

A New Peak Clipping Algorithm for PAPR Reduction in OFDM

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Abstract The peak-to-average-power ratio (PAPR) is one of the major problems of Orthogonal Frequency Division Multiplexing (OFDM) systems. Various PAPR reduction techniques have been given in the past. Among these techniques, the clipping technique has been widely adopted as a practical scheme because of its low computational complexity and simplicity in implementation, while the pre-coding technique is known to provide good PAPR reduction with improved bit error rate (BER) performance. In this paper, a combination of clipping and pre-coding is proposed to reduce PAPR with almost same BER performance as given by individual pre-coding technique. The proposed technique gives good result because the clipping noise of combined scheme would be less than that of single clipping. Also the clipping used in this paper is not hard as used in conventional clipping. A new clipping algorithm is proposed which is based on averaging of high amplitude samples. In addition, another pulse shape called modified Bartlett Hanning (MBH) is proposed to generate pre-coding matrix and its performance is compared with square root raised cosine (SQRC) pulse shape. dB.

Keywords OFDM, PAPR, Clipping, Pre-coding, Window Function

1. Introduction

The Orthogonal Frequency Division Multiplexing (OFDM) has drawn major attention over last decade for its usefulness in broad band wireless communication. Due to its numerous benefits like high data transmission rate, high bandwidth efficiency, robustness against multi-path fading and less complex equalizer, OFDM has been adopted as a major data transmission technique by many wireless communication standards, such as IEEE 802.11a[1], IEEE 802.16a[2] and terrestrial digital video broadcasting (DVB-T)[3] systems in Europe.

However, the major demerit of OFDM system is the high peak-to-average power ratio (PAPR) of transmitter's output signal. When a high PAPR OFDM signal is passed through a nonlinear power amplifier, it reduces the efficiency of power consumption and causes in-band distortion and undesired spectral spreading[4]. Due to this, the bit error rate (BER) performance of the system also degrades. Thus, the PAPR reduction is one of the most important research interests for the OFDM systems. In the absence of PAPR-reduction techniques, the signal is clipped, making the receiver prone to errors

Several PAPR reduction techniques are reviewed in[5]. These techniques include clipping and filtering[6]-[7], coding methods[8]-[11], probabilistic methods like Partial

Transmission Sequence (PTS) and Selective Mapping (SLM)[12]-[14], non linear companding[15]-[16], constellation shaping[17]-[18], pulse shaping[19] and pre-coding[20]-[22].

Clipping is the simplest method for the reduction of PAPR by simultaneously increasing the average value and minimizing the peak value. On the other hand, the pre-coding has been considered as a best among all these techniques because it improves PAPR without increasing much complexity and without destroying the orthogonality between subcarriers. The pre-coding also improves the BER in comparison to normal OFDM system because of diversity gain obtained due to the spreading of data symbol on more than one subcarrier.

With the studies on clipping and pre-coding techniques, an idea emerged to combine the philosophy of pre-coding and peak clipping (in place of clipping of entire symbol). This combination shows good results, because of pre-coding makes the envelope almost constant and then peak clipping reduces the peak value. In this article, the comparison of results obtained by proposed algorithm is made with results reported in[20] and[21], which clearly show the superiority of proposed method. With the proposed algorithm the hardware complexity is also similar as in the case of only pre-coding technique. In[20] and[21], the performance of raised cosine (RC) and square root raised cosine (SQRC) window functions have been evaluated. In the present study, the performance of SQRC is compared with the Modified Bartlett-Hanning (MBH) window function[23]. The BER comparison is done over Rayleigh frequency selective fading channel[24].

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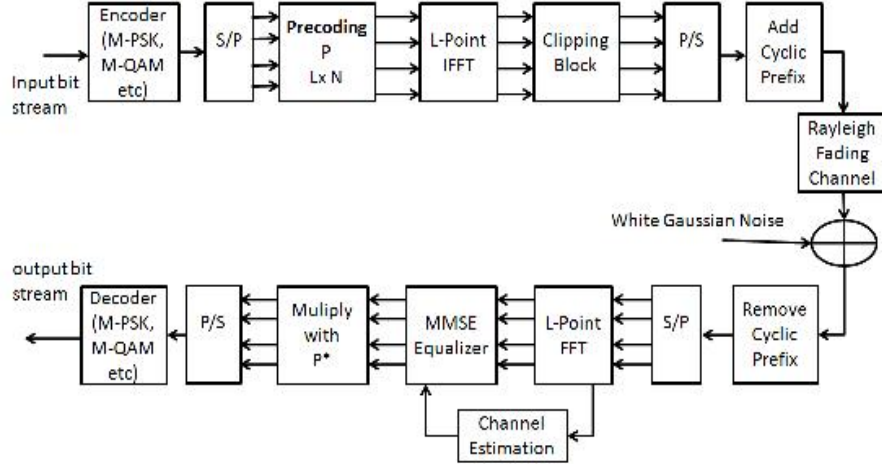


Figure 1. Base-band OFDM System model with N subcarriers

The paper is organized as follows. The system model is given in section-2. The PAPR analysis is given in section-3. The peak clipping algorithm is described in section-4. The performance analysis is done in section-5. Finally, conclusions are given in section-6.

2. System Model

In this study, a discrete time baseband OFDM system with N orthogonal sub-carriers has been considered as shown in Fig.1. It consists of transmitter, channel and receiver blocks which are described below.

2.1. Transmitter

In this system, a block of $\log_2 M$ input bits are mapped in to symbol constellation point D_n , by an M-ary data encoder, and then N such symbols are transferred by the serial-to-parallel converter. Data encoder can use any types of modulation techniques (e.g. BPSK, QPSK, QAM etc.). These complex parallel data symbols $d_1, d_2, d_3, \dots, d_N$ are then fed to the IFFT block for a normal OFDM system. On the other hand for pre-coded OFDM system as shown in Fig.1, these data symbols are first given to pre-coder block and then the IFFT of pre-coded L symbols are taken. The pre-coded OFDM signal with L-subcarrier can be written as

$$x(n) = \sum_{k=0}^{L-1} X(k) e^{j2\pi k \frac{(n-G)}{L}}, n = 0, 1, 2, \dots, L-1. \quad (1)$$

where, G is length of cyclic prefix, $X(k)$ is the output of pre-coder block and it is defined as –

$$X(k) = \sum_{n=0}^{N-1} p_{k,n} D_n, k = 0, 1, 2, \dots, L-1. \quad (2)$$

where, D_n is the complex data symbol of data rate $1/T_s$ and $p_{k,n}$ is the element of pre-coder matrix P of size $L \times N$, which is defined as

$$P = \begin{bmatrix} p_{0,0} & p_{0,1} & \cdot & \cdot & \cdot & p_{0,N-1} \\ p_{1,0} & p_{1,1} & \cdot & \cdot & \cdot & p_{1,N-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{L-1,0} & p_{L-1,1} & \cdot & \cdot & \cdot & p_{L-1,N-1} \end{bmatrix} \quad (3)$$

After IFFT, the pre-coded OFDM signal is given to clipping block, where the signal is clipped according to the algorithm defined in section-4.

2.2. Receiver

The tap-delay line model with Z-path is considered for frequency selective fading channel. The received signal $r(n)$ from a multipath fading channel may be represented as-

$$r(n) = \sum_{m=0}^{Z-1} h(m) x(n-m) + w(n), n = 0, 1, 2, \dots, L-1. \quad (4)$$

where, $h(m)$ is the channel impulse response of mth path and $w(n)$ is a zero-mean, unit variance complex Gaussian noise. After discarding first G sample of the received signal and taking L-point FFT, the output of FFT block $R(k)$ is given as –

$$R(k) = X(k)H(k) + W(k), \text{ for } k = 0, 1, \dots, L-1. \quad (5)$$

The term $H(k)$ is the channel response to the subcarrier frequency k/L . To compensate the fading affect of channel, one-tap equalizer is used and each element of vector $R(k)$ is multiplied by a equalized gain factor $G(k)$, the output of equalizer may be written as-

$$\hat{X}(k) = X(k) H(k) G(k) + W(k) G(k), \quad (6)$$

for $k = 0, 1, \dots, L-1$.

Where, $G(k)$ is defined as –

$$G(k) = \frac{H^*(k)}{|H(k)|^2 + (N_o / E_b)} \quad (7)$$

The output of equalizer is now multiplied with matrix P^* .

where, P^* is the Hermitian transpose of P . After multiplication we get a vector of length N

$$\hat{D}_n = P^* \times \hat{X}(k), \text{ for } n=0,1,\dots,N-1. \quad (8)$$

This vector \hat{D}_n is then given to decoder to recover transmitted symbol (M-PSK / M-QAM).

3. PAPR Analysis

The PAPR of the transmitted signal $x(n)$ can be defined as follows

$$\text{PAPR}[x(n)] = \frac{\max_{0 \leq n < L} [|x(n)|^2]}{E[|x(n)|^2]} \quad (9)$$

where, $|x(n)|^2$ is the instantaneous power and $E[|x(n)|^2]$ is average power of the pre-coded OFDM signal $x(n)$. The element of pre-coding matrix can be determined from the basic function $p(t)$ as given by[20] –

$$p_{k,n} = e^{-j2\pi k \frac{n}{N}} \frac{1}{T} \int_0^T p(t) e^{-j2\pi k \frac{t}{T}} dt \quad (10)$$

After solving (10) –

$$p_{k,n} = (-1)^k e^{-j2\pi k \frac{n}{N}} \frac{1}{T} P(k / NT_s) \quad (11)$$

where, $P(k / NT_s)$ is the Fourier transform of function $p(t)$. From (1) it is clear that the signal $x(n)$ will depend on the elements of pre-coding matrix P , and this can be easily shown that the PAPR of signal $x(n)$ can be controlled by proper selection of $p(t)$ [20].

Two window functions namely SQRC[20] and Modified Bartlett-Hanning (MBH)[23] have been compared in this study of reduction of PAPR. The expression, in the context of PAPR reduction, of SQRC function as given in[20] is –

$$P_{SQRC}(f) = \begin{cases} T_s \text{Sin}\left(\frac{\pi f T_s}{2\beta}\right), & 0 < f \leq \frac{\beta}{T_s} \\ T_s, & \frac{\beta}{T_s} \leq f \leq \frac{1}{T_s} \\ T_s \text{Sin}\left(\frac{\pi(f T_s - 1)}{2\beta} + \frac{\pi}{2}\right), & \frac{1}{T_s} < f \leq \frac{1+\beta}{T_s} \end{cases} \quad (12)$$

The expression for MBH window function after due changes for PAPR improvement is given as

$$P_{MBH}(f) = \begin{cases} T_s \left\{ \beta + \frac{(4\beta-2)}{2\alpha} (f T_s - \alpha) + (1-\beta) \text{Cos}\left(\frac{\pi}{\alpha} (f T_s - \alpha)\right) \right\}, & 0 < f \leq \frac{\alpha}{T_s} \\ T_s, & \frac{\alpha}{T_s} < f \leq \frac{1}{T_s} \\ T_s \left\{ \beta - \frac{(4\beta-2)}{2\alpha} (f T_s - 1) + (1-\beta) \text{Cos}\left(\frac{\pi}{\alpha} (f T_s - 1)\right) \right\}, & \frac{1}{T_s} < f \leq \frac{(1+\alpha)}{T_s} \end{cases} \quad (13)$$

There are two parameters (β and α ; pulse shape parameter and roll-off factor respectively) which decide the shape of

pulse and its performance in an OFDM system. The range of β is from 0.5 to 1.88 as given in[23] and α can take any value between 0 and 1. Fig. 2 shows the comparison between the complementary cumulative distribution function (CCDF) of the PAPR of OFDM signal with $N=64$ subcarriers for SQRC and MBH window function. It is clearly visible from the graph that the pre-coder based on MBH window function with $\beta = 1.5$ performs better than the SQRC based pre-coder.

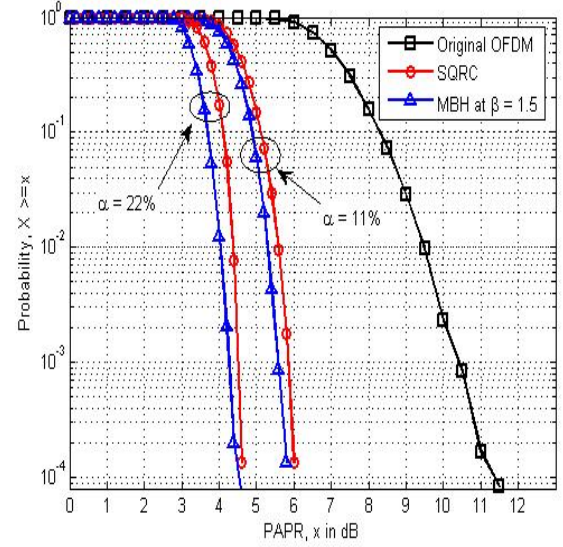


Figure2. Comparison of CCDF of the PAPR of the pre-coded OFDM signal with $N=64$ subcarriers for SQRC and MBH window function

4. Peak Clipping Algorithm

The proposed peak clipping is different than normal peak clipping method because in normal peak clipping only one peak of OFDM symbol is clipped whereas in proposed method, the “Q” high amplitude samples of OFDM symbol are taken. These samples are reduced in amplitude by averaging. The proposed algorithm is given as

New Clipping Algorithm

First arrange the OFDM signal $x(n)$, ($n = 0,1,2, \dots, N - 1$) in ascending order of their amplitude.

- 1: $y(n) \leftarrow \text{sort}(|x(n)|)$ // Arrange $x(n)$ in ascending order of its amplitude //
- 2: $i \leftarrow 1$ // Initialize main loop counter //
- 3: **while** $i \leq NOI$ **do** // NOI is a number of iteration, outer main loop starts//
- 4: $j \leftarrow L$ // Initialize inner loop counter //
- 5: **while** $j > L - Q + 1$ **do** // Q is number of high amplitude sample taken // for averaging //
- 6: $|y(j)| \leftarrow \frac{|y(j)| + |y(j-1)|}{2}$ // average of j^{th} and $(j-1)^{\text{th}}$ samples //
- 7: $j = j - 1$
- 8: **end while** // Inner loop ends //

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9:    $i \leftarrow i + 1$ 
10: end while           // Outer main loop ends //
11:  $z(n) \leftarrow \text{desort}(|y(n)|)$  // re-arrange all
    the samples of  $y(n)$  at their original location.

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After applying peak clipping algorithm, the amplitude of clipped signal $z(n)$ is raised so that the average power is same as before clipping.

5. Performance Evaluation

In this section, the performance of proposed algorithm is evaluated and compared with exponential companding while taking normal and pre-coded OFDM system. The CCDF of PAPR, BER, and power spectral density (PSD) has been taken as performance evaluation parameter. The BER performance is evaluated in frequency selective fading channel and PSD plots are given with four times oversample data. For this simulation, a 64 sub-carrier OFDM system with QPSK modulation is taken. The tap-delay line model with Z-path is considered for frequency selective fading channel[24]. The pre-coder is based on MBH window function with roll-off factor $\alpha = 11\%$ and pulse shape parameter $\beta = 1.5$.

Fig. 3 shows the CCDF plots of normal OFDM system

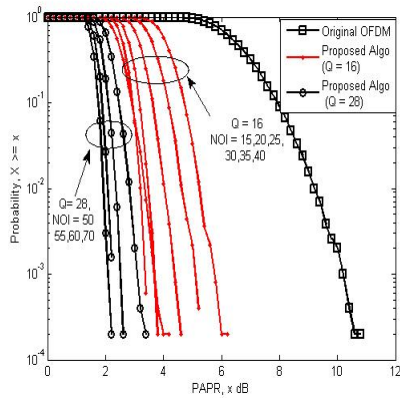


Figure 3. CCDF of the PAPR of OFDM signal with proposed algorithm

with and without peak clipping algorithm. It is clearly visible from the graphs that as the number of iteration (NOI) increases, the PAPR is reduced more. This reduction is not linear because after some iterations (for example, NOI = 40 in our case), the PAPR is not reducing much, this can be considered the saturation level of iteration. For further reduction in PAPR, Q has to be changed.

The proposed algorithm is also compared with exponential companding (EC) method. The comparison is shown in Fig. 4. It is clearly visible from the graphs that proposed method gives better result than EC method ($d=2$). The BER performance of proposed algorithm and EC method is shown in fig. 5. In conclusion, the proposed algorithm gives better the PAPR than EC with almost same BER. The PSD plots support this statement as shown in Fig 6.

Fig. 7 shows the CCDF plots of pre-coded OFDM system with and without peak clipping algorithm. The CCDF of PAPR without peak clipping with $\alpha = 22\%$ is similar to CCDF of PAPR with peak clipping with $\alpha = 11\%$ only. Therefore, a 50% overhead is reduced. The BER performance of pre-coded OFDM system with our proposed algorithm is shown in fig. 8. The conclusion is that with our algorithm the PAPR is reducing more than conventional pre-coding with a small BER degradation.

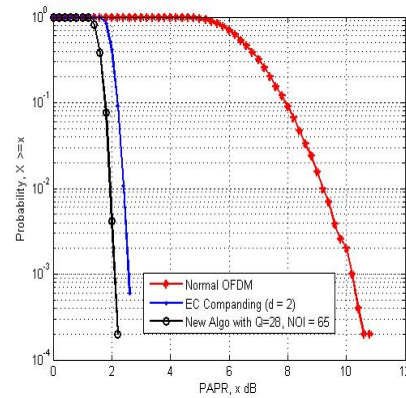


Figure 4. Comparison of CCDF of the PAPR of OFDM system with exponential companding and proposed algorithm

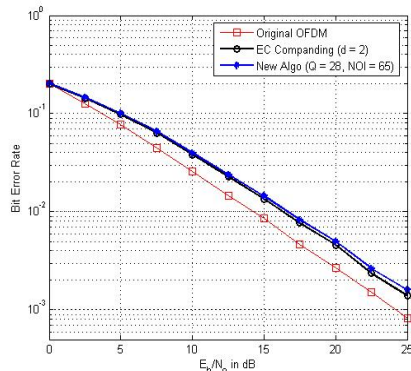


Figure 5. BER for 64-subcarrier QPSK-OFDM over frequency selective Rayleigh fading channel with $Z=10$

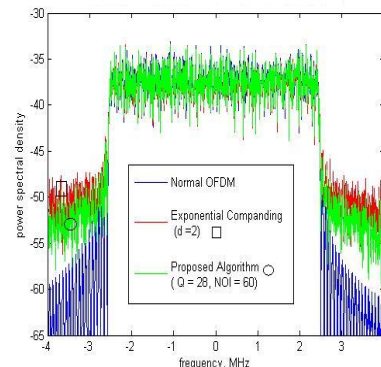


Figure 6. Comparison of Power Spectral density of 64-subcarrier OFDM system with exponential companding and proposed algorithm

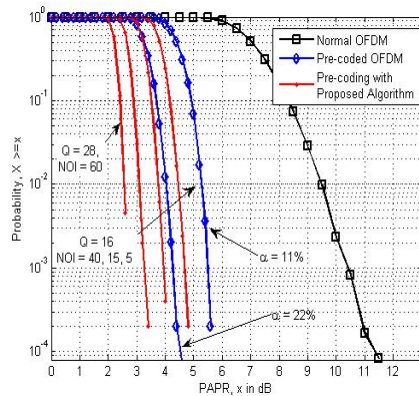


Figure 7. CCDF of the PAPR of the pre-coded OFDM signal N=64 subcarriers with proposed algorithm

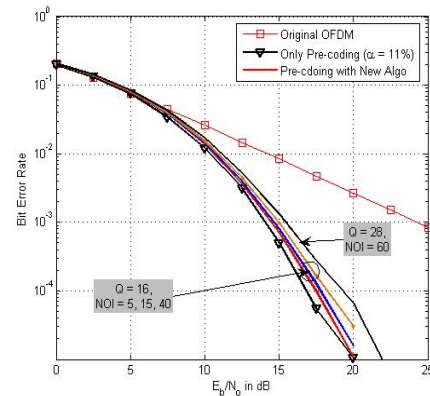


Figure 8. BER for 64-subcarrier pre-coded OFDM system over frequency with selective Rayleigh fading channel with Z=10 with proposed algorithm

6. Conclusions

In this paper, a novel algorithm of peak clipping is proposed in a pre-coded OFDM system to reduce the PAPR. The proposed technique is better than conventional because it does not require any increase in roll off factor to reduce PAPR. Increasing the roll off factor degrades the BER as given in [21]. The peak clipping after pre-coding reduces PAPR and reduced the overhead. The BER performance is not affected in the proposed method.

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