{\bar{Y}}_{p1}\right)=\frac{(1-f)}{n}\bar{Y}\left(\theta_{p1}^2C_x^2-\theta_{p1}C_xC_y\rho\right)$$
$$MSE\left(\hat{\bar{Y}}_{p1}\right)=\frac{(1-f)}{n}\bar{Y}^2\left(C_y^2+\theta_{p1}^2C_x^2-2\theta_{p1}C_xC_y\rho\right)$$
$$\text{where } \theta_{p1}=\frac{\bar{X}C_x}{\bar{X}C_x+M_d} \quad (4)$$

$$B\left(\hat{\bar{Y}}_{p2}\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p2}^2$$
$$MSE\left(\hat{\bar{Y}}_{p2}\right)=\frac{(1-f)}{n}\left(R_{p2}^2S_x^2+S_y^2(1-\rho^2)\right)$$
$$\text{where } R_{p2}=\frac{\bar{Y}}{\bar{X}+M_d} \quad (5)$$

$$B\left(\hat{\bar{Y}}_{p3}\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p3}^2$$
$$MSE\left(\hat{\bar{Y}}_{p3}\right)=\frac{(1-f)}{n}\left(R_{p3}^2S_x^2+S_y^2(1-\rho^2)\right)$$
$$\text{where } R_{p3}=\frac{\bar{Y}C_x}{\bar{X}C_x+M_d} \quad (6)$$

From the expressions given in (2) and (4) we have derived the conditions for which the proposed estimator $\hat{\bar{Y}}_{p1}$ is more efficient than the existing modified ratio estimators given in Class 1, $\hat{\bar{Y}}_i; i=1,2,3,\ldots,9$ and are given below:

$$MSE\left(\hat{\bar{Y}}_{p1}\right)<MSE\left(\hat{\bar{Y}}_i\right) \text{ if } \rho<\frac{(\theta_{p1}+\theta_i)}{2}\frac{C_x}{C_y}; \quad (7)$$
$i=1,2,3,\ldots,9$

From the expressions given in (5), (6) and (4) we have derived the conditions for which the proposed estimator $\hat{\bar{Y}}_{pi}; i=2,3$ is more efficient than the existing modified ratio estimators given in Class 2, $\hat{\bar{Y}}_j; j=1,2,3,\ldots,11$ and are given below:

$$MSE\left(\hat{\bar{Y}}_{pi}\right)<MSE\left(\hat{\bar{Y}}_j\right) \text{ if } R_{pi}<R_j; \quad (8)$$
$i=2,3;\ j=1,2,3,\ldots,11$

4. Numerical Study

The performances of the proposed modified ratio estimators listed in Table 3 are assessed with that of existing modified ratio estimators listed in Table 1 and Table 2 for certain natural populations. In this connection, we have considered three natural populations for the assessment of the performances of the proposed modified ratio estimators with that of existing modified ratio estimators. The population 1 and population 2 are taken from Singh and Chaudhary[8] given in page 177 and population 3 is taken from Murthy[5] given in page 228. The population parameters and the constants computed from the above populations are given below:

The constants, biases and mean squared errors of the existing and proposed modified ratio estimators for the above populations are given from Table 5 to Table 10:

Table 4. Parameters and Constants of the Populations

Parameters	Population 1	Population 2	Population 3
N	34	34	80
n	20	20	20
$\bar{Y}$	856.4117	856.4117	51.8264
$\bar{X}$	208.8823	199.4412	2.8513
ρ	0.4491	0.4453	0.9150
S_y	733.1407	733.1407	18.3569
C_y	0.8561	0.8561	0.3542
S_x	150.5059	150.2150	2.7042
C_x	0.7205	0.7531	0.9484
β_2	0.0978	1.0445	1.3005
β_1	0.9782	1.1823	0.6978
M_d	150.0000	142.5000	1.4800

Table 5. The constants of the (Class 1) existing and proposed modified ratio estimators

Estimator	Constants θ_i		
	Population 1	Population 2	Population 3
$\widehat{\bar{Y}}_1$ Sisodia and Dwivedi[12]	0.9966	0.9962	0.7504
$\widehat{\bar{Y}}_2$ Singh et.al[10]	0.9995	0.9948	0.6868
$\widehat{\bar{Y}}_3$ Yan and Tian[19]	0.9953	0.9941	0.8034
$\widehat{\bar{Y}}_4$ Singh and Tailor[9]	0.9979	0.9978	0.7571
$\widehat{\bar{Y}}_5$ Upadhyaya and Singh[18]	0.9994	0.9931	0.6753
$\widehat{\bar{Y}}_6$ Upadhyaya and Singh[18]	0.9658	0.9964	0.7963
$\widehat{\bar{Y}}_7$ Yan and Tian[19]	0.9542	0.9944	0.8416
$\widehat{\bar{Y}}_8$ Yan and Tian[19]	0.9935	0.9922	0.7949
$\widehat{\bar{Y}}_9$ Subramani and Kumarapandiyan[15]	0.5820	0.5833	0.6583
$\widehat{\bar{Y}}_{p1}$ (Proposed estimator)*	0.5008*	0.5132*	0.6463*

Table 6. The constants of the (Class 2) existing and proposed modified ratio estimators

Estimator	Constants R_i		
	Population 1	Population 2	Population 3
$\widehat{\bar{Y}}_1$ Kadilar and Cingi[2]	4.1000	4.2941	18.1764
$\widehat{\bar{Y}}_2$ Kadilar and Cingi[2]	4.0859	4.2779	13.6396
$\widehat{\bar{Y}}_3$ Kadilar and Cingi[2]	4.0981	4.2717	12.4829
$\widehat{\bar{Y}}_4$ Kadilar and Cingi[2]	3.9598	4.2786	14.4744
$\widehat{\bar{Y}}_5$ Kadilar and Cingi[2]	4.0973	4.2644	12.2737
$\widehat{\bar{Y}}_6$ Yan and Tian[19]	4.0809	4.2688	14.6027
$\widehat{\bar{Y}}_7$ Kadilar and Cingi[3]	4.0912	4.2845	13.7606
$\widehat{\bar{Y}}_8$ Kadilar and Cingi[3]	4.0878	4.2814	13.5810
$\widehat{\bar{Y}}_9$ Kadilar and Cingi[3]	4.0687	4.2579	13.3305
$\widehat{\bar{Y}}_{10}$ Kadilar and Cingi[3]	4.0115	4.2849	14.5790
$\widehat{\bar{Y}}_{11}$ Kadilar and Cingi[3]	4.0957	4.2441	12.1299
$\widehat{\bar{Y}}_{p2}$ (Proposed estimator)*	2.3863*	2.5046*	11.9656*
$\widehat{\bar{Y}}_{p3}$ (Proposed estimator)*	2.0534*	2.2036*	11.7472*

Table 7. The biases of the (Class 1) existing and proposed modified ratio estimators

Estimator	Bias B(.)		
	Population 1	Population 2	Population 3
$\widehat{\bar{Y}}_1$ Sisodia and Dwivedi[12]	4.2233	4.8836	0.5361
$\widehat{\bar{Y}}_2$ Singh et.al[10]	4.2631	4.8621	0.4142
$\widehat{\bar{Y}}_3$ Yan and Tian[19]	4.2070	4.8519	0.6484
$\widehat{\bar{Y}}_4$ Singh and Tailor[9]	4.2406	4.9064	0.5497
$\widehat{\bar{Y}}_5$ Upadhyaya and Singh[18]	4.2607	4.8369	0.3937
$\widehat{\bar{Y}}_6$ Upadhyaya and Singh[18]	3.8212	4.8860	0.6328
$\widehat{\bar{Y}}_7$ Yan and Tian[19]	3.6732	4.8556	0.7355
$\widehat{\bar{Y}}_8$ Yan and Tian[19]	4.1831	4.8236	0.6297
$\widehat{\bar{Y}}_9$ Subramani and Kumarapandiyan[15]	0.2581	0.4499	0.3643
$\widehat{\bar{Y}}_{p1}$ (Proposed estimator)*	0.1502*	0.0361*	0.3441*

Table 8. The biases of the (Class 2) existing and proposed modified ratio estimators

Estimator	Bias B(.)		
	Population 1	Population 2	Population 3
$\widehat{\bar{Y}}_1$ Kadilar and Cingi[2]	9.1539	10.0023	1.7481
$\widehat{\bar{Y}}_2$ Kadilar and Cingi[2]	9.0911	9.9272	0.9844
$\widehat{\bar{Y}}_3$ Kadilar and Cingi[2]	9.1454	9.8983	0.8245
$\widehat{\bar{Y}}_4$ Kadilar and Cingi[2]	8.5387	9.9303	1.1086
$\widehat{\bar{Y}}_5$ Kadilar and Cingi[2]	9.1420	9.8646	0.7971
$\widehat{\bar{Y}}_6$ Yan and Tian[19]	9.0688	9.8847	1.1283
$\widehat{\bar{Y}}_7$ Kadilar and Cingi[3]	9.1147	9.9578	1.0019
$\widehat{\bar{Y}}_8$ Kadilar and Cingi[3]	9.0995	9.9432	0.9759
$\widehat{\bar{Y}}_9$ Kadilar and Cingi[3]	9.0149	9.8348	0.9403
$\widehat{\bar{Y}}_{10}$ Kadilar and Cingi[3]	8.7630	9.9597	1.1246
$\widehat{\bar{Y}}_{11}$ Kadilar and Cingi[3]	9.1349	9.7711	0.7785
$\widehat{\bar{Y}}_{p2}$ (Proposed estimator)*	5.3224*	5.7661*	0.7680*
$\widehat{\bar{Y}}_{p3}$ (Proposed estimator)*	4.5799*	5.0733*	0.7540*

Table 9. The mean squared errors of the (Class 1) existing and proposed modified ratio estimators

Estimator	Mean Squared Error MSE(.)		
	Population 1	Population 2	Population 3
$\widehat{\bar{Y}}_1$ Sisodia and Dwivedi[12]	10514.2250	10929.0458	17.1881
$\widehat{\bar{Y}}_2$ Singh et.al[10]	10535.8620	10916.9080	12.8426
$\widehat{\bar{Y}}_3$ Yan and Tian[19]	10505.3563	10911.1914	21.3660
$\widehat{\bar{Y}}_4$ Singh and Tailor[9]	10523.6171	10941.9491	17.6849
$\widehat{\bar{Y}}_5$ Upadhyaya and Singh[18]	10534.5417	10902.7384	12.1351
$\widehat{\bar{Y}}_6$ Upadhyaya and Singh[18]	10298.4432	10930.3879	20.7801
$\widehat{\bar{Y}}_7$ Yan and Tian[19]	10220.4736	10913.2804	24.6969
$\widehat{\bar{Y}}_8$ Yan and Tian[19]	10492.3779	10895.2039	20.6613
$\widehat{\bar{Y}}_9$ Subramani and Kumarapandiyan[15]	8852.3417	8922.5150	11.1366
$\widehat{\bar{Y}}_{p1}$ (Proposed estimator)*	8842.3621*	8872.0002*	10.4605*

Table 10. The mean squared errors of the (Class 2) existing and proposed modified ratio estimators

Estimator	Mean Squared Error MSE(.)		
	Population 1	Population 2	Population 3
$\hat{\bar{Y}}_1$ Kadilar and Cingi[2]	16673.4489	17437.6451	92.6563
$\hat{\bar{Y}}_2$ Kadilar and Cingi[2]	16619.6435	17373.3111	53.0736
$\hat{\bar{Y}}_3$ Kadilar and Cingi[2]	16666.1389	17348.6192	44.7874
$\hat{\bar{Y}}_4$ Kadilar and Cingi[2]	16146.6142	17376.0389	59.5095
$\hat{\bar{Y}}_5$ Kadilar and Cingi[2]	16663.3064	17319.7468	43.3674
$\hat{\bar{Y}}_6$ Yan and Tian[19]	16600.5393	17336.9770	60.5325
$\hat{\bar{Y}}_7$ Kadilar and Cingi[3]	16639.8457	17399.5196	53.9825
$\hat{\bar{Y}}_8$ Kadilar and Cingi[3]	16626.8702	17387.0811	52.6365
$\hat{\bar{Y}}_9$ Kadilar and Cingi[3]	16554.4002	17294.1864	50.7876
$\hat{\bar{Y}}_{10}$ Kadilar and Cingi[3]	16338.6465	17401.1397	60.3426
$\hat{\bar{Y}}_{11}$ Kadilar and Cingi[3]	16657.1867	17239.6579	42.4051
$\hat{\bar{Y}}_{p2}$ (Proposed estimator)*	11489.7024*	11785.7032*	41.3191*
$\hat{\bar{Y}}_{p3}$ (Proposed estimator)*	10800.4299*	11127.4687*	39.8990*

From the values of Table 7 and Table 8, it is observed that the bias of the proposed modified ratio estimator $\hat{\bar{Y}}_{p1}$ is less than the biases of the (Class 1) existing modified ratio estimators $\hat{\bar{Y}}_i\,; i = 1, 2, 3, \ldots, 9$ and the biases of the proposed modified ratio estimators $\hat{\bar{Y}}_{pi}\,; i = 2,3$ are less than the biases of the (Class 2) existing modified ratio estimators $\hat{\bar{Y}}_j\,; j = 1, 2, 3, \ldots, 11$. Similarly from the values of Table 9 and Table 10, it is observed that the mean squared error of the proposed modified ratio estimator $\hat{\bar{Y}}_{p1}$ is less than the mean squared errors of the (Class 1) existing modified ratio estimators $\hat{\bar{Y}}_i\,; i = 1, 2, 3, \ldots, 9$ and the mean squared errors of the proposed modified ratio estimators $\hat{\bar{Y}}_{pi}\,; i = 2,3$ are less than the mean squared errors of the (Class 2) existing modified ratio estimators $\hat{\bar{Y}}_j\,; j = 1, 2, 3, \ldots, 11$.

5. Conclusions

The existing modified ratio estimators with the known values of the parameters like $\bar{X}$, C_x, β_1, β_2, ρ and M_d are biased but have minimum mean squared errors compared to the classical ratio estimator. In this paper, we use the linear combination of known values of the Co-efficient of variation and Median of the auxiliary variable to improve the ratio estimator. The biases and mean squared errors of the proposed estimators are obtained. Further we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators. We have also assessed the performances of the proposed estimators for some known populations given in the text books [5] and [8]. It is observed that the biases and mean squared errors of the proposed estimators are less than the biases and mean squared errors of the existing modified ratio estimators. Hence we strongly recommend that the proposed modified estimators may be preferred over the existing modified ratio estimators for the use in practical applications.

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