# Performance Analysis of Three Phase Squirrel Cage Induction Motor with Deep Rotor Bars in Transient Behavior

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**Abstract** The rotor parameters, the resistance and the leakage inductance, vary with the currents frequency of rotor bars due to skin effect, and for small frequencies can be negligible (under 10 15 Hz); but for higher frequencies this variation of the parameters cannot be negligible. In this paper, for higher frequencies some analytic expressions are proposed, that result from the skin effect theory, for the rotor resistance and for the leakage inductance. The simulations were done by SYMNAP – SYmbolic Modified Nodal Analysis Program, a program that can determine, in a full-symbolic form the voltages and the currents for the equivalent scheme for one phase of the asynchronous motor. Symbolic expressions for the characteristics quantities of the general regime of the induction motor allow graphic representation for different parameters. These analytic expressions allows a more accurate analysis of asynchronous motor performance for different drive systems. To simulate the dynamic (transient) behavior of the induction motor, Park-Blondel equations have been used; these equations have a nonlinear form due to both their structure and frequency variation of the rotor parameters. The integration of these equations is performed with the help of the integration routines for ordinary differential equations in Matlab programming environment. It was found that the nominal values of the quantities that characterize the induction motor operation, obtained by simulation, practically coincide with the values from the catalogue.

Keywords Induction Motor, State Equations, Transient Behavior, Deep Rotor Bars

## 1. Introduction

Numerous studies have been made on estimating the electrical parameters of the asynchronous machine given both skin effect, and the saturation level[1-9]. For the electrical drives with three phase induction motors with deep rotor bars were developed mathematical models and command strategies for developing high performance systems[6]. In electrical drives it is preferred the vector command in respect to the rotor flux, a flux that cannot be defined using one form for motor with deep rotor bars; as a result, in many cases it is used the air gap flux for the vector command. Using the pseudorotor-flux, it can be achieved similar performances as with the rotor flux orientation, in this case, the equivalent rotor parameters are used. The squirrel cage with deep rotor bars can be equivalent with a double cage, one of these cages having constant parameters [8].

Based on the influence of the skin effect over electrical rotor parameters can be constructed a mathematical model

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for the asynchronous motor with deep rotor bars that allows simulation of different operating modes are presented in this paper. It is considered constant stator current frequency, equal to the network frequency at which the motor is fed. Rotor parameters are considered constant, independent of the rotor current frequency, for values between 0 and 10...15 Hz. For higher frequencies, analytic expressions are proposed, that result from the skin effect theory, for the rotor resistance and leakage inductance. In sinusoidal steady-state, simulations were performed by the SYMNAP - SYmbolic Modified Nodal Analysis[10], a program that allows complete symbolic determination of the branch voltages and currents of the equivalent scheme corresponding to one phase of the induction motor. In order to simulate the induction motor in the transient (dynamic) behaviour we use the Park-Blondel equations. These equations have a nonlinear form due to both their structure and the variation of the rotor parameters in respect of the frequency. The integration of these equations is performed with the help of the integration routines for ordinary differential equations in Matlab programming environment.

# 2. The Mathematical Model of the Induction Motor

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The mathematical model of the induction motor used for any behaviors is elaborated in synchronous coordinate (Park-Blondel equations) and is presented as it follows:

$$u_{sK} = R_s i_{sK} + j\omega_K \psi_{sK} + \frac{d\psi_{sK}}{dt} \Longrightarrow$$
(1a)

$$\Rightarrow \begin{cases} u_{sd} = R_s i_{sd} - \omega_1 \psi_{sq} + \frac{d\psi_{sd}}{dt} \\ u_{sq} = R_s i_{sq} + \omega_1 \psi_{sd} + \frac{d\psi_{sq}}{dt} \end{cases}$$
(1b)

$$\psi_{sK} = L_{s\sigma}i_{sK} + L_{\mu}i_{\mu K} \Rightarrow \begin{cases} \psi_{sd} = L_{s\sigma}i_{sd} + L_{\mu}i_{\mu d} \\ \psi_{sq} = L_{s\sigma}i_{sq} + L_{\mu}i_{\mu q} \end{cases}$$
(1c)

$$0 = R'_{r}i'_{rK} + j(\omega_{K} - \omega)\psi'_{rK} + \frac{d\psi'_{rK}}{dt} \Longrightarrow \qquad (1d)$$

$$0 = R'_{r}i'_{rd} - (\omega_1 - \omega)\psi'_{rq} + \frac{d\psi'_{rd}}{dt}$$

$$0 = R'_{r}i'_{rq} + (\omega_1 - \omega)\psi'_{rd} + \frac{d\psi'_{rq}}{dt}$$
(1e)

$$\psi'_{rK} = \dot{L_{r\sigma}i_{rK}} + L_{\mu}i_{\mu K} \Rightarrow \begin{cases} \psi'_{rd} = \dot{L_{r\sigma}i_{rd}} + L_{\mu}i_{\mu d} \\ \psi'_{rg} = \dot{L_{r\sigma}i_{rg}} + L_{\mu}i_{\mu q} \end{cases}$$
(1f)

$$J\frac{d\Omega}{dt} = \frac{J}{p}\frac{d\omega}{dt} = M - M_2; \qquad (1g)$$

$$M = \frac{3}{2} p \left( \psi_{sd} i_{sq} - \psi_{sq} i_{sd} \right); \quad \omega_K = \omega_1 \qquad (1h)$$

The general coordinate system K, which has the angular velocity  $\omega_K$  in respect to the stator coordinate system, is particularized for the synchronous coordinate with speed  $\omega_1$ , this means that  $\omega_K = \omega_1$ .

In the system equations (1) the rotor cage parameters  $R'_r$ and  $L'_{r\sigma}$  dependent on the rotor bars angular frequency  $\omega_r$ , and the others electrical parameters are constants. If the simulations are performed for only one stator frequency (such as nominal stator frequency) the parameters  $R'_r$  and  $L'_{r\sigma}$  vary with the slip s. For simulations at different stator frequencies is absolutely necessary that the rotor parameters are function of the angular frequency of the currents from the rotor bars  $\omega_r$ , (for the stator frequency of  $f_1 = 5$  Hz, and s = 1, the influence of the skin effect is negligible, that means the

#### **2.1. Rotor Electrical Parameters**

dependence on the slip s is not relevant).

The skin effect analysis shows that the rotor resistance  $R_r'$  reported at the stator increases with  $\sqrt{\omega_r}$ , and leakage inductance  $L_{r\sigma}'$  decreases with  $\sqrt{\omega_r}$ . Therefore, the rotor parameters reported at the stator can be written as it follows:  $R'(\omega_r) = R' = ct$  for  $\omega_r \in (0; \omega_r)$ :

$$\begin{aligned} R_r(\omega_r) &= R_r - ct. \text{ for } \omega_r \in (0, \omega_{rx}), \\ R_r'(\omega_r) &= K_1 + K_2 \sqrt{\omega_r}; \quad \omega_r > \omega_{rx}. \\ L_r'(\omega_r) &= L_r' = ct. \text{ for } \omega_r \in (0; \omega_{rx}); \end{aligned}$$
(2)

$$L_{r}\left(\omega_{r}\right) = K_{3} - \frac{K_{4}}{\sqrt{\omega_{r}}}; \quad \omega_{r} > \omega_{rx}.$$

The constants  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  are determined from the continuity condition at the point  $\omega_r = \omega_{rx}$  and calculation of the electrical parameters for the squirrel cage at starting point (s = 1), these parameters are obtained from the catalogue data. Till the pulsation is  $\omega_r = \omega_{rx}$ , the rotor parameters are considered constant, and for  $\omega_r \ge \omega_{rx}$  the rotor parameters vary with pulsation  $\omega_r$  according to relations (2). These are valid for a wide range of frequency from rotor circuit.

#### 2.1.1. Rotor Parameters at Starting

Rotor squirrel cage is made with deep bars, parameters  $R'_r$  and  $L'_{r\sigma}$  vary in large limits when the rotor angular frequency  $\omega_r$  takes values between 0 and  $\omega_1$ ; at the starting (n= 0), the rotor angular frequency  $\omega_r$  is equal to the stator angular frequency  $\omega_1$ , in this case the skin effect is strong ( $\omega_1 = 2 \cdot \pi \cdot 60$ ).

The catalogue data for the analysed induction motor are presented in table 1 with the known notations.

Table 1. The Catalogue data for the Motor

P <sub>n</sub> [kW]	U <sub>n</sub> [V]	I <sub>n</sub> [A]	n <sub>1</sub> [rot/min]		N [rc	ot/min]	η <sub>n</sub> [%]
$cos\phi_{1n}$ -	i <sub>p</sub> -	m <sub>p</sub> -	m <sub>m</sub> -	J [kgm <sup>2</sup> ]		Star connection	

Based on these data the following quantities can be calculated:  $P_{1n}$ ,  $s_n$ ,  $M_n$ ,  $S_n$ . Next step for the calculation of the starting parameters is: the starting impedance  $Z_p$ , the rotor resistance at starting point  $R'_{rp}$  and the leakage inductance at staring point  $L'_{rp\sigma}$ . In the computation process the rated impedance  $Z_n$  is used. Using the known relations from the asynchronous motor theory, for n = 0, the rotor resistance  $R'_{rp}$  is computed, thus:

$$s = 1 \Rightarrow \omega_{r} = \omega_{l} \Rightarrow Z_{p} = \frac{U_{fn}}{I_{p}} = \frac{U_{fn}}{i_{p} I_{fn}} = \frac{Z_{n}}{i_{p}};$$

$$Z_{n} = \frac{U_{fn}}{I_{fn}}; \quad M = \frac{P_{Jr}}{\Omega_{1}s} = \frac{3R'_{r}I'^{2}}{\Omega_{1}s}$$

$$s = 1 \Rightarrow M_{p} = \frac{3R'_{rp}I'^{2}}{\Omega_{1}} \cong \frac{3R'_{rp}I^{2}_{sp}}{\Omega_{1}};$$

$$(I_{s0} \cong 0 \Rightarrow I'_{rp} \cong I_{sp});$$

$$\frac{M_{p}}{M_{n}} = m_{p} \cong \frac{3R'_{rp}I^{2}_{sp}}{\Omega_{1}M_{n}} \frac{\Omega_{n}}{\Omega_{n}} = \frac{3R'_{rp}I^{2}_{sp}}{P_{n}}(1 - s_{n}) \Rightarrow$$

$$R'_{rp} = \frac{m_{p}P_{n}}{3(1 - s_{n})(i_{p}I_{fn})^{2}}$$
(3a)
(3b)

The values of the parameters  $Z_{p}$ ,  $R_s$ ,  $L_{s\sigma}$ ,  $R'_{rp}$  are known and the leakage inductance  $L'_{rp\sigma}$  can be calculated, at staring point, with the following relation:

$$s = 1 \Longrightarrow X_{p\sigma} = \omega_1 L_{p\sigma} = \omega_1 \left( L_{s\sigma} + L'_{rp\sigma} \right) =$$

$$= \sqrt{Z_p^2 - \left( R_s + c_1 R'_{rp} \right)^2}; \quad c_1 \cong 1$$

$$\Rightarrow L'_{rp\sigma} = \frac{\sqrt{Z_p^2 - \left( R_s + c_1 R'_{rp} \right)^2}}{\omega_1} - L_{s\sigma}$$
(4)

At this stage, in the absence of the skin effect the parameters  $R'_r$  and  $L'_{r\sigma}$  are known and if skin effect is taken into account we will use the parameters  $R'_{rp}$ ,  $L'_{rp\sigma}$ . The calculation of the K<sub>1</sub>, K<sub>2</sub>, K<sub>3</sub> and K<sub>4</sub> constants leads to the next relations for  $R'_r$  and  $L'_{r\sigma}$ .

 $L_{\mu} = ct.$ 

$$R_{r}^{'}(\omega_{r}) = \begin{cases} R_{r}^{'}; \quad \omega_{r} \in [0; \, \omega_{rx}] \\ \frac{R_{r}^{'}\sqrt{\omega_{1}} - R_{rp}^{'}\sqrt{\omega_{rx}}}{\sqrt{\omega_{1}} - \sqrt{\omega_{rx}}} + \frac{R_{rp}^{'} - R_{r}^{'}}{\sqrt{\omega_{1}} - \sqrt{\omega_{rx}}}\sqrt{\omega_{r}}; \\ \omega_{r} > \omega_{rx} \end{cases}$$

$$L_{r\sigma}^{'}(\omega_{r}) = \begin{cases} L_{r\sigma}^{'}; \quad \omega_{r} \in [0; \, \omega_{rx}] \\ \frac{\sqrt{\omega_{rx}} L_{r\sigma}^{'} - \sqrt{\omega_{1}} L_{rp\sigma}^{'}}{\sqrt{\omega_{rx}} - \sqrt{\omega_{1}} - \frac{(L_{r\sigma}^{'} - L_{rp\sigma}^{'})\sqrt{\omega_{1}}\sqrt{\omega_{rx}}}{\sqrt{\omega_{rx}} - \sqrt{\omega_{1}} - \frac{1}{\sqrt{\omega_{rx}}}}; \end{cases}$$

$$(5a)$$

$$(5a)$$

$$L_{r\sigma}^{'}(\omega_{r}) = \frac{\left(L_{r\sigma}^{'}; \omega_{r} \in [0; \omega_{rx}] - \frac{(L_{r\sigma}^{'} - L_{rp\sigma}^{'})\sqrt{\omega_{1}}\sqrt{\omega_{rx}}}{\sqrt{\omega_{rx}} - \sqrt{\omega_{1}} - \frac{1}{\sqrt{\omega_{rx}}}}; \end{cases}$$

The last two relations in system (5) highlight the variation of the rotor parameters  $R'_r(\omega_r)$  and  $L'_{r\sigma}(\omega_r)$  in respect of the  $\omega_r$ , due to skin effect. The mathematical structure of the model (1), remains the same, except that the parameters  $R'_r$ and  $L'_{r\sigma}$  become  $R'_r(\omega_r)$ , and  $L'_{r\sigma}(\omega_r)$ , respectively, according to relation (5).

For the relative rotor resistance reported to the stator  $R'_r$  is known relationship that rotor resistance in relative unit  $r'_r$  is equal to the rated slip  $s_n$  as shown in the following relations:

$$R_{r}^{'} = \frac{s_{n}(P)_{n}}{3I_{r}^{'2}} = \frac{s_{n}(P_{M})_{n}}{3I_{r}^{'2}(1-s_{n})} = \frac{s_{n}(P_{n}+P_{vf})}{3I_{r}^{'2}(1-s_{n})} \cong$$

$$\frac{s_{n}P_{n}}{3I_{r}^{'2}(1-s_{n})} \cong s_{n} \frac{\eta_{n}3U_{sn}I_{sn}\cos\varphi_{sn}}{3(I_{sn}\cos\varphi_{sn})^{2}(1-s_{n})} \Longrightarrow \qquad (6)$$

$$\Rightarrow \frac{R_{r}^{'}}{Z_{b}} = R_{r}^{'} \frac{I_{sn}}{U_{sn}} = r_{r}^{'} = s_{n} \frac{\eta_{n}}{(1-s_{n})\cos\varphi_{sn}};$$

$$\frac{\eta_{n}}{(1-s_{n})\cos\varphi_{sn}} \cong 1 \Rightarrow r_{r}^{'} \cong s_{n}.$$

# 2.2. The Study of Asynchronous Three Phase Motor with Known Parameters

In order to simulate different operating behaviors of the three phase asynchronous motor expressions obtained in section 2.1 we will use the rated values presented in table 2:

Table 2. Rated Values for the Motor

P <sub>n</sub> [kW]	$U_n[V]$	$I_n[A]$	n <sub>1</sub> [rpm]	N [rpm]	η <sub>n</sub> [%]	
100	560	130	1200	1168,8	0,897	
cosq <sub>1n</sub> -	i <sub>p</sub> -	m <sub>p</sub> -	m <sub>m</sub> -	J [kgm <sup>2</sup> ]	Connect.	
0,87	4	1,1	1,8	3,38	star	

U<sub>fn</sub>=323,32 V; I<sub>fn</sub>=130 A; Z<sub>b</sub>=Z<sub>n</sub>=U<sub>fn</sub>/I<sub>fn</sub>=2,48 Ω; f<sub>n</sub>=60 Hz ; p=3; R<sub>s</sub>=0,053 Ω ;  $R'_r$  =0,0657 Ω ; Lsσ =1,034 mH;  $L'_{srr}$  =0,955 mH ; L<sub>u</sub>=28,1 mH .

With the values from table 2 the next quantities are calculated:  $M_n = 817$  Nm;  $M_p = 898,7$  Nm;  $I_p = 520$  A;  $S_n = 128094,8$  VA;  $s_n = 0,026$ ;  $P_{1n} = S_n \cdot \cos \phi_{1n} = 111442,5$  W;  $r'_r = 0,0262 \cong s_n = 0,026$ .

It can be noticed that the last relation (6) is verified with a very good approximation.

Based on relations (5) and the data in table 2 are obtained the numerical values for the electrical parameters reported to the rotor cage for  $\omega_{rx} = 81$  rad/s; electrical parameters of the motor are:

$$L_{s} = 0,29134H; R_{s} = 0,053\Omega; L_{\mu} = 0,0281H$$

$$L_{r\sigma}^{'} = \begin{cases} 0,000955H; \omega_{r} \in [0;81] \\ 0,000155+0,0072\frac{1}{\sqrt{\omega_{r}}}; [81;120\pi]; \omega_{1} = 120 \cdot \pi \end{cases}$$

$$R_{r}^{'}(\omega_{r}) = \begin{cases} 0,065434; \omega_{r} \in [0;81] \\ 0,000904+0,00717\sqrt{\omega_{r}}; [81;120\pi]. \end{cases}$$
(7b)

The rotor cyclic inductance is expressed as:

$$\dot{L_{r}} = 0,0281 + \begin{cases} 0,000955 \, H \, ; \quad \omega_{r} \in [0\,;\,81] \\ 0,000155 + 0,0072 \, \frac{1}{\sqrt{\omega_{r}}} \, ; \\ \omega_{r} \in [81;\,120\pi] \, ; \quad \omega_{1} = 120\pi \end{cases}$$
(8)

With these data we can make any simulation for the motor in any working behavior.

The variations of the parameters  $R'_r$  and  $L'_{r\sigma}$  in respect of the rotor angular frequency  $\omega_r$  are presented in figures 1 and 2.



**Figure 1.** Rotor leakage inductance  $L_{r\sigma}$  variation depending on the pulsation  $\omega_{r}$ .



**Figure 2.** Rotor resistance  $R'_r$  variation depending on the pulsation  $\omega_r$ .

For motor with the data sheet from table 2, the time constants are:

$$T_{s} = \frac{L_{s}}{R_{s}} = 0,5497; \quad T_{s}' = \sigma T_{s} = 0,037s$$

$$T_{r} = \frac{L_{r}'}{R_{r}'} = 0,4422s; \quad T_{r}' = \sigma T_{r} = 0,02976s$$

$$\sigma = 1 - \frac{L_{\mu}^{2}}{L_{s}L_{r}'} = 1 - \frac{L_{m}^{2}}{L_{s}L_{r}} = 0,0672;$$

$$T_{mec} = \frac{J}{M_{m}} \Omega_{1} = 0,325s$$
(9)

where:  $T_s - is$  the time constant of the stator winding;  $T'_s - represents$  the transient time constant of the stator winding;  $T_r - is$  the time constant of the rotor winding;  $T'_r - is$  the transient time constant for the rotor winding, and  $T_{mec}$  is the mechanical constant of the motor. The stator time constants are greater than the rotor time constants. Therefore, the transient behavior in the stator winding will take more time than the transient behavior in the rotor windings.

## 3. Resisting Torque at the Motor Shaft

Load torque is required by the drive equipment (working machine) for a technological operation (turning a piece, forming by pressing, lifting an object) or other operations such as electric traction.

Mechanical friction torques exists, both in electrical machine, and in working machine. It is customary that all torques that work in the opposite direction in the electrical machine–working machine system, distinct from the electromagnetic torque developed by motor, to be summed into one resultant load torque, which is called a resisting torque, a torque that is opposing the movement. The electrical motor develops a torque  $M_2$  at shaft which gives a stronger movement then  $M_r$ ; in steady-state  $M_r = M_2$ .

Mechanical characteristic of the driven system represents the dependence between the load torque (resistance)  $M_r$  at the drive system shaft and a quantity which characterizes the driven installation (the angular velocity  $\Omega$ , the position angle  $\alpha$ , the covered space etc.). In many cases the mechanical characteristic is presented as:  $\Omega = f(M_s)$ .

An analytic expression for the mechanical characteristic can be the following expression:

$$M_s = M_f + M_u ; (10a)$$

$$M_s = M_0 + \left(M_{sn} - M_0\right) \left(\frac{\Omega}{\Omega_n}\right)^a \tag{10b}$$

where:  $\Omega_n$  – the rated angular velocity;  $M_0$  – the initial resisting torque;  $M_{sn}$  – the rated load torque; a – an exponent which can take different values, for example:  $a = 0 \rightarrow$  $M_s = M_{sn}$  =constant (lifting mechanism);  $a=1 \rightarrow M_s$  linear variation with the angular velocity  $\Omega$  (braking with a dc machine with constant excitation);  $a=2 \rightarrow$ mechanical parabolic characteristic, often encountered in practice.

In railway traction, the load torque is based on the variation of traction force  $F_0$  at the wheel rim in respect of the locomotive speed (figure 3), to which is added the dependent drag resistance  $R_L$  (figure 4) function of velocity variation.



**Figure 3.** Variation of the drive force  $F_0$  at the wheel rim according to the movement speed of the locomotive



Figure 4. Variation of the  $R_L$  drags resistance depending on the speed of the locomotive

It is noted that  $R_L << F_0$ ; and on this basis  $M_r$  torque at motor shaft can be approximated as shown in Figure 5.



Figure 5. Variation of the resistive torque  $M_{\rm r}$  at the motor shaft, function of the rotor speed n

## 4. Simulations

#### 4.1. Simulation of the Starting Process with General Mathematical Model for Rated Data and Constant Rotor Parameters

These simulations are preformed for stator currents iA, iB, iC, for rotor speed and electromagnetic torque. These simu-

lations are performed at rated data for a resisting torque equal to the electromagnetic torque and variable rotor parameters. The induction motor can start at rated torque due to the skin effect which increases rotor resistance and the starting torque.

The results of the simulations are presented in figures 6, 7, 8 and 9.



Figure 6. The stator currents iA, iB, iC function of time







The results are similar to those in the literature; at starting point appears that the stator current have high amplitude, and the torque has a greater value than the rated torque, because of the skin effect. Note that the transient process for the stator currents has a longer variation due to the rotor parameters variation with the changing speed. The transient behavior has two steps: the first step is at time t=0 till t<sub>i</sub> $\approx$ 1,3s when the rotor parameters vary in respect to the rotor current angular frequency (figure 6 and figure 7), this variation is maintained

almost till the operating speed; the second step takes place for the constant rotor parameters and it is between  $t_I \cong 1,3s$  la  $t_{II} \cong 1,5s$  (figure 6). For the first step the transient behavior is not changed significantly since the rotor parameters vary continuously and in tight limits. The rotor speed is essentially determined by the motion equation in which the inertial moment J occurs, and only at large oscillations for the amplitude of the electromagnetic torque (figure 8) the speed has small swinging amplitude (figure 7, t < 0,9 s). The electrical transient process takes longer than the mechanical transient process.



Figure 9. Mechanical characteristic the rotor speed in function of the electromagnetic torque

#### 4.2. Results

The motor has the following parameters:  $f_1$ =60 Hz, J=60 kgm<sup>2</sup> and U<sub>1</sub>=U<sub>1n</sub>=323,32 V.

According to the analysis presented in section 3, the load torque at starting point for the locomotive depending on the speed it presents in figure 10; initial torque value is 800 Nm, then decreases linearly to the value of 400 Nm.



Figure 10. Variations of the load torque in respect of the locomotive speed

The motor that drives an electric locomotive works at a higher equivalent inertial moment; total inertial moment  $J_t$  from the motion equations is computed with the expression:

$$J_e = m \left(\frac{V}{\Omega}\right)^2; \ J_t = J + J_e; \ V = R \ [\Omega]; \ R = \frac{D}{2} = \frac{1,11}{2}$$
 (11)

Where: m is the mass of the train which is for one motor, V is the train speed and  $\Omega$  is the angular velocity of the motor

shaft. Mechanical time constant are much greater than the electrical time constants, therefore, permanent regime is dominant.



Figure 13. The time variation of the electromagnetic torque

As expected, at a high inertial moment J, transient time is longer because rotor parameters vary slowly as the rotor speed varies. The stator current magnitude (figure 11), and rotor currents (figure 12) are all bellow 800 A, smaller for low inertial moment J case.

It is found that variation of the electromagnetic torque from the first part of the transient behavior (figure 13) does not make swings for the rotors speed (figure 14).

The transient behavior of the stator currents is longer than

the one of the rotor currents because the time constants for the stator windings are bigger than the ones of the rotor windings. The linear decrease of the resisting torque  $M_r$ leads to a pronounced perturbation of the rotor currents in variation time interval: 9...11s because with the variation of the resisting torque, the electromagnetic torque (depending on the rotor quantities) also varies directly (figure 12). The resisting torque variations influences less the stator currents because of the inductive coupling between the two armature windings, (figure 11).



Figure 14. The time variation of the rotor speed

## 5. Conclusions

This paper presents the influence of skin effect on the electrical parameters of the rotor and on this basis is constructed the mathematical model of the asynchronous machine with deep bars that allows simulation of different operating behaviors. Initial, the stator currents frequency is considered constant, equal to the network frequency. In fact, rotor parameters are considered constant, independently of the rotor frequency, for rotor frequency values between 0 and 10...15 Hz. For higher frequencies analytic expressions are proposed, resulting from skin effect theory, both for the rotor resistance and the leakage inductance. The asynchronous motor simulations in sinusoidal steady state, were performed with the program the SYMNAP - SYmbolic Modified Nodal Analysis, a program that can determine in a full- symbolic form the voltages and the currents for the equivalent scheme for one phase of the asynchronous motor. Symbolic expressions for the characteristics quantities of the general regime of the induction motor allow graphic representation for different parameters.

In simulation of dynamic (transient) behavior of the induction motor, Park-Blondel equations have been used; these equations have a nonlinear form due to both their structure and frequency variation of the rotor parameters. The integration of these equations is performed with the help of the integration routines for ordinary differential equations in Matlab programming environment.

Note that the transient process for the stator currents has a longer variation due to the rotor parameters variation with the changing speed. The transient behavior has two steps: the first step is when the rotor parameters vary in respect to the pulsating currents from the rotor circuit, this variation is maintained almost till the operating speed; the second step takes place for the constant rotor parameters and transient behavior, which are carried out similar to inductive circuits. For the first step the transient behavior is not changed significantly since the rotor parameters vary continuously and in tight limits.

At large oscillations of the electromagnetic torque amplitude, the speed has small swinging magnitude. The electrical transient process takes longer than the mechanical transient process.

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