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Mathematics Solutions for Inverse Tangents

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Abstract Sinusoidal circuit analysis is very common in Alternating Current (AC) electrical circuit analysis and so close to the term of phasor concept using complex frequency, which is mainly use trigonometry figures such as sin, cos, and tangent, for calculations. One of the most important tasks that we will meet e.g. finding phase angles using inverse tangent formula. The author presents this paper to introduce a mathematic solution derived from basic concept of trigonometry, to solve circuit analysis problems related to phase angles and inverse tangents.

Keywords Trigonometry, Inverse Tangent, Sinusoidal, Complex Frequency

1. Introduction

In Electric Circuits, beside learning terms related to sinusoidal circuit analysis, we also would learn analysis problems refer to complex frequency. Of course we still use the phasor concept for calculations, but to solve the complex problems we must use a complex frequency.[1],[3],[4],[10],[11]

While doing some analysis, we will meet one of the specific problems for finding the unknown phase angle. For example, it is being figure out that there are two different kinds of phase angle from a sinusoidal circuit analysis. The sum of inverse tangent for both angles is P_1, and the difference is P_2. The task is to determine the sum of those phase angles. How will you do it? Of course you need to find the value of the two phase angles and then add the two values to get the result that you want.

$$\tan^{-1}x + \tan^{-1}y = P_1 \tag{1}$$

$$\tan^{-1}x - \tan^{-1}y = P_2 \tag{2}$$

P_1 and P_2 $\rightarrow$ known variables

x and y $\rightarrow$ unknown variables (calculated variables, i.e. phase angles)

To solve this model of electric circuit problem, we have to review again the topic related to trigonometric functions, because in here the problem uses parameters related to inverse tangent. We should derive the mathematic solutions for equation (1) and (2) that would be the right formulas for the problem.

1.1. Aims of Study

This paper will explain how to derive a mathematic solution for inverse tangents. I hope it can offer a good solution for students, lecturers, and engineers who had difficulty in solving analysis problems related to these trigonometric functions.

1.2. Research Method

To derive a mathematic solution for an inverse tangent related to phase angle, I did some analysis, derived the final solution from the basic trigonometry formula for tangent and inverse tangent. Then I made few calculations and comparisons to see if the analysis results match the calculated results.

2. Discussion and Results

Let us review and remember again the trigonometric equations for tangent and inverse tangent that we had learned in mathematics or calculus subjects.

2.1. Definition

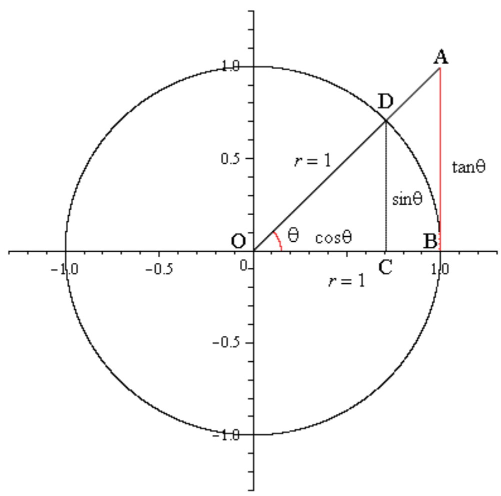

Figure 1. Circular Function of Trigonometry (Source: http://www.mathnotes.org/?pid=28#?pid=27)

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In a coordinate system of a circle, if we draw two lines inside the circle with θ angle, it will form an equation like following:[2],[5],[7],[8],[9],[16]

$$\theta = \frac{s}{r} \tag{3}$$

θ is measured in **radians**

s is the **arc length** and **r** is the **radius**

Taking the r = 1, as P moves around the circle, we define sine θ to be the y-component of the radial arm from the origin to P, and cosine θ to be the x-component of the radial arm from the origin to P. tan θ is defined as the ratio of sine and cosine or geometrically as the line segment of AB:[16]

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{4}$$

The tangent function is an odd symmetric function, where tan (-θ) = - tan (θ) and is periodic.

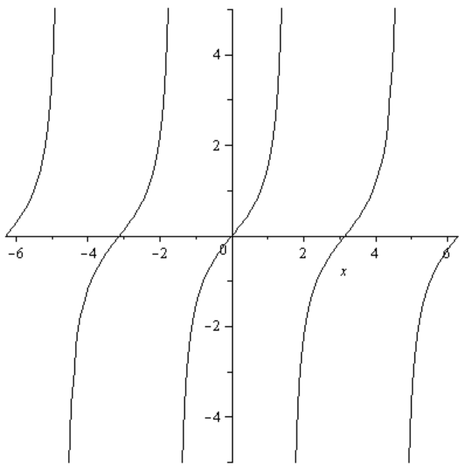

Figure 2. Graph of Tangents (Source: http://www.mathnotes.org/?pid=28#?pid=28)

Since tangent is the ratio of sine and cosine, the value of the tangent is undefined at the values of θ which make cos θ = 0.

To define the inverse tangent, we restrict the value of x:

$$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \tag{5}$$

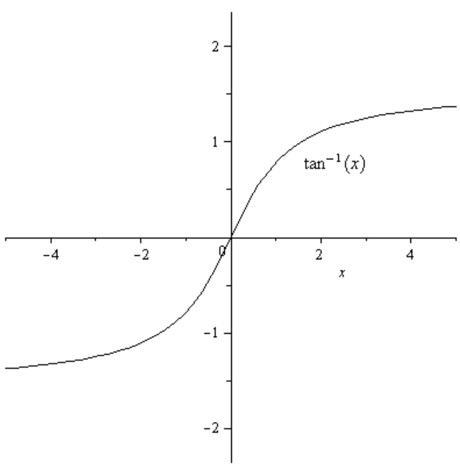

Figure 3. Graph of Inverse Tangents (Source: http://www.mathnotes.org/?pid=28#?pid=28)

2.2. Relationship between Radians and Degrees

Sinusoidal circuits are familiar with radians and degree as the units of θ.[6],[12],[13]

$$1 \text{ radian} = 180^{o} / \pi = 57.2957795 \tag{6}$$

$$1^{o} = \pi / 180 \text{ radians} = 0.01745329 \tag{7}$$

2.3. Tangent and Inverse Tangent Functions

Inverse tangent is the inverse function of a tangent, so it can be written in a relationship:

$$x = \tan A$$

$$A = \tan^{-1} x$$

$$\tan(\tan^{-1} x) = x \tag{8}$$

Using trigonometric functions for sine and cosine, tangent functions for two added values or difference values are given as follows:[12],[14],[15]

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \tag{9}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \tag{10}$$

2.4. Derive the Mathematic Solutions

From these basic trigonometry formulas, we will derive another simple formula for the equation (1) and (2) that we are trying to solve. We will solve them one by one, respectively.

I. $\tan^{-1}x + \tan^{-1}y$

Firstly we will find the solution for equation (1). It should be started from the equation (9).

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Assume:

$$x = \tan A$$

$$A = \tan^{-1} x$$

$$y = \tan B$$

$$B = \tan^{-1} y$$

so we will get:

$$\tan\left(\tan^{-1} x + \tan^{-1} y\right) = \frac{x+y}{1-xy}$$

$$\tan\left(\tan^{-1} x + \tan^{-1} y\right) = \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

Eliminating the tan function outside the brackets, we finally have the result:

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \quad 11)$$

If, $\tan^{-1}x + \tan^{-1}y = P_1$, then

$$P_1 = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \tag{12}$$

$$\left(\frac{x+y}{1-xy}\right) = \tan \quad P_1 \tag{13}$$

II. $\tan^{-1}x - \tan^{-1}y$

Secondly we will find the solution for equation (2). This time it should be started from the equation (10).

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$\begin{aligned} &Assume: \\ &x = \tan A \\ &A = \tan^{-1} x \\ &y = \tan B \\ &B = \tan^{-1} y \end{aligned}$$

so we will get:

$$\tan\left(\tan^{-1} x - \tan^{-1} y\right) = \frac{x-y}{1+xy}$$

$$\tan\left(\tan^{-1} x - \tan^{-1} y\right) = \tan\left[\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

Eliminating the tan function outside the brackets, we finally have the result:

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right) \tag{14}$$

If, $\tan^{-1}x - \tan^{-1}y = P_1$, then

$$P_2 = \tan^{-1}\left(\frac{x-y}{1+xy}\right) \tag{15}$$

$$\left(\frac{x-y}{1+xy}\right) = \tan \quad P_2 \tag{16}$$

If P_1 and P_2 are known variables, by eliminating and substituting equation (13) and (16), we could derive the value for x and y. To prove the derived solutions, let's try to conduct some calculations to make a comparison for the result we will get using the final solutions that have been described above.

I. $\tan^{-1}x + \tan^{-1}y$

Using electronic calculator for trigonometric functions, a mathematic analysis has been taken for both side of equations.[6]

For the equation of $\tan^{-1}x + \tan^{-1}y$, values of variable x and y are selected randomly to derive the calculated results as follows:

$\tan^{-1}1 + \tan^{-1}2 = 108{,}434949^{o}$
$\tan^{-1}2 + \tan^{-1}4 = 139{,}398705^{o}$
$\tan^{-1}3 + \tan^{-1}6 = 152{,}102729^{o}$
$\tan^{-1}4 + \tan^{-1}8 = 158{,}838740^{o}$
$\tan^{-1}5 + \tan^{-1}10 = 162{,}979474^{o}$
$\tan^{-1}6 + \tan^{-1}12 = 165{,}774036^{o}$
$\tan^{-1}1 + \tan^{-1}3 = 116{,}565051^{o}$
$\tan^{-1}4 + \tan^{-1}1 = 120{,}963757^{o}$
$\tan^{-1}6 + \tan^{-1}30 = 168{,}628525^{o}$

With the same values of variable x and y which have selected randomly, the calculated results are derived for the equation of $\tan^{-1}((x+y)/(1-xy))$ as follows:

$\tan^{-1}(-3/1) = 108{,}434949^{o}$
$\tan^{-1}(-6/7) = 139{,}398705^{o}$
$\tan^{-1}(-9/17) = 152{,}102729^{o}$
$\tan^{-1}(-12/31) = 158{,}838740^{o}$
$\tan^{-1}(-15/49) = 162{,}979474^{o}$
$\tan^{-1}(-18/71) = 165{,}774036^{o}$
$\tan^{-1}(-4/2) = 116{,}565051^{o}$
$\tan^{-1}(-5/3) = 120{,}963757^{o}$
$\tan^{-1}(-36/179) = 168{,}628525^{o}$

II. $\tan^{-1}x - \tan^{-1}y$

Again, for the equation of $\tan^{-1}x - \tan^{-1}y$, values of variable x and y are selected randomly to derive the calculated results as follows:

$\tan^{-1}2 - \tan^{-1}1 = 18{,}434949^{o}$
$\tan^{-1}4 - \tan^{-1}2 = 12{,}528808^{o}$
$\tan^{-1}6 - \tan^{-1}3 = 8{,}972627^{o}$
$\tan^{-1}8 - \tan^{-1}4 = 6{,}911227^{o}$
$\tan^{-1}10 - \tan^{-1}5 = 5{,}599339^{o}$
$\tan^{-1}1 - \tan^{-1}2 = -18{,}434949^{o}$
$\tan^{-1}2 - \tan^{-1}4 = -12{,}58808^{o}$
$\tan^{-1}3 - \tan^{-1}6 = -8{,}972627^{o}$
$\tan^{-1}4 - \tan^{-1}8 = -6{,}911227^{o}$
$\tan^{-1}5 - \tan^{-1}10 = -5{,}599339^{o}$
$\tan^{-1}8 - \tan^{-1}6 = 2{,}337306^{o}$
$\tan^{-1}10 - \tan^{-1}7 = 2{,}419509^{o}$
$\tan^{-1}20 - \tan^{-1}10 = 2{,}848188^{o}$
$\tan^{-1}45 - \tan^{-1}23 = 1{,}216523^{o}$
$\tan^{-1}6 - \tan^{-1}8 = -2{,}337306^{o}$
$\tan^{-1}7 - \tan^{-1}10 = -2{,}419509^{o}$
$\tan^{-1}10 - \tan^{-1}20 = -2{,}848188^{o}$
$\tan^{-1}23 - \tan^{-1}45 = -1{,}216523^{o}$

With the same values of variable x and y which have selected randomly, the calculated results are derived for the equation of $\tan^{-1}((x-y)/(1+xy))$ as follows:

$\tan^{-1}(1/3) = 18{,}434949^{o}$
$\tan^{-1}(2/9) = 12{,}528808^{o}$
$\tan^{-1}(3/19) = 8{,}972627^{o}$
$\tan^{-1}(4/33) = 6{,}911227^{o}$
$\tan^{-1}(5/51) = 5{,}599339^{o}$
$\tan^{-1}(-1/3) = -18{,}434949^{o}$
$\tan^{-1}(-2/9) = -12{,}528808^{o}$
$\tan^{-1}(-3/19) = -8{,}972627^{o}$
$\tan^{-1}(-4/33) = -6{,}911227^{o}$
$\tan^{-1}(-5/51) = -5{,}599339^{o}$
$\tan^{-1}(2/49) = 2{,}337306^{o}$
$\tan^{-1}(3/71) = 2{,}419509^{o}$
$\tan^{-1}(10/201) = 2{,}848188^{o}$
$\tan^{-1}(22/1036) = 1{,}216523^{o}$
$\tan^{-1}(-2/49) = -2{,}337306^{o}$
$\tan^{-1}(-3/71) = -2{,}419509^{o}$
$\tan^{-1}(-10/201) = -2{,}848188^{o}$
$\tan^{-1}(-22/1036) = -1{,}216523^{o}$

From the mathematical analysis being given above, it shows and clearly states that the statement of equation (11) and equation (14) related to inverse tangents are true and can be used for deriving phase angle solutions related to sinusoidal circuit analysis problems.

Suppose a sequential circuit analysis problem only states two variables, $P_1 = 108{,}434949\,^{o}$ and $P_2 = 18{,}434949\,^{o}$. Our task is to determine the value of x and y. With equation (13) and (16), we could derive the value for x and y.

$$\left(\frac{x+y}{1-xy}\right) = \tan\ 108{,}434949 = \text{-}2{,}99 \qquad (17)$$

$$x + y = -(2{,}99 + 2{,}99xy)$$

$$x + y + 2{,}99xy = -2{,}99$$

$$\left(\frac{x-y}{1+xy}\right) = \tan 18{,}434949 = 0{,}33 \qquad (18)$$

$$x - y = 0{,}33 + 0{,}33xy$$

$$x - y - 0{,}33xy = 0{,}33$$

The rest would be quite easy to find the result using elimination, substitution, and square equations for x and y.

3. Conclusions

The research derives two mathematic solutions for inverse tangent. First, the addition of two inverse tangents equals to the addition of variable x and y, divided by the difference of one and multiplication of variable x and y. Second, the difference of two inverse tangents equals to the difference of variable x and y, divided by the addition of one and multiplication of variable x and y.

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