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Control Charts for Variables with Specified Process Capability Indices

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Abstract Control charts, also known as Shewhart charts in statistical process control are the statistical tools used to determine whether a manufacturing process is in a state of statistical control or not. If analysis of the control chart indicates that the process is currently under statistical control (i.e. is stable, with variation only coming from sources common to the process) then no corrections or changes to process control parameters are needed or desirable. In addition, data from the controlled process can be used to predict the future performance of the process with the help of process capability indices. That is, the suitability and the performance of the manufacturing process are assessed in two stage processes namely control charts and process capability indices. This paper deals with the techniques namely, capability based control charts which combines the two stage processes into a single stage process in industrial applications for on line process control. The relative performance of the capability based control charts for variables is assessed with that of existing usual control charts for variables, namely $\bar{X}-Charts$ and $R-Charts$. It is observed that the proposed method is simple to apply and does not warrant any tedious computations both for control charts and for computing process capability indices. For the sake of convenience we have presented the control charts constants for constructing capability based $R-Charts$ and $\bar{X}-Charts$. The proposed method is also illustrated with the help of a numerical example.

Keywords Control Charts, Industrial Applications, Process Capability Index, Quality Characteristics

1. Introduction

The present scenario of total quality management demands the effective use of statistical tools for analyzing quality problems and improving the performance of production process. Measuring, monitoring and maintaining several quality characteristics to achieve further improvements are required in every production process. Many of the quality characteristics are measurable in nature and can be expressed in terms of numerical measurement by measuring them using micrometer, vernier calipers etc. For example, length, diameter, weight, pocket width, distance and groove diameter of a shaft are some of the quantifiable quality characteristics encountered during the production of shafts. When dealing with measurable quality characteristics, it is usually necessary to monitor the behaviour of the mean value of the quality characteristics and its variability. To study the behaviour of the production process and to take a necessary action on the process, control charts for variables are very much used which requires to collect random samples from the production process and to compute the estimate of the quality characteristics. Further one has to show that the process is under statistical control before computing the process capability indices. Nowadays process capability analysis is a mandatory requirement and to have a minimum prescribed value for the process capability indices C_p and C_{pk} to be a supplier for original manufacturing companies, particularly in the field of automobile industries. Process capability indices (PCI) have proliferated in both use and variety during the last three decades. Their widespread and often uncritical use led to improvements in quality.

Process capability indices are considered as a practical tool by many advocates of the statistical process control industry. They are used to determine whether the manufacturing process is capable of producing units within the product or process specifications. A capability index is a dimensionless measure based on the process parameters and the process specifications, designed to quantify, in a simple and easily understood way, the performance of the process. Several process capability indices have been developed among which the basic and widely used indices are C_p and C_{pk}. (see Kane[8]). A detailed review of the process capability indices can be seen in Kotz and Johnson[9].

The process capability indices are unit less measures showing the capability of a manufacturing process whether it is capable of operating within its specifications limits. If the value of the PCI exceeds one, then the process is considered

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to be satisfactory or capable. However this approach ignores the fact that these PCIs are random variables with distributions. Confidence limits play a major role for the correct interpretation of PCIs. Recently, techniques and tables were developed to construct confidence limits for the process capability indices. These techniques are based on the assumption that the underlying process is normal. Moreover, since many processes can frequently have skewed or heavily tailed distributions. In practice, the interval estimation technique that is free from the assumption of distribution is desirable. The various other properties and the behaviour of process capability indices at different type of distributions were studied by many authors. For example, the 95% confidence limits for the process capability indices C_p and C_{pk} were constructed by Chou, Owen and Borrego[5]. As their limits on C_{pk} can produce 97% or 98% lower confidence limits (instead of 95%) making them conservative, an approximation presented by Bissell[4] is recommended. Franklin and Wasserman[6] presented an initial study of the properties of these three bootstrap confidence intervals for C_{pk}. Franklin and Wasserman[7] constructed the confidence limits for some basic capability indices and examined their behaviors when the underlying process is normal, skewed or heavily tailed distributions. Balamurali and Kalyanasundaram[3] constructed bootstrap confidence limits for the capability indices C_p, C_{pk} and C_{pm} based on lognormal and chi-squared distributions. Balamurali[2] considered the above capability indices under short run production processes and constructed the confidence limits.

However the problems one has to face are the following: There is a considerable time taken for establishing and monitoring the process through control charts for variables; inadequate technical personnel to correctly interpret the outcome of the control charts; unnecessary inventory and delay due to the computation of process capability indices and to take a decision on the process based on the resulting values of process capability indices. Hence it is felt to have an alternative method to achieve the benefit of both control charts for variables and the process capability analysis. As a result we have proposed new type of control charts for variables based on the required process capability indices C_p and C_{pk}, called as "*control charts for variables with specified value for process capability indices*". Further we have presented the control charts constants to construct the proposed control charts and are explained with the help of a numerical example. For a more detailed discussion on the control charts for variables and process capability analysis, the readers are referred to Montgomery[10], Spiring[11], Sarkar and Pal[1], Subramani[12, 13], Subramani and Balamurali[14] and the references cited therein. In this paper, we propose a capability based control chart for on line process control. The relative performance of this control charts for variables is assessed with that of existing usual control charts for variables. It is observed that the proposed method is simple to apply and does not warrant any tedious computations both for control charts and for computing process capability indices. We are also presenting the control charts constants for constructing capability based control charts. The proposed method is also illustrated with a numerical example.

2. Control Charts and Process Capability Analysis

2.1. Usual Method

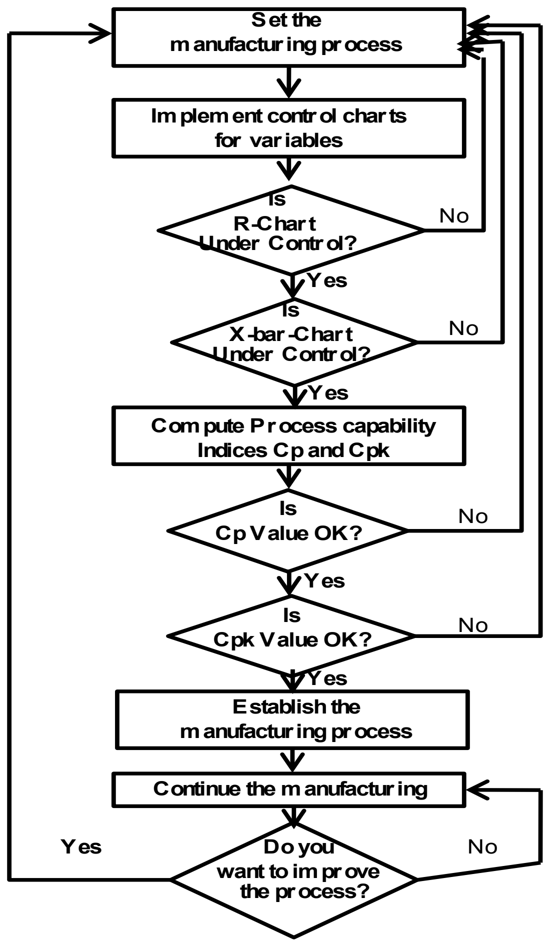

Figure 1. Present Manufacturing System

For the sake of convenience to the readers, we have presented the control charts for variables for checking whether the process is under statistical control or not. That is, to take a decision whether the process can be allowed further without making any adjustment or to take corrective actions if any, to bring back the process under statistical control. Once the process has been brought under the state of statistical control by both $R-Chart$ and $\overline{X}-Chart$ then the process capability indices $C_p\ and\ C_{pk}$ can be computed. If the values of the process capability indices are at the satisfactory level, the process can be allowed further without making any change or adjustment in the process. Otherwise suitable adjustments have to be made so that the process shall satisfy both the requirements of control charts and the process capability indices. The present manufacturing process can be explained in the flow chart as shown in Figure 1.

The following are the various computational formulae involved in constructing the control charts and for the computation of process capability indices.

$\overline{X}_i = \frac{1}{n}\sum_{i=1}^{n} x_i$: Mean of ith sample

$\overline{\overline{X}} = \frac{1}{N}\sum_{i=1}^{N} x_i$: Grand Mean

$R_i = Max(x_i) - Min(x_i)$: Range of ith sample

$\overline{R} = \frac{1}{k}\sum_{i=1}^{k} R_i$: Mean of Range values

$UCL_{\overline{x}} = \overline{\overline{X}} + A_2\overline{R}$: Upper Control Limit for $\overline{X}$ -Chart

$LCL_{\overline{x}} = \overline{\overline{X}} - A_2\overline{R}$: Lower Control Limit for $\overline{X}$ -Chart

$UCL_R = D_4\overline{R}$: Upper Control Limit for R-Chart

$LCL_R = D_3\overline{R}$: Lower Control Limit for R-Chart

$C_p = \frac{USL - LSL}{6\overline{R}/d_2}$: Process Capability

$C_{pu} = \frac{USL - \overline{\overline{X}}}{3\overline{R}/d_2}$: Upper Process Capability Index

$C_{pl} = \frac{\overline{\overline{X}} - LSL}{3\overline{R}/d_2}$: Lower Process Capability Index

$C_{pk} = Min(C_{pu}, C_{pl})$: Process Capability Index

$$Tolerance = T = USL - LSL$$

$$Target = M = \frac{USL + LSL}{2}$$

LSL: Lower Specification Limit of the Quality Characteristics

USL: Upper Specification Limit of the Quality Characteristics

When the sample size n=5, the values for the control chart constants obtained from the table are as given below:

$D_4 = 2.114$, $D_3 = 0$, $d_2 = 2.326$ and $A_2 = 0.577$

The Control limits for constructing $R - Charts$ and $\overline{X} - Charts$ are as given below:

Control limits for $R - Chart$:

$$CL_R = \overline{R}$$

$$LCL_R = D_3\overline{R}$$

$$UCL_R = D_4\overline{R}$$

Control limits for $\overline{X} - Chart$:

$$CL_{\overline{x}} = \overline{\overline{X}}$$

$$LCL_{\overline{x}} = \overline{\overline{X}} - A_2\overline{R}$$

$$UCL_{\overline{x}} = \overline{\overline{X}} + A_2\overline{R}$$

2.2. Proposed Method- Control Charts for Variables with Specified C_p Value

Suppose that a customer wants to have products produced in a process with a specified value for the process capability index C_p. To achieve this, the manufacturer has to establish a process under statistical control and then the process capability index has to be computed. If the computed value of C_p is more than the specified value given by the customer, the process can be run without any adjustment. Otherwise suitable actions have to be taken to meet the required process capability values. The drawback of this method is that the manufacturer has to keep all the produced items in the inventory until the computation of the process capability index and to take a decision on the process based on the process capability index. It is a time consuming procedure as well as requires additional skilled personnel. To avoid this, we have proposed control charts for variables with specified process capability indices. In the proposed method, the process is under statistical control means that it satisfies the required process capability index. Further it is not necessary to compute the process capability indices separately. The proposed manufacturing process can be explained in the flow chart shown in Figure 2.

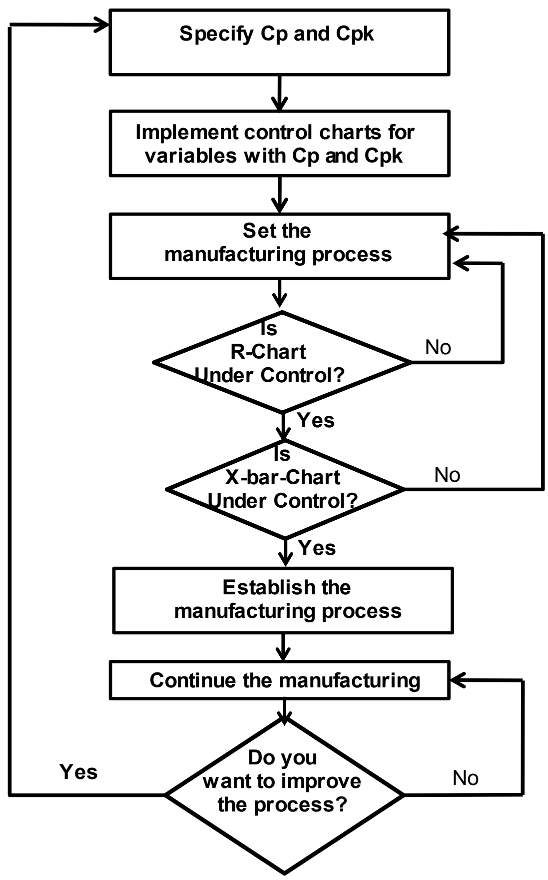

Figure 2. Present Manufacturing System

The following are the derivation of various computational formulae involved in constructing the proposed control charts with specified process capability index C_p.

Consider $C_p = \frac{USL - LSL}{6\sigma}$

$$\Rightarrow C_p = \frac{T}{6\overline{R}/d_2}$$

If C_p value is given then $\overline{R}$ value can be obtained from the above formula as $\overline{R} = \frac{d_2 T}{6C_p}$

$$\Rightarrow \overline{R} = D^* \frac{T}{C_p} \text{ Where } D^* = \frac{d_2}{6}$$

Similarly one can obtain the lower control limit (LCL) and upper control limit (UCL) of the $R-Chart$ as given below:

Center Line= $CL_R = \overline{R} = D^* \frac{T}{C_p}$

$$LCL_R = D_3 \overline{R}$$

$$\Rightarrow LCL_R = \frac{D_3 d_2 T}{6C_p}$$

$$\Rightarrow LCL_R = D_3^* \frac{T}{C_p}$$

Where $D_3^* = \frac{D_3 d_2}{6}$

$$UCL_R = D_4 \overline{R}$$

$$\Rightarrow UCL_R = \frac{D_4 d_2 T}{6C_p}$$

In the similar manner one can compute the control limits to construct $\overline{X}-Chart$ with specified C_p value as given below.

Center line= $CL_{\bar{x}} = \overline{\overline{X}}$

Lower control limit for the proposed $\overline{X}-Chart$ is obtained as given below:

$$LCL_{\bar{x}} = \overline{\overline{X}} - A_2 \overline{R}$$

$$\Rightarrow LCL_{\bar{x}} = \overline{\overline{X}} - A_2 D^* \frac{T}{C_p}$$

$$\Rightarrow LCL_{\bar{x}} = \overline{\overline{X}} - A_2^* \frac{T}{C_p} \text{ Where } A_2^* = A_2 D^*$$

$$\Rightarrow UCL_R = D_4^* \frac{T}{C_p} \text{ Where } D_4^* = \frac{D_4 d_2}{6}$$

Table 1. Control Charts Constants for specified C_p value

n	d_2	D_3	D_4	A_2	D^*	D_3^*	D_4^*	A_2^*
2	1.128	0	3.267	1.880	0.1880	0.0000	0.6142	0.3534
3	1.693	0	2.574	1.023	0.2822	0.0000	0.7263	0.2887
4	2.059	0	2.282	0.729	0.3432	0.0000	0.7831	0.2502
5	2.326	0	2.115	0.557	0.3877	0.0000	0.8199	0.2159
6	2.534	0	2.004	0.483	0.4223	0.0000	0.8464	0.2040
7	2.704	0.076	1.924	0.419	0.4507	0.0343	0.8671	0.1888
8	2.847	0.136	1.864	0.373	0.4745	0.0645	0.8845	0.1770
9	2.970	0.184	1.816	0.337	0.4950	0.0911	0.8989	0.1668
10	3.078	0.223	1.777	0.308	0.5130	0.1144	0.9116	0.1580
11	3.173	0.256	1.744	0.285	0.529	0.135	0.922	0.151
12	3.258	0.283	1.717	0.266	0.543	0.154	0.932	0.144
13	3.336	0.307	1.693	0.249	0.556	0.171	0.941	0.138
14	3.407	0.328	1.672	0.235	0.568	0.186	0.949	0.133
15	3.472	0.347	1.653	0.223	0.579	0.201	0.957	0.129
16	3.532	0.363	1.637	0.212	0.589	0.214	0.964	0.125
17	3.588	0.378	1.622	0.203	0.598	0.226	0.970	0.121
18	3.640	0.391	1.608	0.194	0.607	0.237	0.976	0.118
19	3.689	0.403	1.597	0.187	0.615	0.248	0.982	0.115
20	3.735	0.415	1.585	0.180	0.623	0.258	0.987	0.112
21	3.778	0.425	1.575	0.173	0.630	0.268	0.992	0.109
22	3.819	0.434	1.566	0.167	0.637	0.276	0.997	0.106
23	3.858	0.443	1.557	0.162	0.643	0.285	1.001	0.104
24	3.895	0.451	1.548	0.157	0.649	0.293	1.005	0.102
25	3.931	0.459	1.541	0.153	0.655	0.301	1.010	0.100

One can use the above control limits to construct $R - Chart$ with specified C_p value.

Similarly one may obtain the upper control limit for the proposed $\overline{X} - Chart$ as

$$UCL_{\bar{x}} = \overline{\overline{X}} + A_2 \overline{R}$$

$$\Rightarrow UCL_{\bar{x}} = \overline{\overline{X}} + A_2 D^* \frac{T}{C_p}$$

$$\Rightarrow UCL_{\bar{x}} = \overline{\overline{X}} + A_2^* \frac{T}{C_p} \quad \text{Where} \quad A_2^* = A_2 D^*$$

Thus the control limits for the control charts with specified C_p value are as given below:

Control limits for proposed $R - Chart$:

$$CL_R = D^* \frac{T}{C_p}$$

$$LCL_R = D_3^* \frac{T}{C_p}$$

$$UCL_R = D_4^* \frac{T}{C_p}$$

Control limits for proposed $\overline{X} - Chart$:

$$CL_{\bar{x}} = \overline{\overline{X}}$$

$$LCL_{\bar{x}} = \overline{\overline{X}} - A_2^* \frac{T}{C_p}$$

$$UCL_{\bar{x}} = \overline{\overline{X}} + A_2^* \frac{T}{C_p}$$

One can use the above control limits to construct the $R - Chart$ and $\overline{X} - Chart$ with specified C_p value. For the sample size n $(2 \le n \le 25)$, we have presented the values of the control charts constants $D^*, D_2^*, D_3^* \; and \; A_2^*$ in the Table 1 given below.

The advantageous of the proposed control charts is that the observed range values are not required for computing the control limits. Further if the process is under the state of statistical control means that it automatically satisfies the required conditions regarding the values of process capability index C_p.

2.3. Proposed Method- Control Charts for Variables with Specified C_{pk} value

In the section 2.2, we have introduced a method for constructing control charts for variables with specified value for the process capability index C_p. In the similar manner we have developed a method for constructing control charts for variables with specified value for the process capability index C_{pk}. The following are the derivation of various computational formulae involved in constructing control charts for variables with specified process capability index C_{pk}

Consider $C_{pk} = Min\left\{C_{pl}, C_{pu}\right\}$

Where $C_{pu} = \dfrac{USL - \overline{\overline{X}}}{3\overline{R}/d_2}$ and $C_{pl} = \dfrac{\overline{\overline{X}} - LSL}{3\overline{R}/d_2}$

After a little algebra, one can rewrite the C_{pk} as

$$C_{pk} = \frac{\frac{T}{2} - \left|\overline{\overline{X}} - M\right|}{3\overline{R}/d_2}$$

If C_{pk} value is given then $\overline{R}$ value can be obtained from the above formula as

$$\overline{R} = \frac{\frac{T}{2} - \left|\overline{\overline{X}} - M\right|}{3C_{pk}/d_2} = \frac{d_2\left[\frac{T}{2} - \left|\overline{\overline{X}} - M\right|\right]}{3C_{pk}} = \frac{d_2}{3}\frac{\left[\frac{T}{2} - \left|\overline{\overline{X}} - M\right|\right]}{C_{pk}}$$

$$\Rightarrow \overline{R} = D_k^* \frac{\left[\frac{T}{2} - \left|\overline{\overline{X}} - M\right|\right]}{C_{pk}} \quad \text{Where} \quad D_k^* = \frac{d_2}{3}$$

Similarly one can obtain the lower control limit (LCL) and upper control limit (UCL) of the $R - Chart$ as given below:

$$CenterLine = CL_R = \overline{R} = D_k^* \frac{\left[\frac{T}{2} - \left|\overline{\overline{X}} - M\right|\right]}{C_{pk}}$$

$$LCL_R = D_3 \overline{R}$$

$$\Rightarrow LCL_R = D_3 D_k^* \frac{\left[\frac{T}{2} - \left|\overline{\overline{X}} - M\right|\right]}{C_{pk}}$$

$$\Rightarrow LCL_R = D_{3k}^* \frac{\left[\frac{T}{2} - \left|\overline{\overline{X}} - M\right|\right]}{C_{pk}}$$

Where $D_{3k}^* = \dfrac{D_3 d_2}{3} = D_3 D_k^*$ $\quad UCL_R = D_4 \overline{R}$

$$\Rightarrow UCL_R = D_4 D_k^* \frac{\left[\frac{T}{2} - \left|\overline{\overline{X}} - M\right|\right]}{C_{pk}}$$

$$\Rightarrow UCL_R = D_{4k}^* \frac{\left[\frac{T}{2} - \left|\overline{\overline{X}} - M\right|\right]}{C_{pk}} \quad \text{where}$$

$$D_{4k}^* = \frac{D_4 d_2}{3} = D_4 D_k^*$$

One can use the above control limits to construct

$R-Chart$ with specified C_{pk} value.

In the similar manner one can compute the control limits to construct $\overline{X}-Chart$ with specified C_{pk} value as given below.

Center line= $CL_{\bar{x}} = \overline{\overline{X}}$

Lower control limit for the proposed $\overline{X}-Chart$ is obtained as given below:

$$\Rightarrow \overline{R} = D_k^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}} \quad LCL_{\bar{x}} = \overline{\overline{X}} - A_2\overline{R}$$

$$\Rightarrow LCL_{\bar{x}} = \overline{\overline{X}} - A_2 D_k^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}}$$

$$\Rightarrow LCL_{\bar{x}} = \overline{\overline{X}} - A_{2k}^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}} \text{ Where } A_{2k}^* = A_2 D_k^*$$

Similarly one may obtain the upper control limit for the proposed $\overline{X}-Chart$ as

$$UCL_{\bar{x}} = \overline{\overline{X}} + A_2\overline{R}$$

$$\Rightarrow UCL_{\bar{x}} = \overline{\overline{X}} + A_2 D_k^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}}$$

$$\Rightarrow UCL_{\bar{x}} = \overline{\overline{X}} + A_{2k}^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}} \text{ Where } A_{2k}^* = A_2 D_k^*$$

Thus the control limits for the proposed control charts with specified C_{pk} value are as given below:

Control limits for $R-Chart$ with specified C_{pk} value

$$CL_R = \overline{R} = D_k^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}}$$

$$LCL_R = D_{3k}^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}}$$

$$UCL_R = D_{4k}^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}}$$

Control limits for the proposed $\overline{X}-Chart$ with specified C_{pk} value:

$$CL_{\bar{x}} = \overline{\overline{X}}$$

$$LCL_{\bar{x}} = \overline{\overline{X}} - A_{2k}^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}}$$

$$UCL_{\bar{x}} = \overline{\overline{X}} + A_{2k}^* \frac{\left[\frac{T}{2}-\left|\overline{\overline{X}}-M\right|\right]}{C_{pk}}$$

The above control limits can be further simplified by substituting the values of T and M, which lead to the following two cases:

Case 1: When $\overline{\overline{X}} > M$

Control limits for $R-Chart$ with specified C_{pk} value

$$CL_R = \overline{R} = D_k^* \left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right]$$

$$LCL_R = D_{3k}^* \left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right]$$

$$UCL_R = D_{4k}^* \left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right]$$

Control limits for the proposed $\overline{X}-Chart$ with specified C_{pk} value:

$$CL_{\bar{x}} = \overline{\overline{X}}$$

$$LCL_{\bar{x}} = \overline{\overline{X}} - A_{2k}^* \left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right]$$

$$UCL_{\bar{x}} = \overline{\overline{X}} + A_{2k}^* \left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right]$$

Case 2: When $\overline{\overline{X}} < M$

Control limits for $R-Chart$ with specified C_{pk} value

$$CL_R = \overline{R} = D_k^* \left[\frac{\overline{\overline{X}}-LSL}{C_{pk}}\right]$$

$$LCL_R = D_{3k}^* \left[\frac{\overline{\overline{X}}-LSL}{C_{pk}}\right]$$

$$UCL_R = D_{4k}^* \left[\frac{\overline{\overline{X}}-LSL}{C_{pk}}\right]$$

Control limits for the proposed $\overline{X}-Chart$ with specified C_{pk} value:

$$CL_{\bar{x}} = \overline{\overline{X}}$$

$$LCL_{\bar{x}} = \overline{\overline{X}} - A_{2k}^* \left[\frac{\overline{\overline{X}}-LSL}{C_{pk}}\right]$$

$$UCL_{\bar{x}} = \overline{\overline{X}} + A_{2k}^* \left[\frac{\overline{\overline{X}}-LSL}{C_{pk}}\right]$$

Table 2. Control Charts Constants for specified C_{pk} value

n	d_2	D_3	D_4	A_2	D_k^*	D_{3k}^*	D_{4k}^*	A_{2k}^*
2	1.128	0	3.267	1.880	0.376	0.000	1.228	0.707
3	1.693	0	2.574	1.023	0.564	0.000	1.453	0.577
4	2.059	0	2.282	0.729	0.686	0.000	1.566	0.500
5	2.326	0	2.115	0.557	0.775	0.000	1.640	0.432
6	2.534	0	2.004	0.483	0.845	0.000	1.693	0.408
7	2.704	0.076	1.924	0.419	0.901	0.069	1.734	0.378
8	2.847	0.136	1.864	0.373	0.949	0.129	1.769	0.354
9	2.970	0.184	1.816	0.337	0.990	0.182	1.798	0.334
10	3.078	0.223	1.777	0.308	1.026	0.229	1.823	0.316
11	3.173	0.256	1.744	0.285	1.058	0.271	1.845	0.301
12	3.258	0.283	1.717	0.266	1.086	0.307	1.865	0.289
13	3.336	0.307	1.693	0.249	1.112	0.341	1.883	0.277
14	3.407	0.328	1.672	0.235	1.136	0.372	1.899	0.267
15	3.472	0.347	1.653	0.223	1.157	0.402	1.913	0.258
16	3.532	0.363	1.637	0.212	1.177	0.427	1.927	0.250
17	3.588	0.378	1.622	0.203	1.196	0.452	1.940	0.243
18	3.640	0.391	1.608	0.194	1.213	0.474	1.951	0.235
19	3.689	0.403	1.597	0.187	1.230	0.496	1.964	0.230
20	3.735	0.415	1.585	0.180	1.245	0.517	1.973	0.224
21	3.778	0.425	1.575	0.173	1.259	0.535	1.983	0.218
22	3.819	0.434	1.566	0.167	1.273	0.552	1.994	0.213
23	3.858	0.443	1.557	0.162	1.286	0.570	2.002	0.208
24	3.895	0.451	1.548	0.157	1.298	0.586	2.010	0.204
25	3.931	0.459	1.541	0.153	1.310	0.601	2.019	0.200

One can use the above control limits to construct the $R-Chart$ and $\overline{X}-Chart$ with specified C_{pk} value. For the given sample size n $(2 \le n \le 25)$, we have presented the values of the control charts constants $D_k^*, D_{2k}^*, D_{3k}^*$ *and* A_{2k}^* in the Table 2 given below.

The advantageous of the proposed control charts with specified C_p and C_{pk} values, is that the observed range values are not required for computing the control limits. Further if the process is under the state of statistical control means that it automatically satisfies the required conditions regarding the values of process capability indices C_p and C_{pk}.

3. Numerical Example

Consider the data given below in Table 3 is the Example 5.1 (Montgomery[10], page 213). The data is pertaining to the manufacturing of Piston Rings for an automotive engine produced by a forging process. Twenty five samples, each of size five have been taken and the inside diameter is measured. The resulting data together with the sample means and sample range values are given below in Table 3.

From the above values we have obtained the following:

$\overline{\overline{X}} = 74.0011$, $\overline{R} = 0.0232$ and

$$\sigma = \overline{R}/d_2 = 0.0099$$

Case 1: Usual Method of Control Charts and the Computation of Process Capability Indices

Control Limits for R-Chart are obtained as:

$$CL_R = \overline{R} = 0.02324$$

$$LCL_R = D_3 * \overline{R} = 0 * 0.02324 = 0$$

$$UCL_R = D_4 * \overline{R} = 2.115 * 0.02324 = 0.0492$$

Control Limits for $\overline{X}$-Chart are obtained as:

$$CL_{\overline{x}} = \overline{\overline{X}} = 74.00118$$

$$LCL_{\overline{x}} = \overline{\overline{X}} - A_2 * \overline{R}$$
$$= 74.00118 - 0.577 * 0.02324 = 73.98777$$

$$UCL_{\overline{x}} = \overline{\overline{X}} + A_2 * \overline{R}$$
$$= 74.00118 - 0.577 * 0.02324 = 74.01459$$

By plotting the control limits, sample ranges and sample means, one may get $R-Chart$ and $\overline{X}-Chart$ as given in Figure 3 and Figure 4. From the $R-$ Chart, we observe that all the plotted sample range values are falling within the control limits. Hence we may conclude that the variations are under control. Similarly from the $\overline{X}-Chart$, we observe that all the plotted sample means are falling inside the control limits. Hence we may conclude that the averages are also under the statistical control.

Table 3. Data of Inside Diameter of Piston Rings (Spec: 74.000±0.05 mm)

Sample Number	x1	x2	x3	x4	x5	Sample Mean	Range R
1	74.030	74.002	74.019	73.992	74.008	74.0102	0.038
2	73.995	73.992	74.001	74.011	74.004	74.0006	0.019
3	73.988	74.024	74.021	74.005	74.002	74.0080	0.036
4	74.002	73.996	73.993	74.015	74.009	74.0030	0.022
5	73.992	74.007	74.015	73.989	74.014	74.0034	0.026
6	74.009	73.994	73.997	73.985	73.993	73.9956	0.024
7	73.995	74.006	73.994	74.000	74.005	74.0000	0.012
8	73.985	74.003	73.993	74.015	73.988	73.9968	0.030
9	74.008	73.995	74.009	74.005	74.004	74.0042	0.014
10	73.998	74.000	73.990	74.007	73.995	73.9980	0.017
11	73.994	73.998	73.994	73.995	73.990	73.9942	0.008
12	74.004	74.000	74.007	74.000	73.996	74.0014	0.011
13	73.983	74.002	73.998	73.997	74.012	73.9984	0.029
14	74.006	73.967	73.994	74.000	73.984	73.9902	0.039
15	74.012	74.014	73.998	73.999	74.007	74.0060	0.016
16	74.000	73.984	74.005	73.998	73.996	73.9966	0.021
17	73.994	74.012	73.986	74.005	74.007	74.0008	0.026
18	74.006	74.010	74.018	74.003	74.000	74.0074	0.018
19	73.984	74.002	74.003	74.005	73.997	73.9982	0.021
20	74.000	74.010	74.013	74.020	74.003	74.0092	0.020
21	73.982	74.001	74.015	74.005	73.996	73.9998	0.033
22	74.004	73.999	73.990	74.006	74.009	74.0016	0.019
23	74.010	73.989	73.990	74.009	74.014	74.0024	0.025
24	74.015	74.008	73.993	74.000	74.010	74.0052	0.022
25	73.982	73.984	73.995	74.017	74.013	73.9982	0.035
Average						74.00118	0.02324

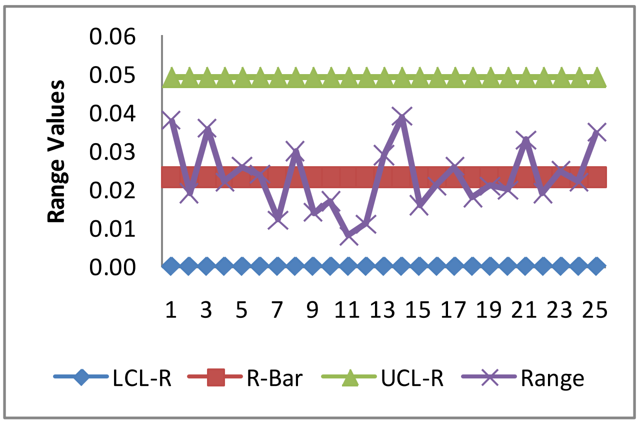

Figure 3. R-Chart- Usual Method

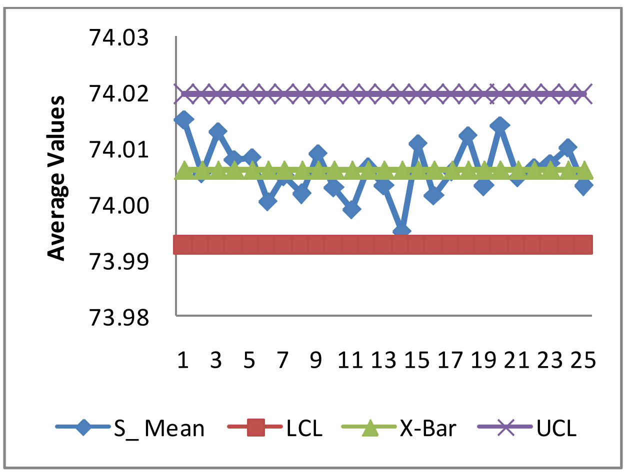

Figure 4. X-Bar Chart- Usual Method

The manufacturing capability of a process can normally be evaluated in terms of process capability indices C_p and C_{pk}. The process capability indices obtained from the above values are given below:

$$C_p = \frac{USL - LSL}{6\bar{R}/d_2} = \frac{74.05 - 73.95}{6*0.2324/2.326} = 1.6681$$

$$C_{pu} = \frac{USL - \bar{\bar{X}}}{3\bar{R}/d_2} = \frac{74.05 - 74.00118}{3*0.2324/2.326} = 1.6287$$

$$C_{pl} = \frac{\bar{\bar{X}} - LSL}{3\bar{R}/d_2} = \frac{74.00118 - 73.95}{3*0.2324/2.326} = 1.7075$$

$$C_{pk} = Min(C_{pu}, C_{pl}) = Min(1.6287, 1.7075) = 1.6287$$

If the customer's requirement of the process capability index C_p is 1.5, then one may conclude from the above capability indices that the process is an efficient one. However If the customer's requirement of the process capability index C_p is 2.0, then one may conclude from the above capability indices that the process is not an efficient one and hence the process has to be adjusted so as the resulting process capability index C_p is at least 2.0.

Case 2: Control Charts with specified $C_p = 1.5$

In this case we construct the control limits for the given value of process capability index C_p and hence separate computation of the process capability index is not required. Further the proposed control charts states that the process is under the statistical control means that the process satisfies the requirement of the process capability index. Hence one can construct several control limits for different values of process capability index C_p. If any plotted points falls outside the control limits with specified C_p= 1.5 means that the process does not satisfy the required process capability index C_p= 1.5.

Let the given process capability index C_p= 1.5. Then the Control limits for $R-Chart$ with specified C_p= 1.5 are obtained as given below:

$$CL_R = D^*.\frac{T}{C_p} = 0.3877\left[\frac{0.1}{1.5}\right] = 0.2585$$

$$LCL_R = D_3^*.\frac{T}{C_p} = 0\left[\frac{0.1}{1.5}\right] = 0$$

$$UCL_R = D_4^*.\frac{T}{C_p} = 0.8199\left[\frac{0.1}{1.5}\right] = 0.0547$$

Control limits for $\overline{X}-Chart$ with specified $C_p = 1.5$

$$CL_{\overline{x}} = \overline{\overline{X}} = 74.00118$$

$$LCL_{\overline{x}} = \overline{\overline{X}} - A_2^*.\frac{T}{C_p} = 74.00118 - 0.2159\left[\frac{0.1}{1.5}\right] = 73.9868$$

$$UCL_{\overline{x}} = \overline{\overline{X}} + A_2^*.\frac{T}{C_p} = 74.00118 + 0.2159\left[\frac{0.1}{1.5}\right] = 74.0156$$

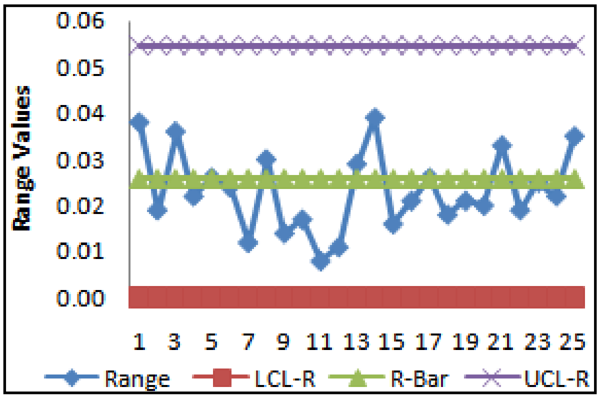

Figure 5. R- Chart with Specified Cp=1.5

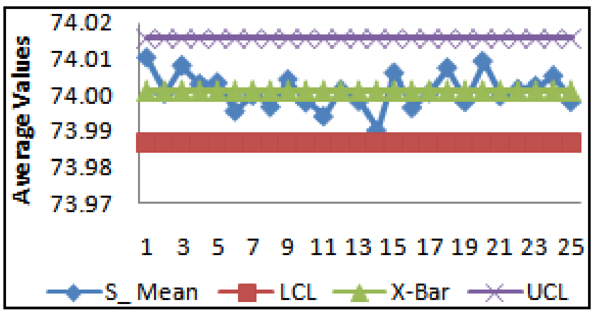

Figure 6. X-Bar Chart with Specified *Cp*=1.5

By plotting the control limits, sample ranges and sample means, one may get $R-Chart$ and $\overline{X}-Chart$ with specified $C_p = 1.5$ as given in Figure 5 and Figure 6. From the $R-Chart$, we observe that all the plotted sample range values are falling within the control limits. Hence we may conclude that the variations are under control. Similarly from the $\overline{X}-Chart$, we observe that all the plotted sample means are falling inside the control limits. Hence we may conclude that the averages are also under the statistical control. Further we conclude that the process capability index for the given process is at least 1.5 and satisfies the customer's requirements.

Case 3: Control Charts with specified C_p=1.5

In this case we construct the control limits for the given value of process capability index C_{pk} and hence separate computation of the process capability index is not required. Further the proposed control charts states that the process is under the statistical control means, the process satisfies the requirement of the process capability index. Hence one can construct several control limits for different values of process capability index. If any plotted points falls outside the control limits with specified C_{pk}=1.5 means that the process does not satisfy the required process capability index. C_{pk}=1.5.

Let the given process capability index C_p = 1.5. Further $\overline{\overline{X}} = 74.00118 > M = 74.00$ the control limits with specified process capability index C_{pk}=1.5 are obtained as given below:

Control limits for $R-Chart$ with specified C_{pk} value

$$CL_R = \overline{R} = D_k^*\left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right] = 0.775\left[\frac{74.05-74.00118}{1.5}\right] = 0.025224$$

$$LCL_R = D_{3k}^*\left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right] = 0\left[\frac{74.05-74.00118}{1.5}\right] = 0$$

$$UCL_R = D_{4k}^*\left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right] = 1.64\left[\frac{74.05-74.00118}{1.5}\right] = 0.053377$$

Control limits for the proposed $\overline{X}-Chart$ with specified $C_{pk} = 1.5$ value are obtained as given below:

$$CL_{\overline{x}} = \overline{\overline{X}} = 74.00118$$

$$LCL_{\overline{x}} = \overline{\overline{X}} - A_{2k}^*\left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right]$$

$$= 74.00118 - 0.432\left[\frac{74.05-74.00118}{1.5}\right]$$

$$= 73.98712$$

$$UCL_{\overline{x}} = \overline{\overline{X}} + A_{2k}^*\left[\frac{USL-\overline{\overline{X}}}{C_{pk}}\right]$$

$$= 74.00118 + 0.432\left[\frac{74.05-74.00118}{1.5}\right]$$

$$= 74.01524$$

By plotting the control limits, sample ranges and sample means, one may get $R-Chart$ and $\overline{X}-Chart$ with specified C_{pk}=1.5 as given in Figure 7 and Figure 8. From the $R-Chart$, we observe that all the plotted sample range values are falling within the control limits. Hence we may conclude that the variations are under control. Similarly from the $\overline{X}-Chart$, we observe that all the plotted sample means are falling inside the control limits. Hence we may conclude that the averages are also under the statistical control. Further we conclude that the process capability index C_{pk} for the given process is at least 1.5 and satisfies the customer's requirements.

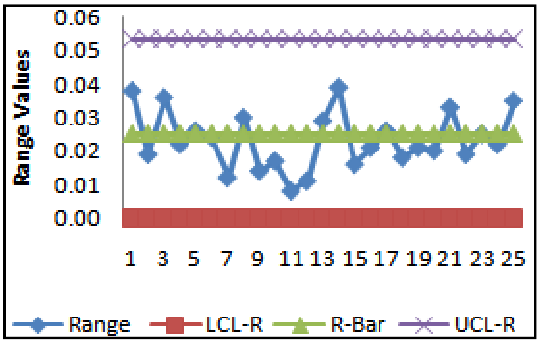

Figure 7. R-Chart with Specified Cpk=1.5

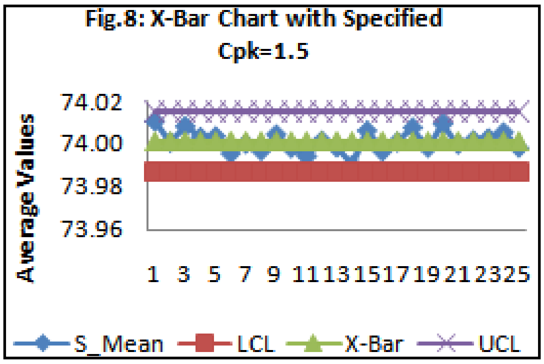

Figure 8. X-Bar Chart with Specified Cpk=1.5

4. Conclusions

In this paper, we have proposed a control chart which is based on the process capability indices C_p and C_{pk} for on line process control, which have the benefit of the usual control charts and the process capability indices. That is, the proposed control charts combines the two stage controlling mechanism namely, Control Charts and the process capability indices into a single stage controlling mechanism to monitor the process online and to assess the suitability of the manufacturing process. It has been shown that the relative performance of the proposed control charts for variables can be assessed with that of usual variable control charts. It has also been shown that the proposed control chart is simple to apply and does not warrant any tedious computations both for control charts and for computing process capability indices. Moreover, we have also presented tables for the control charts constants for computing the control limits of the capability based control charts. The proposed method has also been illustrated with a numerical example. This study can also be extended to develop control charts based on other process capability indices.

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