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Three Dimensional Radiative Heat and Mass Transfer Periodic Flow through a Vertical Porous Channel with Transpiration Cooling and Slip Boundary Conditions

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Abstract In this paper we have studied a free and forced convective flow of a viscous incompressible fluid through a vertical porous channel bounded by two vertical plates moving with same velocity but in opposite directions with slip parameters. The wall temperature and the mass concentration are assumed to be spanwise consinusoidal. Expressions for velocity, temperature and concentration profiles along with skin friction and Nusselt number are obtained and comparitive study is made to analyze the effects of different parameters. We observe that skin friction ($|C_f|$) is lower for water (Pr = 7, Sc = 0.61) than for air (Pr = 0.71, Sc = 0.22).

Keywords Free and Forced Convection, Mass Transfer, Three Dimensional Flow, Transpiration Cooling, Slip

1. Introduction

Transpiration cooling or effusion cooling is the process of injecting a fluid into a porous material which can be served as very efficient cooling method for protecting solid surfaces that are exposed to high heat flux, high temperature from environments such as hypersonic vehicle combustors, rocket nozzles, gas turbine blades etc. Andoh and Lips[2] did prediction of porous wall thermal protection by effusion or transpiration cooling. Kamiuto[11] studied thermal characteristics of transpiration cooling system using open-cellular porous material in a radiative environment where as Jain and Sharma[10] studied three dimensional free convection couette flow with transpiration cooling and temperature jump boundary conditions.

In most of the studies investigators have restricted themselves to two-dimensional flows only by assuming either constant or time dependent suction velocity at the plate. But there may arise situation when the flow field may essentially be three dimensional. Chaudhary and Sharma[6] have studied three dimensional unsteady convection and mass transfer flow through porous medium where as Jain and Khandelwal[9] have studied three dimensional free convection polar flow with radiation and sinusoidal temperature along a porous plate in slip flow regime. Recently Vishalakshi et al.[14] have studied three dimensional couette flow of a dusty fluid through a porous medium with heat transfer.

During the last decade many research workers have studied mixed convection in channels, which is a phenomenon in many technological processes, such as design of solar collectors, thermal design of buildings, air conditioning etc. Barletta and Celli[3] studied mixed convection MHD flow in a vertical channel with effects of Joule heating and viscous dissipation where as Bhoite et al.[4] studied mixed convection in a shallow enclosure with a series of heat generation components. Working on a horizontal channel, Rahman et al.[13] and Brown and Lai[5] have studied conjugated effects of joule heating and magnetohydrodynamic on double diffusive mixed convection in a horizontal channel with an open cavity and correlation for combined heat and mass transfer from an open cavity respectively.

It is a well known fact that in case of many polymeric liquids when the weight of the molecules is high, the molecules at the boundary show slip. In many problems like thin film problems, rarefied fluid problems, fluid containing concentrated suspension the no slip boundary condition fails to work. Hayat et al.[8] have studied the influence of slip on the peristaltic motion of third order fluid in an assyminetric channel. Moreover, Farhad et al.[7] have made studies on accelerated MHD flow in a porous medium with slip condition and Makinde and Osalusi[12] have studied MHD steady flow in a channel with slip at the permeable boundaries.

In the present paper, we have analyzed a problem on mixed convection heat and mass transfer in a channel filled with porous material bounded by two vertical plates moving in opposite direction with respect to each other. The

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temperature of the upward moving plate and the mass concentration are taken to be spanwise co-sinusoidal with slip at both the plates. Effects of different parameters entering into the problem are shown graphically on velocity, temperature, concentration, skin friction and Nusselt number. We observe that increasing the thermal Grashof number (Gr) increases the skin friction (|Cf|) and in case of no slip (h1=0,h2 = 0) skin friction (|Cf|) is more as compared to the case with slip at the boundary for both the basic fluids air (Pr=0.71, Sc=0.22) and water (Pr=7, Sc=0.61).2.

2. Formulation of the Problem

We consider the flow of a viscous incompressible fluid of density ρ and viscosity μ through a vertical channel formed by two parallel plates moving with equal velocities in opposite direction at a distance d apart. The co-ordinate axis are so chosen that x and z axes are on the plane of the plate at y = 0, where x-axis is in vertically upward direction and y-axis is along the normal to the plane of the plates. The temperature of the plate situated at y = 0 and the mass concentration are considered to be co-sinusoidal. The gap between the two plates is filled with a porous material. There is suction at plate y = 0 and equal injection on plate y = d.

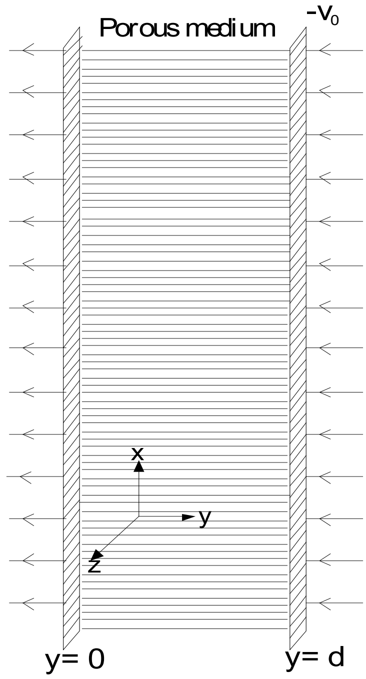


Figure 1. Schematic Diagram

Let (u, v, w) be the components of velocity in the directions (x, y, z) respectively. The plate being considered infinite in x direction, hence all the physical quantities are independent of x. Thus following Acharya and Padhy[1], w is independent of z and equation of continuity gives $v = -v_0$ throughout. Using the Boussinesq approximation the momentum equation, energy equation including the viscous dissipative term and the concentration equation are given by:

$$-V_0\frac{\partial u}{\partial y}=V\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial Z^2}\right)+g\beta\left(T-T_d\right)+g\beta^*\left(C-C_d\right)-\frac{v}{K}u, \tag{1}$$

$$V_0\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_P}\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial Z^2}\right)+\frac{\mu}{\rho C_P}\left\{\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial Z}\right)^2\right\}-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial z} \tag{2}$$

$$-V_0\frac{\partial C}{\partial y}=D\left(\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right), \tag{3}$$

where g is acceleration due to gravity, β is coefficient of thermal expansion, β* is coefficient of expansion due to concentration, v, κ, C_p and D are kinematic viscosity, thermal conductivity, specific heat at constant pressure and diffusion coefficient respectively.

The boundary conditions are given by:

$$\left.\begin{aligned}
y=0: u&=U+L_1\frac{\partial u}{\partial y},\quad T=T_d+\left(T_d-T_0\right)\left(1+\in\cos\frac{\pi z}{\ell}\right),\\
&\qquad C=C_d+\left(C_d-C_0\right)\left(1+\in\cos\frac{\pi z}{\ell}\right),\\
y=d: u&=-U+L_2\frac{\partial u}{\partial y},\quad T=T_d\quad,\quad C=C_d,
\end{aligned}\right\} \tag{4}$$

where U and –U are the velocities of the plates and

$$L_i = \left(\frac{2 - a_i}{a_i} \right) L, \ for \ i = 1, 2$$

where L is mean free path and a_i the Maxwell's reflection coefficient. $\in > 0$ is a small number, ℓ is the wave length, T_d the constant temperature of the plate at y = d and T_0 some constant reference temperature. The local radiant for the case of optically thin gray gas is expressed by:

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma^* \left(T_d^4 - T^4 \right), \tag{5}$$

we assume that the temperature differences within the flow are sufficiently small such that T_4 may be expressed as a linear function of the temperature. This is accomplished by expanding T_4 in a Taylor series about T_d and neglecting the higher order, thus

$$T^4 \cong 4T_d^3 T - 3T_d^4, \tag{6}$$

by using (5) and (6) we obtain

$$\frac{\partial q_r}{\partial y} = -16\, a^* \sigma^* T_d^3 \left(T_d - T \right), \tag{7}$$

where σ* in Stephen-Boltzmann constant and a* is absorption coefficient.

On introducing the following non-dimensional quantities:

$$u^* = \frac{u}{U}, y^* = \frac{y}{d}, z^* = \frac{z}{d}, T^* = \frac{T - T_d}{T_d - T_0}, C^* = \frac{C - C_d}{C_d - C_0}, \lambda = \frac{d}{\ell}, K^* = \frac{K}{d^2},$$

and parameters:

$$\mathrm{Re} = \frac{v_0 d}{v} \ \text{(cross flow Reynold's number)},$$

$$h_1 = \frac{L_1}{d} \ \text{(velocity slip parameter on plate at y = 0)},$$

$$h_2 = \frac{L_2}{d} \ \text{(velocity slip parameter on plate at y = 1)},$$

$$\mathrm{Pr} = \frac{\mu C_p}{\kappa} \ \text{(Prandtl number)},$$

$$\mathrm{Sc} = \frac{v_0 d}{D} \ \text{(Schmidt number)},$$

$$G_r = \frac{g\beta (T_d - T_0) v}{v_0^2 U} \ (thermal\ Grashof\ number),$$

$$Gc = \frac{g\beta^* (C_d - C_0) v}{v_0^2 U} \ (mass\ Grashof\ number),$$

$$Ec = \frac{U^2}{C_P (T_d - T_0)} \ (Eckert\ number),$$

$$R = \frac{16 a^* \sigma^* v^2 T_d^3}{v_0^2 K} \ (Radiation\ parameter)$$

equations (1) to (3), using (7), in non dimensional form after dropping asteriks are:

$$\frac{\partial u}{\partial y} = -\frac{1}{\mathrm{Re}} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \mathrm{Re}\ \mathrm{Gr}\ T - \mathrm{Re}\ \mathrm{Gc}\ C + \frac{1}{\mathrm{Re}\ K} u, \tag{8}$$

$$\frac{\partial T}{\partial y} = -\frac{1}{\mathrm{Pr}.\mathrm{Re}} \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) - \frac{\mathrm{Ec}}{\mathrm{Re}} \left(\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 \right) + \frac{\mathrm{R}.\mathrm{Re}}{\mathrm{Pr}} T, \tag{9}$$

$$\frac{\partial C}{\partial y} = -\frac{1}{Sc}\left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right), \tag{10}$$

with boundary conditions:

$$\left.\begin{aligned} &y=0: \ u = 1 + h_1\frac{\partial u}{\partial y}, \ T = 1 + \in \cos\lambda\pi z, \ C = 1 + \in \cos\lambda\pi z, \\ &y=1: \ u = -1 + h_2\frac{\partial u}{\partial y}, \quad T = 0 \quad , \quad C = 0. \end{aligned}\right\} \tag{11}$$

3. Solution of the Problem

Since the amplitude, $\in(<<1)$, of the plate temperature is very small, we represent the velocity, temperature and concentration in the neighbourhood of the plate as:

$$f(y,z) = f_0(y) + \in f_1(y,z) + 0(\in^2) + \ldots\ldots\ldots\ldots, \tag{12}$$

where f stands for u, T and C.

Now substituting equation (12) in equation (8) to (10) we get the following set of equations by equating like powers $\in$, neglecting those $\in^2$ and higher orders

$$u_0'' + Re\, u_0' - \frac{1}{K}u_0 = -Re^2\, Gr\, T_0 - Re^2\, Gc\, C_0, \tag{13}$$

$$T_0'' + Pr\, Re\, T_0' - R\, Re^2\, T_0 = -Ec \quad Pr \quad u_0'^{\,2} \quad , \tag{14}$$

$$C_0'' + Sc\, C_0' = 0, \tag{15}$$

with boundary conditions as:

$$\left.\begin{aligned} &y=0: \quad u_0 = 1 + h_1 u_0', \qquad T_0 = 1, \quad C_0 = 1, \\ &y=1: \quad u_0 = -1 + h_2 u_0', \quad T_0 = 0, \quad C_0 = 0, \end{aligned}\right\} \tag{16}$$

and

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial Z^2} + Re\frac{\partial u_1}{\partial y} - \frac{1}{K}u_1 = -Re^2\, GrT_1 - Re^2\, GcC_1 \tag{17}$$

$$\frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2} + Pr.Re\frac{\partial T_1}{\partial y} - R.Re^2\, T_1 = -2Ec.Pr\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y}, \tag{18}$$

$$\frac{\partial^2 C_1}{\partial y^2} + \frac{\partial^2 C_1}{\partial z^2} + Sc\frac{\partial C_1}{\partial y} = 0, \tag{19}$$

with boundary conditions as:

$$\left.\begin{aligned} &y=0: \quad u_1 = h_1\frac{\partial u_1}{\partial y}, \quad T_1 = \cos\lambda\pi z, \quad C_1 = \cos\lambda\pi z, \\ &y=1: \quad u_1 = h_2\frac{\partial u_1}{\partial y}, \quad T_1 = 0, \qquad\quad T_1 = 0. \end{aligned}\right\} \tag{20}$$

In view of equation (20) the perturbed part of velocity, temperature and concentration are assumed to be of the form:

$$\left.\begin{aligned} &u_1 = V(y)\cos\lambda\pi z\,, \\ &T_1 = \theta(y)\cos\lambda\pi z\,, \\ \text{and} \quad &C_1 = \phi(y)\cos\lambda\pi z. \end{aligned}\right\} \tag{21}$$

Now using (21) in equations (17) to (19) we get:

$$V'' + Re\, V' - \left(\frac{1}{K} + \lambda^2\pi^2\right)V = -Re^2\, Gr\, \theta - Re^2\, Gc\, \phi, \tag{22}$$

$$\theta'' + \Pr . \mathrm{Re}\,\theta' - \left(\mathrm{R}\ \mathrm{Re}^2 + \lambda^2\pi^2\right)\theta = -2\mathrm{Ec}\ \Pr\ u_0'\ V', \tag{23}$$

$$\phi'' + \mathrm{Sc}\ \phi' - \lambda^2\pi^2\phi = 0. \tag{24}$$

with boundary conditions as:

$$\begin{aligned} y = 0 &: \quad V = h_1 V' \quad , \quad \theta = 1 \quad , \quad \phi = 1, \\ y = 1 &: \quad V = h_2 V' \quad , \quad \theta = 0 \quad , \quad \phi = 0, \end{aligned} \tag{25}$$

Equations (13) to (15) and (22) to (24) are coupled thus approximate solution is obtained by perturbation technique for small values of Eckert number (Ec), (Ec<<1),

$$\left.\begin{aligned} f_0 &= f_{00} + \mathrm{Ec} f_{01} + O\left(\mathrm{Ec}^2\right) \\ g &= g_0 + \mathrm{Ec} g_1 + O\left(\mathrm{Ec}^2\right) \end{aligned}\right\} \tag{26}$$

where f_0 stands for u_0, T_0, C_0 and g stands for V, θ and φ.

Using (26) in (13) to (15) and (22) to (24) we get:

$$\left.\begin{aligned} & u_{00}'' + \mathrm{Re}\, u_{00}' - \frac{1}{K} u_{00} = -\mathrm{Re}^2\ \mathrm{Gr}\ T_{00} - \mathrm{Re}^2\ \mathrm{Gc}\ C_{00}, \\ & T_{00}'' + \Pr\ \mathrm{Re}\ T_{00}' - \mathrm{R}\ \mathrm{Re}^2\ T_{00} = 0, \\ & C_{00}'' + \mathrm{Sc}\ C_{00}' = 0, \\ & V_0'' + \mathrm{Re}\ V_0' - \left(\frac{1}{K} + \lambda^2\pi^2\right) V_0 = -\mathrm{Re}^2\ \mathrm{Gr}\ \theta_0 - \mathrm{Re}^2\ \mathrm{Gc}\ \phi_0, \\ & \theta_0'' + \Pr\ \mathrm{Re}\ \theta_0' - \left(\mathrm{R.}\ \mathrm{Re}^2 + \lambda^2\pi^2\right)\theta_0 = 0, \\ & \phi_0'' + \mathrm{Sc}\ \phi_0' - \lambda^2\pi^2\phi_0 = 0, \end{aligned}\right\} \tag{27}$$

with boundary conditions as:

$$\left.\begin{aligned} y = 0 : u_{00} &= 1 + h_1 u_{00}' \quad , T_{00} = 1, \quad C_{00} = 1, \\ V_0 &= h_1 V_0' \qquad , \theta_0 = 1\ , \quad \phi_0 = 1, \\ y = 1 : u_{00} &= -1 + h_2 u_{00}'\ , \quad T_{00} = 0, C_{00} = 0, \\ V_0 &= h_2 V_0' \qquad , \theta_0 = 0, \quad \phi_0 = 0, \end{aligned}\right\} \tag{28}$$

and

$$\left.\begin{aligned} & u_{01}'' + \mathrm{Re}\, u_{01}' - \frac{1}{K} u_{01} = -\mathrm{Re}^2\ \mathrm{Gr}\, T_{01} - \mathrm{Re}^2\ \mathrm{Gc}\, C_{01}, \\ & T_{01}'' + \Pr\ \mathrm{Re}\, T_{01}' - \mathrm{R}\,\mathrm{Re}^2\ T_{01} = -\Pr\ u_{00}'^2, \\ & C_{01}'' + \mathrm{Sc}\, C_{01}' = 0, \\ & V_1'' + \mathrm{Re}\, V_1' - \left(\frac{1}{K} + \lambda^2\pi^2\right) V_1 = -\mathrm{Re}^2\ \mathrm{Gr}\,\theta_1 - \mathrm{Re}^2\ \mathrm{Gc}\,\phi_1, \\ & \theta_1'' + \Pr\ \mathrm{Re}\,\theta_1' - \left(\mathrm{R.}\,\mathrm{Re}^2 + \lambda^2\pi^2\right)\theta_1 = -2\Pr\ u_{00}' V_0', \\ & \phi_1'' + \mathrm{Sc}\,\phi_1' - \lambda^2\pi^2\phi_1 = 0, \end{aligned}\right\} \tag{29}$$

with boundary conditions as:

$$\left.\begin{aligned} y = 0:\ u_{01} &= h_1 u_{01}',\ T_{01} = 0, \quad C_{01} = 0, \\ V_1 &= h_1 V_1',\ \theta_1 = 0, \quad \phi_1 = 0, \\ y = 1:\ u_{01} &= h_2 u_{01}',\ T_{01} = 0, \quad C_{01} = 0, \\ V_1 &= h_2 V_1',\ \theta_1 = 0, \quad \phi_1 = 0, \end{aligned}\right\} \tag{30}$$

here prime denotes differentiation with respect y throughout.

Equations (27) and (29) are linear in nature, their final solutions are obtained with the help of corresponding boundary conditions and then are substituted back in (12), we obtain the result as:

$$
\begin{aligned}
u &= u_0 + \in u_1 = u_{00} + Ecu_{01} + \in (V_0 + Ec\,V_1)\cos\lambda\pi z \\
&= m_5 e^{x_3 y} + m_6 e^{x_4 y} + A_1 e^{x_1 y} + A_2 e^{x_2 y} + A_3 + A_4 \overline{e}^{scy} + Ec\{m_{17} e^{x_{13} y} + m_{18} e^{x_{14} y} + \\
&A_{39} e^{x_{11} y} + A_{40} e^{x_{12} y} + A_{41} e^{2x_3 y} + A_{42} e^{2x_4 y} + A_{43} e^{2x_1 y} + A_{44} e^{2x_2 y} + A_{45} \overline{e}^{2Scy} + A_{46} e^{(x_3+x_4)y} + \\
&A_{47} e^{(x_3+x_1)y} + A_{48} e^{(x_3+x_2)y} + A_{49} e^{(x_3-Sc)y} + A_{50} e^{(x_4+x_1)y} + A_{51} e^{(x_4+x_2)y} + A_{52} e^{(x_4-Sc)y} + \\
&A_{53} e^{(x_1+x_2)y} + A_{54} e^{(x_1-Sc)y} + A_{55} e^{(x_2-Sc)y}\} + \in \cos\lambda\pi z\Big[\Big(m_{11} e^{x_9 y} + m_{12} e^{x_{10} y} + A_5 e^{x_5 y} + A_6 e^{x_6 y} \\
&+ A_7 e^{x_7 y} + A_8 e^{x_8 y}\Big) + Ec\Big(m_{23} e^{x_{19} y} + m_{24} e^{x_{20} y} + s_1 e^{x_{15} y} + s_2 e^{x_{16} y} + s_3 e^{(x_3+x_9)y} + s_4 e^{(x_3+x_{10})y} + \\
&s_5 e^{(x_3+x_5)y} + s_6 e^{(x_3+x_6)y} + s_7 e^{(x_3+x_7)y} + s_8 e^{(x_3+x_8)y} + s_9 e^{(x_4+x_9)y} + s_{10} e^{(x_4+x_{10})y} + s_{11} e^{(x_4+x_5)y} + \\
&s_{12} e^{(x_4+x_6)y} + s_{13} e^{(x_4+x_7)y} + s_{14} e^{(x_4+x_8)y} + s_{15} e^{(x_1+x_9)y} + s_{16} e^{(x_1+x_{10})y} + s_{17} e^{(x_1+x_5)y} + s_{18} e^{(x_1+x_6)y} + \\
&s_{19} e^{(x_1+x_7)y} + s_{20} e^{(x_1+x_8)y} + s_{21} e^{(x_2+x_9)y} + s_{22} e^{(x_2+x_{10})y} + s_{23} e^{(x_2+x_5)y} + s_{24} e^{(x_2+x_6)y} + s_{25} e^{(x_2+x_7)y} + \\
&s_{26} e^{(x_2+x_8)y} + s_{27} e^{x_9 y} + s_{28} e^{x_{10} y} + s_{29} e^{x_5 y} + s_{30} e^{x_6 y} + s_{31} e^{x_7 y} + s_{32} e^{x_8 y} + s_{33} e^{(x_9-Sc)y} + s_{34} e^{(x_{10}-Sc)y} + \\
&s_{35} e^{(x_5-Sc)y} + s_{36} e^{(x_6-Sc)y} + s_{37} e^{(x_7-Sc)y} + s_{38} e^{(x_8-Sc)y}.
\end{aligned}
$$

$$
\begin{aligned}
T &= T_0 + \in T_1 = T_{00} + EcT_{01} + \in \cos\lambda\pi z(\theta_0 + Ec\theta_1) \\
&= \Big(m_1 e^{x_1 y} + m_2 e^{x_2 y}\Big) + Ec\Big(m_{13} e^{x_{11} y} + m_{14} e^{x_{12} y} + A_{24} e^{2x_3 y} + A_{25} e^{2x_4 y} + A_{26} e^{2x_1 y} + A_{27} e^{2x_2 y} + \\
&A_{28} e^{-2Scy} + A_{29} e^{(x_3+x_4)y} + A_{30} e^{(x_3+x_1)y} + A_{31} e^{(x_3+x_2)y} + A_{32} e^{(x_3-Sc)y} + A_{33} e^{(x_4+x_1)y} + A_{34} e^{(x_4+x_2)y} + \\
&A_{35} e^{(x_4-Sc)y} + A_{36} e^{(x_1+x_2)y} + A_{37} e^{(x_1-Sc)y} + A_{38} e^{(x_2-Sc)y}\Big) + \in \cos\lambda\pi z\Big[\Big(m_7 e^{x_5 y} + m_8 e^{x_6 y}\Big) + \\
&Ec\Big(m_{19} e^{x_{15} y} + m_{20} e^{x_{16} y} + A_{56} e^{(x_3+x_9)y} + A_{57} e^{(x_3+x_{10})y} + A_{58} e^{(x_3+x_5)y} + A_{59} e^{(x_3+x_8)y} + A_{60} e^{(x_3+x_7)y} + \\
&A_{61} e^{(x_3+x_8)y} + A_{62} e^{(x_4+x_9)y} + A_{63} e^{(x_4+x_{10})y} + A_{64} e^{(x_4+x_5)y} + A_{65} e^{(x_4+x_6)y} + A_{66} e^{(x_4+x_7)y} + A_{67} e^{(x_4+x_8)y} + \\
&A_{68} e^{(x_1+x_9)y} + A_{69} e^{(x_1+x_{10})y} + A_{70} e^{(x_1+x_5)y} + A_{71} e^{(x_1+x_6)y} + A_{72} e^{(x_1+x_7)y} + A_{73} e^{(x_1+x_8)y} + A_{74} e^{(x_2+x_9)y} + \\
&A_{75} e^{(x_2+x_{10})y} + A_{76} e^{(x_2+x_5)y} + A_{77} e^{(x_2+x_6)y} + A_{78} e^{(x_2+x_7)y} + A_{79} e^{(x_2+x_8)y} + A_{80} e^{x_9 y} + A_{81} e^{x_{10} y} + A_{82} e^{x_5 y} \\
&+ A_{83} e^{x_6 y} + A_{84} e^{x_7 y} + A_{85} e^{x_8 y} + A_{86} e^{(x_9-Sc)y} + A_{87} e^{(x_{10}-Sc)y} + A_{88} e^{(x_5-Sc)y} + A_{89} e^{(x_6-Sc)y} + A_{90} e^{(x_7-Sc)y} \\
&+ A_{91} e^{(x_8-Sc)y}\Big)\Big].
\end{aligned}
$$

and

$$
\begin{aligned}
C &= C_0 + \in C_1 = C_{00} + Ec\,C_{01} + \in \cos\lambda\pi z(\phi_0 + Ec\phi_1). \\
&= \left(m_3 + m_4 e^{-scy}\right) + \in \cos\lambda\pi z\left(m_9 e^{x_7 y} + m_{10} e^{x_8 y}\right).
\end{aligned}
$$

4. Skin Friction

From the velocity component u, we can now calculate an important parameter of skin friction. In non-dimensional form it is given by (on the plate $y = 0$):

$$
C_f = \frac{\tau_w d}{\mu U} = \left(\frac{\partial u}{\partial y}\right)_{y=0}
$$

$$\begin{aligned} C_f &= m_5x_3 + m_6x_4 + A_1x_1 + A_2x_2 - A_4Sc + Ec\{m_{17}x_{13} + m_{18}x_{14} + A_{39}x_{11} + A_{40}x_{12} \\ &+A_{41}x_3 + 2A_{42}x_4 + 2A_{43}x_1 + 2A_{44}x_2 - 2A_{45}Sc + A_{46}(x_3 + x_4) + A_{47}(x_3 + x_1) \\ &+A_{48}(x_3 + x_2) + A_{49}(x_3 - Sc) + A_{50}(x_4 + x_1) + A_{51}(x_4 + x_2) + A_{52}(x_4 - Sc) \\ &+A_{53}(x_1 + x_2) + A_{54}(x_1 - Sc) + A_{55}(x_2 - Sc)\} + \in \cos\lambda\pi z\Big[(m_{11}x_9 + m_{12}x_{10} + \\ &A_5x_5 + A_6x_6 + A_7x_7 + A_8x_8) + Ec(m_{23}x_{19} + m_{24}x_{20} + s_1x_{15} + s_2x_{16} + s_3(x_3 + x_9 \\ &+s_4(x_3 + x_{10}) + s_5(x_3 + x_5) + s_6(x_3 + x_6) + s_7(x_3 + x_7) + s_8(x_3 + x_8) \\ &+s_9(x_4 + x_9) + s_{10}(x_4 + x_{10}) + s_{11}(x_4 + x_5) + s_{12}(x_4 + x_6) + s_{13}(x_4 + x_7) + \\ &s_{14}(x_4 + x_8) + s_{15}(x_1 + x_9) + s_{16}(x_1 + x_{10}) + s_{17}(x_1 + x_5) + s_{18}(x_1 + x_6) + \\ &s_{19}(x_1 + x_7) + s_{20}(x_1 + x_8) + s_{21}(x_2 + x_9) + s_{22}(x_2 + x_{10}) + s_{23}(x_2 + x_5) + \\ &s_{24}(x_2 + x_6) + s_{25}(x_2 + x_7) + s_{26}(x_2 + x_8) + s_{27}x_9 + s_{28}x_{10} + s_{29}x_5 + s_{30}x_6 + \\ &s_{31}x_7 + s_{32}x_8 + s_{33}(x_9 - Sc) + s_{34}(x_{10} - Sc) + s_{35}(x_5 - Sc) + s_{36}(x_6 - Sc) + \\ &+s_{37}(x_7 - Sc) + s_{38}(x_8 - Sc))\Big]. \end{aligned} \tag{31}$$

5. Nusselt Number

Another important physical parameter of interest viz. Nusselt number in dimensionless form is:

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$\begin{aligned} Nu &= -\Big[m_1x_1 + m_2x_2 + Ec(m_{13}x_{11} + m_{14}x_{12} + 2A_{24}x_3 + 2A_{25}x_4 + 2A_{26}x_1 + 2A_{27}x_2 - \\ &2A_{28}Sc + A_{29}(x_3 + x_4) + A_{30}(x_3 + x_1) + A_{31}(x_3 + x_2) + A_{32}(x_3 - Sc) \\ &+A_{33}(x_4 + x_1) + A_{34}(x_4 + x_2) + A_{35}(x_4 - Sc) + A_{36}(x_1 + x_2) + A_{37}(x_1 - Sc) \\ &+A_{38}(x_2 - Sc)) + \in \cos\lambda\pi z\{(m_7x_5 + m_8x_6) + Ec(m_{19}x_{15} + m_{20}x_{16} + \\ &A_{56}(x_3 + x_9) + A_{57}(x_3 + x_{10}) + A_{58}(x_3 + x_5) + A_{59}(x_3 + x_6) + A_{60}(x_3 + x_7) \\ &+A_{61}(x_3 + x_8) + A_{62}(x_4 + x_9) + A_{63}(x_4 + x_{10}) + A_{64}(x_4 + x_5) + A_{65}(x_4 + x_6) \\ &+A_{66}(x_4 + x_7) + A_{67}(x_4 + x_8) + A_{68}(x_1 + x_9) + A_{69}(x_1 + x_{10}) + A_{70}(x_1 + x_5) \\ &+A_{71}(x_1 + x_6) + A_{72}(x_1 + x_7) + A_{73}(x_1 + x_8) + A_{74}(x_2 + x_9) + A_{75}(x_2 + x_{10}) \\ &+A_{76}(x_2 + x_5) + A_{77}(x_2 + x_6) + A_{78}(x_2 + x_7) + A_{79}(x_2 + x_8) + A_{80}x_9 + A_{81}x_{10} \\ &+A_{82}x_5 + A_{83}x_6 + A_{84}x_7 + A_{85}x_8 + A_{86}(x_9 - Sc) + A_{87}(x_{10} - Sc) + A_{88}(x_5 - Sc) \\ &+A_{89}(x_6 - Sc) + A_{90}(x_7 - Sc) + A_{91}(x_8 - Sc))\}\Big]. \end{aligned} \tag{32}$$

Where,

$$x_1, x_2 = \frac{-\Pr \operatorname{Re} \mp \sqrt{\Pr^2 \operatorname{Re}^2 + 4R \operatorname{Re}^2}}{2}, \quad x_3, x_4 = \frac{-\operatorname{Re} \mp \sqrt{\operatorname{Re}^2 + 4\left(1/K\right)}}{2}$$

$$x_5, x_6 = \frac{-\Pr \operatorname{Re} \mp \sqrt{\Pr^2 \operatorname{Re}^2 + 4\left(R \operatorname{Re}^2 + \lambda^2\pi^2\right)}}{2}, \quad x_7, x_8 = \frac{-Sc \mp \sqrt{Sc^2 + 4\lambda^2\pi^2}}{2}$$

$$x_9, x_{10} = \frac{-\operatorname{Re} \mp \sqrt{\operatorname{Re}^2 + 4\left(1/K + \lambda^2\pi^2\right)}}{2}, \quad x_{11}, x_{12} = \frac{-\Pr \operatorname{Re} \mp \sqrt{\Pr^2 \operatorname{Re}^2 + 4R \operatorname{Re}^2}}{2}$$

$$m_1 = \frac{-e^{x_2}}{e^{x_1} - e^{x_2}}, m_2 = \frac{e^{x_1}}{e^{x_1} - e^{x_2}}, m_3 = (1 - m_4), m_4 = \frac{1}{1 - e^{-Sc}}$$

$$m_5 = \frac{\left\{1 - m_6(1 - h_1x_4) - A_1(1 - h_1x_1) - A_2(1 - h_1x_2) - A_3 - A_4(1 + h_1Sc)\right\}}{(1 - h_1x_3)}$$

$$m_6 = \frac{\begin{array}{l}\left\{(1-h_1x_3) + e^{x_3}(1-h_2x_3) - A_1e^{x_3}(1-h_1x_1)(1-h_2x_3) - A_2(1-h_1x_2)e^{x_3}(1-h_2x_3)\right.\\ -A_3e^{x_3}(1-h_2x_3) - A_4e^{x_3}(1+h_1Sc)(1-h_2x_3) + A_1e^{x_1}(1-h_2x_1)(1-h_1x_3) - A_2e^{x_2}\\ \left.(1-h_2x_2)(1-h_1x_3) + A_3(1-h_1x_3) + A_4\bar{e}^{Sc}(1+h_2Sc)(1-h_1x_3)\right\}\end{array}}{\left\{e^{x_3}(1-h_1x_4)(1-h_2x_3) - e^{x_4}(1-h_2x_4)(1-h_1x_3)\right\}}$$

$$m_7 = (1 - m_8), m_8 = \frac{e^{x_5}}{e^{x_5} - e^{x_6}}, m_9 = (1 - m_{10}), m_{10} = \frac{e^{x_7}}{e^{x_7} - e^{x_8}},$$

$$m_{11} = \frac{\left\{m_{12}(1 - h_1x_{10}) + A_5(1 - h_1x_5) + A_6(1 - h_1x_6) + A_7(1 - h_1x_7) + A_8(1 - h_1x_8)\right\}}{(h_1x_9 - 1)}$$

$$m_{12} = \frac{\begin{array}{l}\left\{A_5e^{x_9}(1-h_1x_5)(1-h_2x_9) + A_6e^{x_9}(1-h_1x_6)(1-h_2x_9) + A_7e^{x_9}\right.\\ (1-h_1x_7)(1-h_2x_9) + A_8e^{x_9}(1-h_1x_8)(1-h_2x_9) + A_5e^{x_5}(1-h_2x_5)(h_1x_9-1)\\ +A_6e^{x_6}(1-h_2x_6)(h_1x_9-1) + A_7e^{x_7}(1-h_2x_7)(h_1x_9-1) + A_8e^{x_8}(1-h_2x_8)\\ \left.(h_1x_9-1)\right\}\end{array}}{\left\{e^{x_9}(1-h_1x_{10})(1-h_2x_9) + e^{x_{10}}(1-h_2x_{10})(h_1x_9-1)\right\}}$$

$$A_1 = \frac{-\operatorname{Re}^2 Gr\, m_1}{(x_1 - x_3)(x_1 - x_4)}, \; A_2 = \frac{-\operatorname{Re}^2 Gr\, m_2}{(x_2 - x_3)(x_2 - x_4)}, \; A_3 = \frac{-\operatorname{Re}^2 Gc\, m_3}{x_3.x_4},$$

$$A_4 = \frac{-\operatorname{Re}^2 Gc\, m_4}{(Sc + x_3)(Sc + x_4)}, \; A_5 = \frac{-\operatorname{Re}^2 Gr\, m_7}{(x_5 - x_9)(x_5 - x_{10})}, \; A_6 = \frac{-\operatorname{Re}^2 Gr\, m_8}{(x_6 - x_9)(x_6 - x_{10})},$$

$$A_7 = \frac{-\operatorname{Re}^2 Gc\, m_9}{(x_7 - x_9)(x_7 - x_{10})}, \; A_8 = \frac{-\operatorname{Re}^2 Gc\, m_{10}}{(x_8 - x_9)(x_8 - x_{10})}, \; A_9 = m_5^2\, x_3^2,$$

$$A_{10} = m_6^2\, x_4^2,\;\; A_{11} = A_1^2\, x_1^2,\;\; A_{12} = A_2^2\, x_2^2,\;\; A_{13} = A_4^2\, Sc^2,\; A_{14} = 2m_5x_3m_6x_4,$$

$$A_{15} = 2m_5x_3A_1x_1,\;\; A_{16} = 2A_2x_2m_5x_3,\;\; A_{17} = -2m_5x_3A_4Sc,\;\; A_{18} = 2m_6x_4A_1x_1,$$

$$A_{19} = 2m_6x_4A_2x_2,\; A_{20} = -2m_6x_4A_4Sc,\;\; A_{21} = 2A_1x_1A_2x_2,$$

$$A_{22} = -2A_1x_1A_4Sc,\; A_{23} = -2A_2x_2A_4Sc,\;\; A_{24} = \frac{-\Pr A_9}{(2x_3 - x_{11})(2x_3 - x_{12})},$$

$$A_{25} = \frac{-\Pr A_{10}}{(2x_4 - x_{11})(2x_4 - x_{12})},\; A_{26} = \frac{-\Pr A_{11}}{(2x_1 - x_{11})(2x_1 - x_{12})},\; A_{27} = \frac{-\Pr A_{12}}{(2x_2 - x_{11})(2x_2 - x_{12})}$$

$$A_{28} = \frac{-\Pr .A_{13}}{(2Sc + x_{11})(2Sc + x_{12})},\; A_{29} = \frac{-\Pr A_{14}}{(x_3 + x_4 - x_{11})(x_3 + x_4 - x_{12})},$$

$$A_{30} = \frac{-\Pr A_{15}}{(x_3 + x_1 - x_{11})(x_3 + x_1 - x_{12})},\; A_{31} = \frac{-\Pr A_{16}}{(x_3 + x_2 - x_{11})(x_3 + x_2 - x_{12})},$$

$$A_{32} = \frac{-\Pr A_{17}}{(x_3 - Sc - x_{11})(x_3 - Sc - x_{12})},\; A_{33} = \frac{-\Pr A_{18}}{(x_4 + x_1 - x_{11})(x_4 + x_1 - x_{12})},$$

$$A_{34} = \frac{-\Pr A_{19}}{(x_4 + x_2 - x_{11})(x_4 + x_2 - x_{12})},\;\; A_{35} = \frac{-A_{20}\Pr}{(x_4 - Sc - x_{11})(x_4 - Sc - x_{12})},$$

$$A_{36} = \frac{-\Pr A_{21}}{(x_1 + x_2 - x_{11})(x_1 + x_2 - x_{12})},\; A_{37} = \frac{-\Pr A_{22}}{(x_1 - Sc - x_{11})(x_1 - Sc - x_{12})},$$

$$A_{38} = \frac{-\Pr A_{23}}{(x_2 - Sc - x_{11})(x_2 - Sc - x_{12})},$$

$$m_{13} = -\left\{ m_{14} + \sum_{i=24}^{38} A_i \right\}$$

$$m_{14} = \frac{\begin{aligned}&\left\{A_{24}\left(e^{2x_3} - e^{x_{11}}\right) + A_{25}\left(e^{2x_4} - e^{x_{11}}\right) + A_{26}\left(e^{2x_1} - e^{x_{11}}\right) + A_{27}\left(e^{2x_2} - e^{x_{11}}\right) + \right.\\ &A_{28}\left(\overline{e}^{2Sc} - e^{x_{11}}\right) + A_{29}\left(e^{(x_3+x_4)} - e^{x_{11}}\right) + A_{30}\left(e^{(x_3+x_1)} - e^{x_{11}}\right) + A_{31}\left(e^{(x_3+x_2)} - e^{x_{11}}\right)\\ &+ A_{32}\left(e^{(x_3-Sc)} - e^{x_{11}}\right) + A_{33}\left(e^{(x_4+x_1)} - e^{x_{11}}\right) + A_{34}\left(e^{(x_4+x_2)} - e^{x_{11}}\right) + A_{35}\left(e^{(x_4-Sc)} - e^{x_{11}}\right)\\ &\left.+ A_{36}\left(e^{(x_1+x_2)} - e^{x_{11}}\right) + A_{37}\left(e^{(x_1-Sc)} - e^{x_{11}}\right) + A_{38}\left(e^{(x_2-Sc)} - e^{x_{11}}\right)\right\}\end{aligned}}{\left(e^{x_{11}} - e^{x_{12}}\right)},$$

$m_{15} = -m_{16} = 0.$

$$A_{39} = \frac{-\mathrm{Re}^2\,\mathrm{Gr}}{(x_{11}-x_{13})(x_{11}-x_{14})},\ A_{40} = \frac{-\mathrm{Re}^2\,\mathrm{Gr}}{(x_{12}-x_{13})(x_{12}-x_{14})},\ A_{41} = \frac{A_{24}}{(2x_3-x_{13})(2x_3-x_{14})},$$

$$A_{42} = \frac{A_{25}}{(2x_4-x_{13})(2x_4-x_{14})}, A_{43} = \frac{A_{26}}{(2x_1-x_{13})(2x_1-x_{14})},\ A_{44} = \frac{A_{27}}{(2x_2-x_{13})(2x_2-x_{14})},$$

$$A_{45} = \frac{A_{28}}{(2\mathrm{Sc}+x_{13})(2\mathrm{Sc}+x_{14})}, A_{46} = \frac{A_{29}}{(x_3+x_4-x_{13})(x_3+x_4-x_{14})}$$

$$A_{47} = \frac{A_{30}}{(x_3+x_1-x_{13})(x_3+x_1-x_{14})}, A_{48} = \frac{A_{31}}{(x_3+x_2-x_{13})(x_3+x_2-x_{14})},$$

$$A_{49} = \frac{A_{32}}{(x_3-\mathrm{Sc}-x_{13})(x_3-\mathrm{Sc}-x_{14})}, A_{50} = \frac{A_{33}}{(x_4+x_1-x_{13})(x_4+x_1-x_{14})},$$

$$A_{51} = \frac{A_{34}}{(x_4+x_2-x_{13})(x_4+x_2-x_{14})},\ A_{52} = \frac{A_{35}}{(x_4-\mathrm{Sc}-x_{13})(x_4-\mathrm{Sc}-x_{14})},$$

$$A_{53} = \frac{A_{36}}{(x_1+x_2-x_{13})(x_1+x_2-x_{14})}, A_{54} = \frac{A_{37}}{(x_1-\mathrm{Sc}-x_{13})(x_1-\mathrm{Sc}-x_{14})},$$

$$A_{55} = \frac{A_{38}}{(x_2-\mathrm{Sc}-x_{13})(x_2-\mathrm{Sc}-x_{14})}, f_1 = A_{39}(1-h_1x_{11}), f_2 = A_{40}(1-h_1x_{12}),$$

$$f_3 = A_{41}(1-h_1 2x_3), f_4 = A_{42}(1-2h_1x_4), f_5 = A_{43}(1-2h_1x_1),$$

$$f_6 = A_{44}(1-2h_1x_2), f_7 = A_{45}(1+2h_1\mathrm{Sc}),\ f_8 = A_{46}(1-h_1(x_3+x_4)),$$

$$f_9 = A_{47}(1-h_1(x_3+x_1)), f_{10} = A_{48}(1-h_1(x_3+x_2)), f_{11} = A_{49}(1-h_1(x_3-\mathrm{Sc})),$$

$$f_{12} = A_{50}(1-h_1(x_4+x_1)), f_{13} = A_{51}(1-h_1(x_4+x_2)), f_{14} = A_{52}(1-h_1(x_4-\mathrm{Sc})),$$

$$f_{15} = A_{53}(1-h_1(x_1+x_2)), f_{16} = A_{54}(1-(x_1-\mathrm{Sc})h_1), f_{17} = A_{55}(1-h_1(x_2-\mathrm{Sc})),$$

$$f_{18} = A_{39}e^{x_{11}}(1-h_2x_{11})(h_1x_{13}-1), f_{19} = A_{40}e^{x_{12}}(1-h_2x_{12})(h_1x_{13}-1),$$

$$f_{20} = A_{41}e^{2x_3}(1-2h_2x_3)(h_1x_{13}-1), f_{21} = A_{42}e^{2x_4}(1-2h_2x_4)(h_1x_{13}-1),$$

$$f_{22} = A_{43}e^{2x_1}(1-2h_2x_1)(h_1x_{13}-1), f_{23} = A_{44}e^{2x_2}(1-2h_2x_2)(h_1x_{13}-1),$$

$$f_{24} = A_{45}e^{-2\mathrm{Sc}}(1+2h_2\mathrm{Sc})(h_1x_{13}-1), f_{25} = A_{46}e^{(x_3+x_4)}(1-h_2(x_3+x_4))(h_1x_{13}-1),$$

$$f_{26} = A_{47}e^{(x_3+x_1)}(1-h_2(x_3+x_1))(h_1x_{13}-1), f_{27} = A_{48}e^{(x_3+x_2)}(1-h_2(x_3+x_2))(h_1x_{13}-1),$$

$$f_{28} = A_{49}e^{(x_3-\mathrm{Sc})}(1-h_2(x_3-\mathrm{Sc}))(h_1x_{13}-1), f_{29} = A_{50}e^{(x_4+x_1)}(1-h_2(x_4+x_1))(h_1x_{13}-1),$$

$$f_{30} = A_{51}e^{(x_4+x_2)}(1-h_2(x_4+x_2))(h_1x_{13}-1), f_{31} = A_{52}e^{(x_4-\mathrm{Sc})}(1-h_2(x_4-\mathrm{Sc}))(h_1x_{13}-1),$$

$$f_{32} = A_{53}e^{(x_1+x_2)}(1-h_2(x_1+x_2))(h_1x_{13}-1), f_{33} = A_{54}e^{(x_1-\mathrm{Sc})}(1-h_2(x_1-\mathrm{Sc}))(h_1x_{13}-1),$$

$$f_{34} = A_{55}e^{(x_2-\mathrm{Sc})}(1-h_2(x_2-\mathrm{Sc}))(h_1x_{13}-1).$$

$$m_{17} = \frac{\left\{m_{18}(1-h_1x_{14}) + \sum_{j=1}^{17} f_j\right\}}{(h_1x_{13}-1)},\quad m_{18} = \frac{\left\{\sum_{j=1}^{17} f_j\right\}e^{x_{13}}(1-h_2x_{13}) + \left\{\sum_{k=18}^{34} f_k\right\}}{e^{x_{14}}(h_2x_{14}-1)(h_1x_{13}-1) - m_{18}e^{x_{13}}(1-h_1x_{14})(1-h_2x_{13})}.$$

$$A_{56}=\frac{-2\Pr m_5 x_3 m_{11} x_9}{(x_3+x_9-x_{16})(x_3+x_9-x_{15})},\ A_{57}=\frac{-2\Pr m_5 x_3 m_{12} x_{10}}{(x_3+x_{10}-x_{16})(x_3+x_{10}-x_{15})},$$

$$A_{58}=\frac{-2\Pr m_5 x_3 A_5 x_5}{(x_3+x_5-x_{16})(x_3+x_5-x_{15})},\ A_{59}=\frac{-2\Pr m_5 x_3 A_6 x_6}{(x_3+x_6-x_{16})(x_3+x_6-x_{15})},$$

$$A_{60}=\frac{-2\Pr m_5 x_3 A_7 x_7}{(x_3+x_7-x_{16})(x_3+x_7-x_{15})},\ A_{61}=\frac{-2\Pr m_5 x_3 A_8 x_8}{(x_3+x_8-x_{15})(x_3+x_8-x_{16})},$$

$$A_{62}=\frac{-2\Pr m_6 x_4 m_{11} x_9}{(x_4+x_9-x_{15})(x_4+x_9-x_{16})},\ A_{63}=\frac{-2\Pr m_6 x_4 m_{12} x_{10}}{(x_4+x_{10}-x_{15})(x_4+x_{10}-x_{16})},$$

$$A_{64}=\frac{-2\Pr m_6 x_4 A_5 x_5}{(x_4+x_5-x_{15})(x_4+x_5-x_{16})},\ A_{65}=\frac{-2\Pr m_6 x_4 A_6 x_6}{(x_4+x_6-x_{15})(x_4+x_6-x_{16})},$$

$$A_{66}=\frac{-2\Pr m_6 x_4 A_7 x_7}{(x_4+x_7-x_{15})(x_4+x_7-x_{16})},\ A_{67}=\frac{-2\Pr m_6 x_4 A_8 x_8}{(x_4+x_8-x_{15})(x_4+x_8-x_{16})},$$

$$A_{68}=\frac{-2\Pr A_1 x_1 m_{11} x_9}{(x_1+x_9-x_{15})(x_1+x_9-x_{16})},\ A_{69}=\frac{-2\Pr A_1 x_1 m_{12} x_{10}}{(x_1+x_{10}-x_{15})(x_1+x_{10}-x_{16})},$$

$$A_{70}=\frac{-2\Pr A_1 x_1 A_5 x_5}{(x_1+x_5-x_{15})(x_1+x_5-x_{16})},\ A_{71}=\frac{-2\Pr A_1 x_1 A_6 x_6}{(x_1+x_6-x_{15})(x_1+x_6-x_{16})},$$

$$A_{72}=\frac{-2\Pr A_1 x_1 A_7 x_7}{(x_1+x_7-x_{15})(x_1+x_7-x_{16})},\ A_{73}=\frac{-2\Pr A_1 x_1 A_8 x_8}{(x_1+x_8-x_{15})(x_1+x_8-x_{16})},$$

$$A_{74}=\frac{-2\Pr A_2 x_2 m_{11} x_9}{(x_2+x_9-x_{15})(x_2+x_9-x_{16})},\ A_{75}=\frac{-2\Pr A_2 x_2 m_{12} x_{10}}{(x_2+x_{10}-x_{15})(x_2+x_{10}-x_{16})},$$

$$A_{76}=\frac{-2\Pr A_2 x_2 A_5 x_5}{(x_2+x_5-x_{15})(x_2+x_5-x_{16})},\ A_{77}=\frac{-2\Pr A_2 x_2 A_6 x_6}{(x_2+x_6-x_{15})(x_2+x_6-x_{16})},$$

$$A_{78}=\frac{-2\Pr A_2 x_2 A_7 x_7}{(x_2+x_7-x_{15})(x_2+x_7-x_{16})},\ A_{79}=\frac{-2\Pr A_2 x_2 A_8 x_8}{(x_2+x_8-x_{15})(x_2+x_8-x_{16})},$$

$$A_{80}=\frac{-2\Pr A_3 m_{11} x_9}{(x_9-x_{15})(x_9-x_{16})},\ A_{81}=\frac{-2\Pr A_3 m_{12} x_{10}}{(x_{10}-x_{15})(x_{10}-x_{16})},$$

$$A_{82}=\frac{-2\Pr A_3 A_5 x_5}{(x_5-x_{15})(x_5-x_{16})},\ A_{83}=\frac{-2\Pr A_3 A_6 x_6}{(x_6-x_{15})(x_6-x_{16})},$$

$$A_{84}=\frac{-2\Pr A_3 A_7 x_7}{(x_7-x_{15})(x_7-x_{16})},\ A_{85}=\frac{-2\Pr A_3 A_8 x_8}{(x_8-x_{15})(x_8-x_{16})},$$

$$A_{86}=\frac{-2\Pr A_4 Sc\, m_{11} x_9}{(x_9-Sc-x_{15})(x_9-Sc-x_{16})},\ A_{87}=\frac{-2\Pr A_4 Sc\, m_{12} x_{10}}{(x_{10}-Sc-x_{15})(x_{10}-Sc-x_{16})},$$

$$A_{88}=\frac{-2\Pr A_4 Sc A_5 x_5}{(x_5-Sc-x_{15})(x_5-Sc-x_{16})},\ A_{89}=\frac{-2\Pr A_4 Sc A_6 x_6}{(x_6-Sc-x_{15})(x_6-Sc-x_{16})},$$

$$A_{90}=\frac{-2\Pr A_4 Sc A_7 x_7}{(x_7-Sc-x_{15})(x_7-Sc-x_{16})},\ A_{91}=\frac{-2\Pr A_4 Sc A_8 x_8}{(x_8-Sc-x_{15})(x_8-Sc-x_{16})},$$

$$m_{19}=-\left\{m_{20}+\sum_{a=56}^{91}A_a\right\}$$

$$m_{20}=\frac{\begin{array}{l}\Big\{A_{56}\left(e^{(x_3-x_9)}-e^{x_{15}}\right)+A_{57}\left(e^{(x_3+x_{10})}-e^{x_{15}}\right)+A_{58}\left(e^{(x_3+x_5)}-e^{x_{15}}\right)+\\ A_{59}\left(e^{(x_3+x_6)}-e^{x_{15}}\right)+A_{60}\left(e^{(x_3+x_7)}-e^{x_{15}}\right)+A_{61}\left(e^{(x_3+x_8)}-e^{x_{15}}\right)+\\ A_{62}\left(e^{(x_4+x_9)}-e^{x_{15}}\right)+A_{63}\left(e^{(x_4+x_{10})}-e^{x_{15}}\right)+A_{64}\left(e^{(x_4+x_5)}-e^{x_{15}}\right)+\\ A_{65}\left(e^{(x_4+x_6)}-e^{x_{15}}\right)+A_{66}\left(e^{(x_4+x_7)}-e^{x_{15}}\right)+A_{67}\left(e^{(x_4+x_8)}-e^{x_{15}}\right)+\\ A_{68}\left(e^{(x_1+x_9)}-e^{x_{15}}\right)+A_{69}\left(e^{(x_1+x_{10})}-e^{x_{15}}\right)+A_{70}\left(e^{(x_1+x_5)}-e^{x_{15}}\right)+\\ A_{71}\left(e^{(x_1+x_6)}-e^{x_{15}}\right)+A_{72}\left(e^{(x_1+x_7)}-e^{x_{15}}\right)+A_{73}\left(e^{(x_1+x_8)}-e^{x_{15}}\right)+\\ A_{74}\left(e^{(x_2+x_9)}-e^{x_{15}}\right)+A_{75}\left(e^{(x_2+x_{10})}-e^{x_{15}}\right)+A_{76}\left(e^{(x_2+x_5)}-e^{x_{15}}\right)+\\ A_{77}\left(e^{(x_2+x_6)}-e^{x_{15}}\right)+A_{78}\left(e^{(x_2+x_7)}-e^{x_{15}}\right)+A_{79}\left(e^{(x_2+x_8)}-e^{x_{15}}\right)+\\ A_{80}\left(e^{x_9}-e^{x_{15}}\right)+A_{81}\left(e^{x_{10}}-e^{x_{15}}\right)+A_{82}\left(e^{x_5}-e^{x_{15}}\right)+A_{83}\left(e^{x_6}-e^{x_{15}}\right)+\\ A_{84}\left(e^{x_7}-e^{x_{15}}\right)+A_{85}\left(e^{x_8}-e^{x_{15}}\right)+A_{86}\left(e^{(x_9-Sc)}-e^{x_{15}}\right)+A_{87}\left(e^{(x_{10}-Sc)}-e^{x_{15}}\right)\\ +A_{88}\left(e^{(x_5-Sc)}-e^{x_{15}}\right)+A_{89}\left(e^{(x_6-Sc)}-e^{x_{15}}\right)+A_{90}\left(e^{(x_7-Sc)}-e^{x_{15}}\right)+\\ A_{91}\left(e^{(x_8-Sc)}-e^{x_{15}}\right)\Big\}\end{array}}{\left(e^{x_{15}}-e^{x_{16}}\right)}$$

$$m_{21}=-m_{22}=0.$$

$$s_1=\frac{-Re^2\,Gr\,m_{19}}{(x_{15}-x_{19})(x_{15}-x_{20})},\ s_2=\frac{-Re^2\,Gr\,m_{20}}{(x_{16}-x_{19})(x_{16}-x_{20})},$$

$$s_3=\frac{-Re^2\,Gr\,A_{56}}{(x_3+x_9-x_{19})(x_3+x_9-x_{20})},\ s_4=\frac{-Re^2\,Gr\,A_{57}}{(x_3+x_{10}-x_{19})(x_3+x_{10}-x_{20})},$$

$$s_5=\frac{-Re^2\,Gr\,A_{58}}{(x_3+x_5-x_{19})(x_3+x_5-x_{20})},\ s_6=\frac{-Re^2\,Gr\,A_{59}}{(x_3+x_6-x_{19})(x_3+x_6-x_{20})},$$

$$s_7=\frac{-Re^2\,Gr\,A_{60}}{(x_3+x_7-x_{19})(x_3+x_7-x_{20})},\ s_8=\frac{-Re^2\,Gr\,A_{61}}{(x_3+x_8-x_{19})(x_3+x_8-x_{20})},$$

$$s_9=\frac{-Re^2\,Gr\,A_{62}}{(x_4+x_9-x_{19})(x_4+x_9-x_{20})},\ s_{10}=\frac{-Re^2\,Gr\,A_{63}}{(x_4+x_{10}-x_{19})(x_4+x_{10}-x_{20})},$$

$$s_{11}=\frac{-Re^2\,Gr\,A_{64}}{(x_4+x_5-x_{19})(x_4+x_5-x_{20})},\ s_{12}=\frac{-Re^2\,Gr\,A_{65}}{(x_4+x_6-x_{19})(x_4+x_6-x_{20})},$$

$$s_{13}=\frac{-Re^2\,Gr\,A_{66}}{(x_4+x_7-x_{19})(x_4+x_7-x_{20})},\ s_{14}=\frac{-Re^2\,Gr\,A_{67}}{(x_4+x_8-x_{19})(x_4+x_8-x_{20})},$$

$$s_{15}=\frac{-Re^2\,Gr\,A_{68}}{(x_1+x_9-x_{19})(x_1+x_9-x_{20})},\ s_{16}=\frac{-Re^2\,Gr\,A_{69}}{(x_1+x_{10}-x_{19})(x_1+x_{10}-x_{20})},$$

$$s_{17}=\frac{-Re^2\,Gr\,A_{70}}{(x_1+x_5-x_{19})(x_1+x_5-x_{20})},\ s_{18}=\frac{-Re^2\,Gr\,A_{71}}{(x_1+x_6-x_{19})(x_1+x_6-x_{20})},$$

$$s_{19}=\frac{-Re^2\,Gr\,A_{72}}{(x_1+x_7-x_{19})(x_1+x_7-x_{20})},\ s_{20}=\frac{-Re^2\,Gr\,A_{73}}{(x_1+x_8-x_{19})(x_1+x_8-x_{20})},$$

$$s_{21}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{74}}{\left(x_2+x_9-x_{19}\right)\left(x_2+x_9-x_{20}\right)},\ s_{22}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{75}}{\left(x_2+x_{10}-x_{19}\right)\left(x_2+x_{10}-x_{20}\right)},$$

$$s_{23}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{76}}{\left(x_2+x_5-x_{19}\right)\left(x_2+x_5-x_{20}\right)},\ s_{24}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{77}}{\left(x_2+x_9-x_{19}\right)\left(x_2+x_9-x_{20}\right)},$$

$$s_{25}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{78}}{\left(x_2+x_7-x_{19}\right)\left(x_2+x_7-x_{20}\right)},\ s_{26}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{79}}{\left(x_2+x_8-x_{19}\right)\left(x_2+x_8-x_{20}\right)},$$

$$s_{27}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{80}}{\left(x_9-x_{19}\right)\left(x_9-x_{20}\right)},\ s_{28}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{81}}{\left(x_{10}-x_{19}\right)\left(x_{10}-x_{20}\right)},$$

$$s_{29}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{82}}{\left(x_5-x_{19}\right)\left(x_5-x_{20}\right)},\ s_{30}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{83}}{\left(x_6-x_{19}\right)\left(x_6-x_{20}\right)},$$

$$s_{31}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{84}}{\left(x_7-x_{19}\right)\left(x_7-x_{20}\right)},\ s_{32}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{85}}{\left(x_8-x_{19}\right)\left(x_8-x_{20}\right)},$$

$$s_{33}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{86}}{\left(x_9-\mathrm{Sc}-x_{19}\right)\left(x_9-\mathrm{Sc}-x_{20}\right)},\ s_{34}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{87}}{\left(x_{10}-\mathrm{Sc}-x_{19}\right)\left(x_{10}-\mathrm{Sc}-x_{20}\right)},$$

$$s_{35}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{88}}{\left(x_5-\mathrm{Sc}-x_{19}\right)\left(x_5-\mathrm{Sc}-x_{20}\right)},\ s_{36}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{89}}{\left(x_6-\mathrm{Sc}-x_{19}\right)\left(x_6-\mathrm{Sc}-x_{20}\right)},$$

$$s_{37}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{90}}{\left(x_7-\mathrm{Sc}-x_{19}\right)\left(x_7-\mathrm{Sc}-x_{20}\right)},\ s_{38}=\frac{-\mathrm{Re}^2\,\mathrm{Gr}\,A_{91}}{\left(x_8-\mathrm{Sc}-x_{19}\right)\left(x_8-\mathrm{Sc}-x_{20}\right)},$$

$$s_{39}=s_1\left(1-h_1x_{15}\right),\ s_{40}=s_2\left(1-h_1x_{16}\right),\ s_{41}=s_3\left(1-h_1\left(x_3+x_9\right)\right),$$

$$s_{42}=s_4\left(1-h_1\left(x_3+x_{10}\right)\right),\ s_{43}=s_5\left(1-h_1\left(x_3+x_5\right)\right),\ s_{44}=s_6\left(1-h_1\left(x_3+x_6\right)\right),$$

$$s_{45}=s_7\left(1-h_1\left(x_3+x_7\right)\right),\ s_{46}=s_8\left(1-h_1\left(x_3+x_8\right)\right),\ s_{47}=s_9\left(1-h_1\left(x_4+x_9\right)\right),$$

$$s_{48}=s_{10}\left(1-h_1\left(x_4+x_{10}\right)\right),\ s_{49}=s_{11}\left(1-h_1\left(x_4+x_5\right)\right),\ s_{50}=s_{12}\left(1-h_1\left(x_4+x_6\right)\right),$$

$$s_{51}=s_{13}\left(1-h_1\left(x_4+x_7\right)\right),\ s_{52}=s_{14}\left(1-h_1\left(x_4+x_8\right)\right),\ s_{53}=s_{15}\left(1-h_1\left(x_1+x_9\right)\right),$$

$$s_{54}=s_{16}\left(1-h_1\left(x_1+x_{10}\right)\right),\ s_{55}=s_{17}\left(1-h_1\left(x_1+x_5\right)\right),\ s_{56}=s_{18}\left(1-h_1\left(x_1+x_6\right)\right),$$

$$s_{57}=s_{19}\left(1-h_1\left(x_1+x_7\right)\right),\ s_{58}=s_{20}\left(1-h_1\left(x_1+x_8\right)\right),\ s_{59}=s_{21}\left(1-h_1\left(x_2+x_9\right)\right),$$

$$s_{60}=s_{22}\left(1-h_1\left(x_2+x_{10}\right)\right),\ s_{61}=s_{23}\left(1-h_1\left(x_2+x_5\right)\right),\ s_{62}=s_{24}\left(1-h_1\left(x_2+x_6\right)\right),$$

$$s_{63}=s_{25}\left(1-h_1\left(x_2+x_7\right)\right),\ s_{64}=s_{26}\left(1-h_1\left(x_2+x_8\right)\right),\ s_{65}=s_{27}\left(1-h_1x_9\right),$$

$$s_{66}=s_{28}\left(1-h_1x_{10}\right),\ s_{67}=s_{29}\left(1-h_1x_5\right),\ s_{68}=s_{30}\left(1-h_1x_6\right),\ s_{69}=s_{31}\left(1-h_1x_7\right),$$

$$s_{70}=s_{32}\left(1-h_1x_8\right),\ s_{71}=s_{33}\left(1-h_1\left(x_9-Sc\right)\right),\ s_{72}=s_{34}\left(1-h_1\left(x_{10}-Sc\right)\right),$$

$$s_{73}=s_{35}\left(1-h_1\left(x_5-Sc\right)\right),\ s_{74}=s_{36}\left(1-h_1\left(x_6-Sc\right)\right),\ s_{75}=s_{37}\left(1-h_1\left(x_7-Sc\right)\right),$$

$$s_{76}=s_{38}\left(1-h_1\left(x_8-Sc\right)\right),\ s_{77}=s_1\left(1-h_2x_{15}\right),\ s_{78}=s_2\left(1-h_2x_{16}\right),$$

$$s_{79}=s_3\left(1-h_2\left(x_3-x_9\right)\right),\ s_{80}=s_4\left(1-h_2\left(x_3+x_{10}\right)\right),\ s_{81}=s_5\left(1-h_2\left(x_3+x_5\right)\right),$$

$$s_{82}=s_6\left(1-h_2\left(x_3+x_6\right)\right),\ s_{83}=s_7\left(1-h_2\left(x_3+x_7\right)\right),\ s_{84}=s_8\left(1-h_2\left(x_3+x_8\right)\right),$$

$$s_{85}=s_9\left(1-h_2\left(x_4+x_9\right)\right),\ s_{86}=s_{10}\left(1-h_2\left(x_4+x_{10}\right)\right),\ s_{87}=s_{11}\left(1-h_2\left(x_4+x_5\right)\right),$$

$$s_{88}=s_{12}\left(1-h_2\left(x_4+x_6\right)\right),\ s_{89}=s_{13}\left(1-h_2\left(x_4+x_7\right)\right),\ s_{90}=s_{14}\left(1-h_2\left(x_4+x_8\right)\right),$$

$$s_{91}=s_{15}\left(1-h_2\left(x_1+x_9\right)\right),\ s_{92}=s_{16}\left(1-h_2\left(x_1+x_{10}\right)\right),\ s_{93}=s_{17}\left(1-h_2\left(x_1+x_5\right)\right),$$
$$s_{94}=s_{18}\left(1-h_2\left(x_1+x_6\right)\right),\ s_{95}=s_{19}\left(1-h_2\left(x_1+x_7\right)\right),\ s_{96}=s_{20}\left(1-h_2\left(x_1+x_8\right)\right),$$

$$s_{97}=s_{21}\left(1-h_2\left(x_2+x_9\right)\right),\ s_{98}=s_{22}\left(1-h_2\left(x_2+x_{10}\right)\right),\ s_{99}=s_{23}\left(1-h_2\left(x_2+x_5\right)\right),$$
$$s_{100}=s_{24}\left(1-h_2\left(x_2+x_6\right)\right),\ s_{101}=s_{25}\left(1-h_2\left(x_2+x_7\right)\right),\ s_{102}=s_{26}\left(1-h_2\left(x_2+x_8\right)\right),$$
$$s_{103}=s_{27}\left(1-h_2x_9\right),\ s_{104}=s_{28}\left(1-h_2x_{10}\right),\ s_{105}=s_{29}\left(1-h_2x_5\right),\ s_{106}=s_{30}\left(1-h_2x_6\right),$$
$$s_{107}=s_{31}\left(1-h_2x_7\right),\ s_{108}=s_{32}\left(1-h_2x_8\right),\ s_{109}=s_{33}\left(1-h_2\left(x_9-Sc\right)\right),$$

$$s_{110}=s_{34}\left(1-h_2\left(x_{10}-Sc\right)\right),\ s_{111}=s_{35}\left(1-h_2\left(x_5-Sc\right)\right),\ s_{112}=s_{36}\left(1-h_2\left(x_6-Sc\right)\right),$$

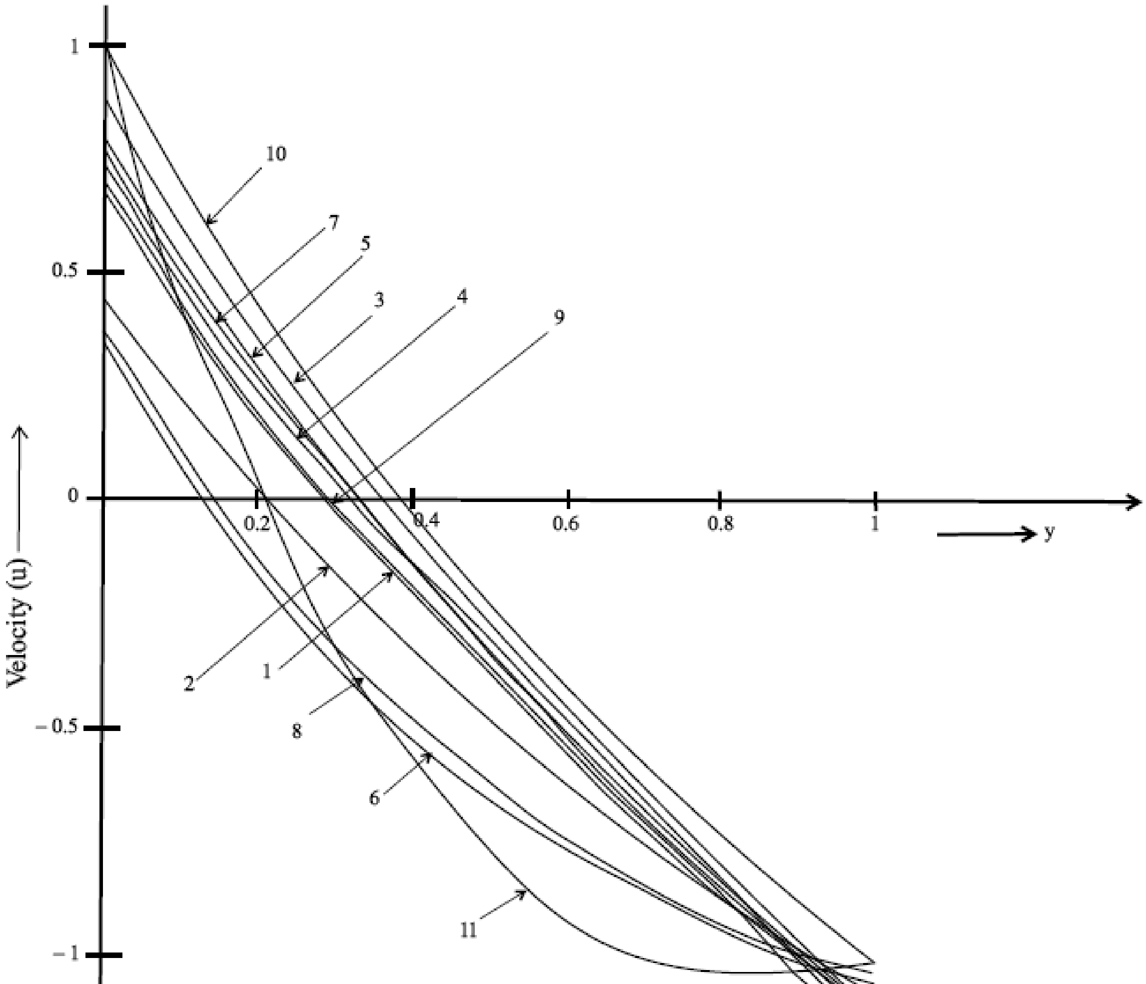
$$s_{113}=s_{37}\left(1-h_2\left(x_7-Sc\right)\right),\ s_{114}=s_{38}\left(1-h_2\left(x_8-Sc\right)\right).$$

$$m_{23}=\frac{\left[m_{24}\left(1-h_1x_{20}\right)+\sum_{i=39}^{76}s_i\right]}{\left(h_1x_{19}-1\right)}$$

$$m_{24}=\frac{\left[-\left\{\sum_{i=39}^{76}s_i\right\}e^{x_{19}}\left(h_2x_{19}-1\right)+\left\{\sum_{j=77}^{114}s_j\right\}e^{x_{15}}\left(h_1x_{19}-1\right)\right]}{\left\{e^{x_{20}}\left(h_2x_{20}-1\right)\left(h_1x_{19}-1\right)+e^{x_{19}}\left(h_2x_{19}-1\right)\left(1-h_1x_{20}\right)\right\}}.$$

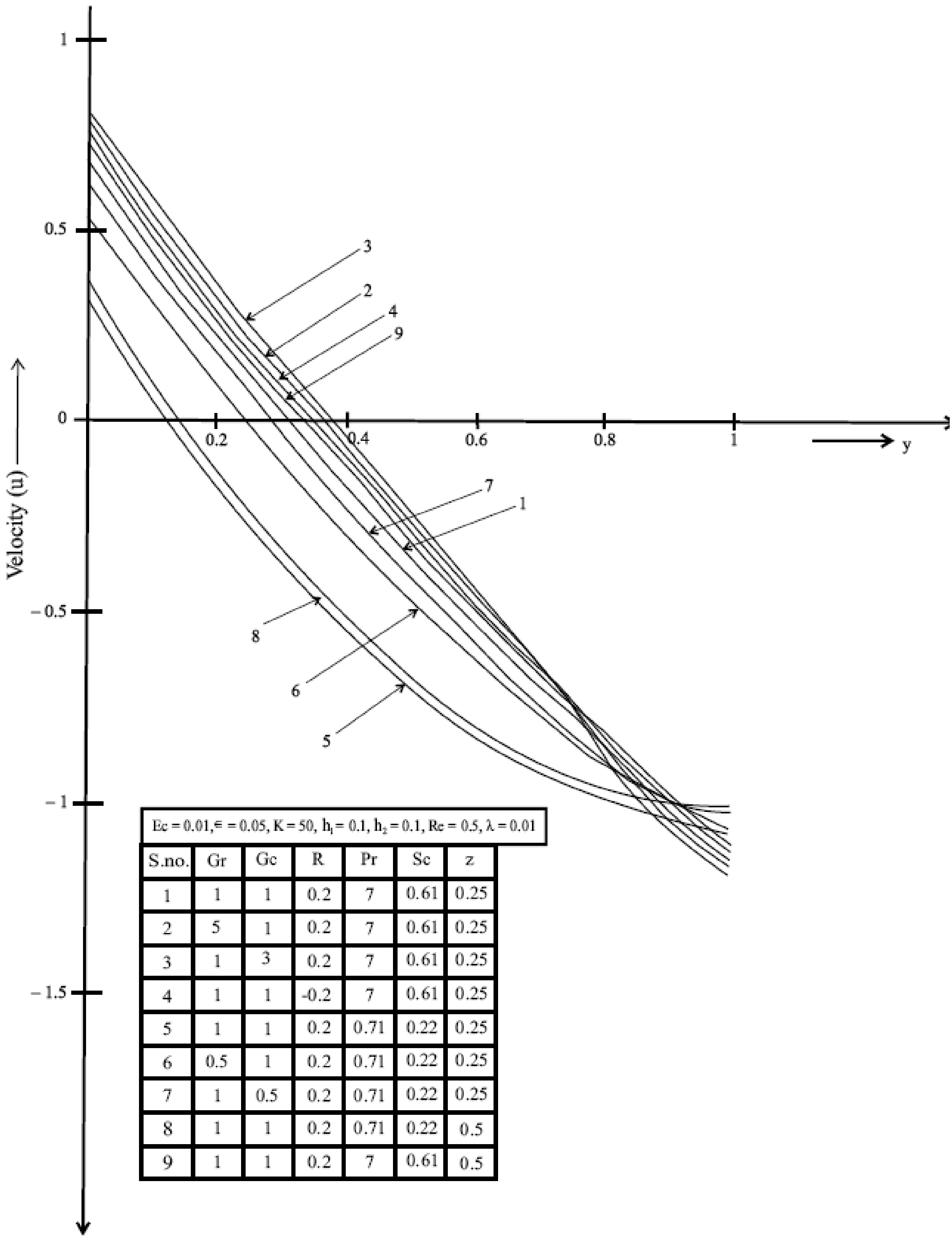
6. Result and Discussion

In order to understand the physical importance of the flow between the two plates, calculations have been carried out for velocity, temperature, concentration, skin friction and the rate of heat transfer. Effects for different values of permeability parameter (K), the velocity slip parameter (h_1) on plate y = 0, the velocity slip parameter (h_2) on plate y = 1, the cross flow Reynold's number (Re), the radiation parameter (R), the thermal Grashof number (Gr), the mass Grashof number (Gc), the Prandtl number (Pr), the Schmidt number (Sc), z and λ are shown graphically. We specially observe the case of free flow ($K = \infty$) along with no slip ($h_1 = 0, h_2 = 0$) for water (Pr = 7, Sc = 0.61) and for air (Pr=0.71, Sc=0.22). We fix Ec = 0.01, $\in$ = 0.05 through out our calculations.



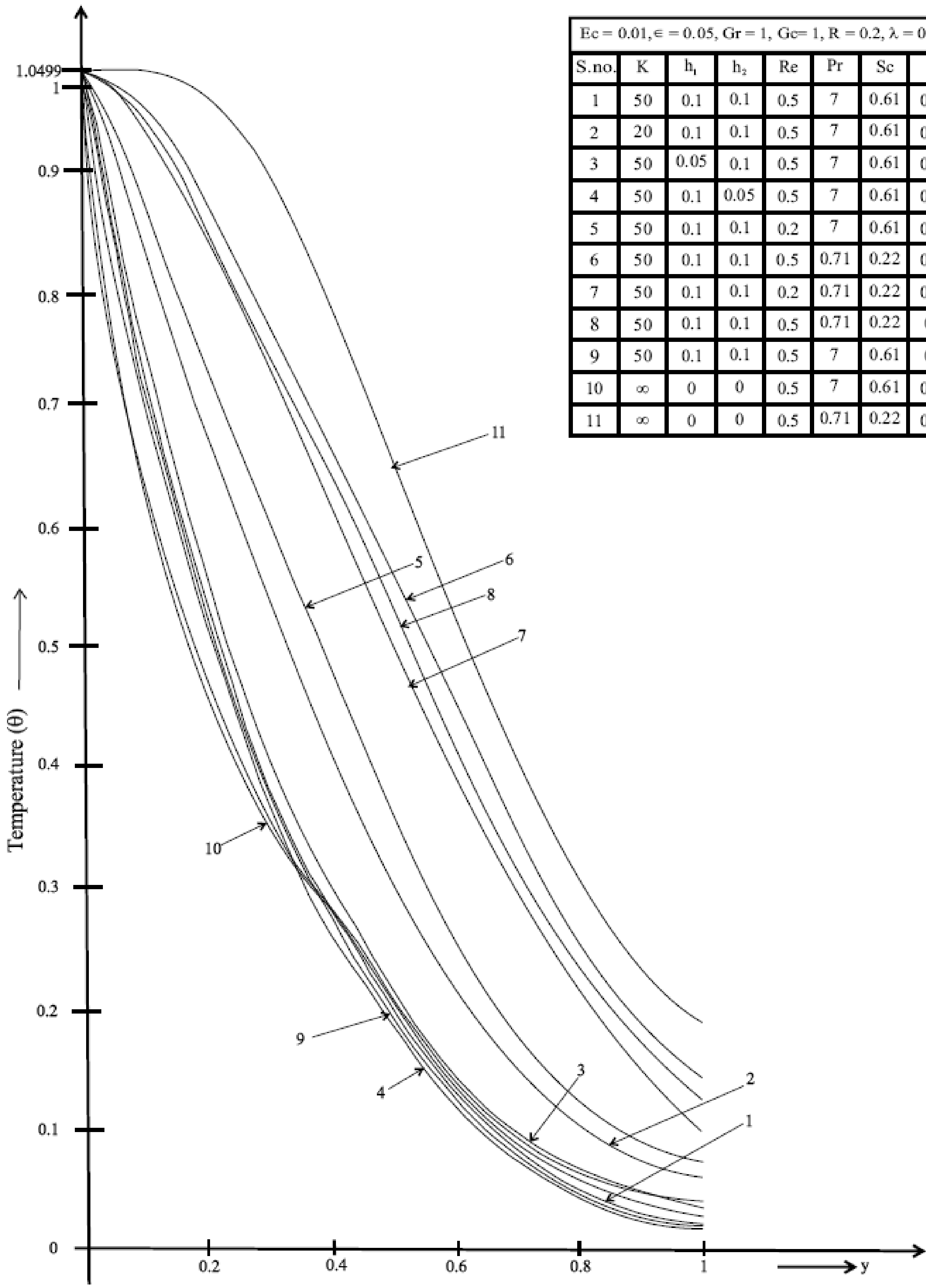
Ec = 0.01, ∈ = 0.05, Gr = 1, Gc= 1, R = 0.2, λ = 0.01							
S.no.	K	h_1	h_2	Re	Pr	Sc	z
1	50	0.1	0.1	0.5	7	0.61	0.25
2	20	0.1	0.1	0.5	7	0.61	0.25
3	50	0.05	0.1	0.5	7	0.61	0.25
4	50	0.1	0.05	0.5	7	0.61	0.25
5	50	0.1	0.1	0.2	7	0.61	0.25
6	50	0.1	0.1	0.5	0.71	0.22	0.25
7	50	0.1	0.1	0.2	0.71	0.22	0.25
8	50	0.1	0.1	0.5	0.71	0.22	0.5
9	50	0.1	0.1	0.5	7	0.61	0.5
10	∞	0	0	0.5	7	0.61	0.25
11	∞	0	0	0.5	0.71	0.22	0.25

Figure 2. Velocity profiles plotted against y for different values of K, h_1, h_2, Re, Pr, Sc and z



Ec = 0.01, ∈ = 0.05, K = 50, h_1 = 0.1, h_2 = 0.1, Re = 0.5, λ = 0.01						
S.no.	Gr	Gc	R	Pr	Sc	z
1	1	1	0.2	7	0.61	0.25
2	5	1	0.2	7	0.61	0.25
3	1	3	0.2	7	0.61	0.25
4	1	1	-0.2	7	0.61	0.25
5	1	1	0.2	0.71	0.22	0.25
6	0.5	1	0.2	0.71	0.22	0.25
7	1	0.5	0.2	0.71	0.22	0.25
8	1	1	0.2	0.71	0.22	0.5
9	1	1	0.2	7	0.61	0.5

Figure 3. Velocity profiles plotted against y for different values of Gr, Gc, R, Pr, Sc and z



Ec = 0.01, ∈ = 0.05, Gr = 1, Gc= 1, R = 0.2, λ = 0.01							
S.no.	K	h_1	h_2	Re	Pr	Sc	z
1	50	0.1	0.1	0.5	7	0.61	0.25
2	20	0.1	0.1	0.5	7	0.61	0.25
3	50	0.05	0.1	0.5	7	0.61	0.25
4	50	0.1	0.05	0.5	7	0.61	0.25
5	50	0.1	0.1	0.2	7	0.61	0.25
6	50	0.1	0.1	0.5	0.71	0.22	0.25
7	50	0.1	0.1	0.2	0.71	0.22	0.25
8	50	0.1	0.1	0.5	0.71	0.22	0.5
9	50	0.1	0.1	0.5	7	0.61	0.5
10	∞	0	0	0.5	7	0.61	0.25
11	∞	0	0	0.5	0.71	0.22	0.25

Figure 4. Temperature profiles plotted against y for different values of K, h_1, h_2, Re, Pr, Sc and z

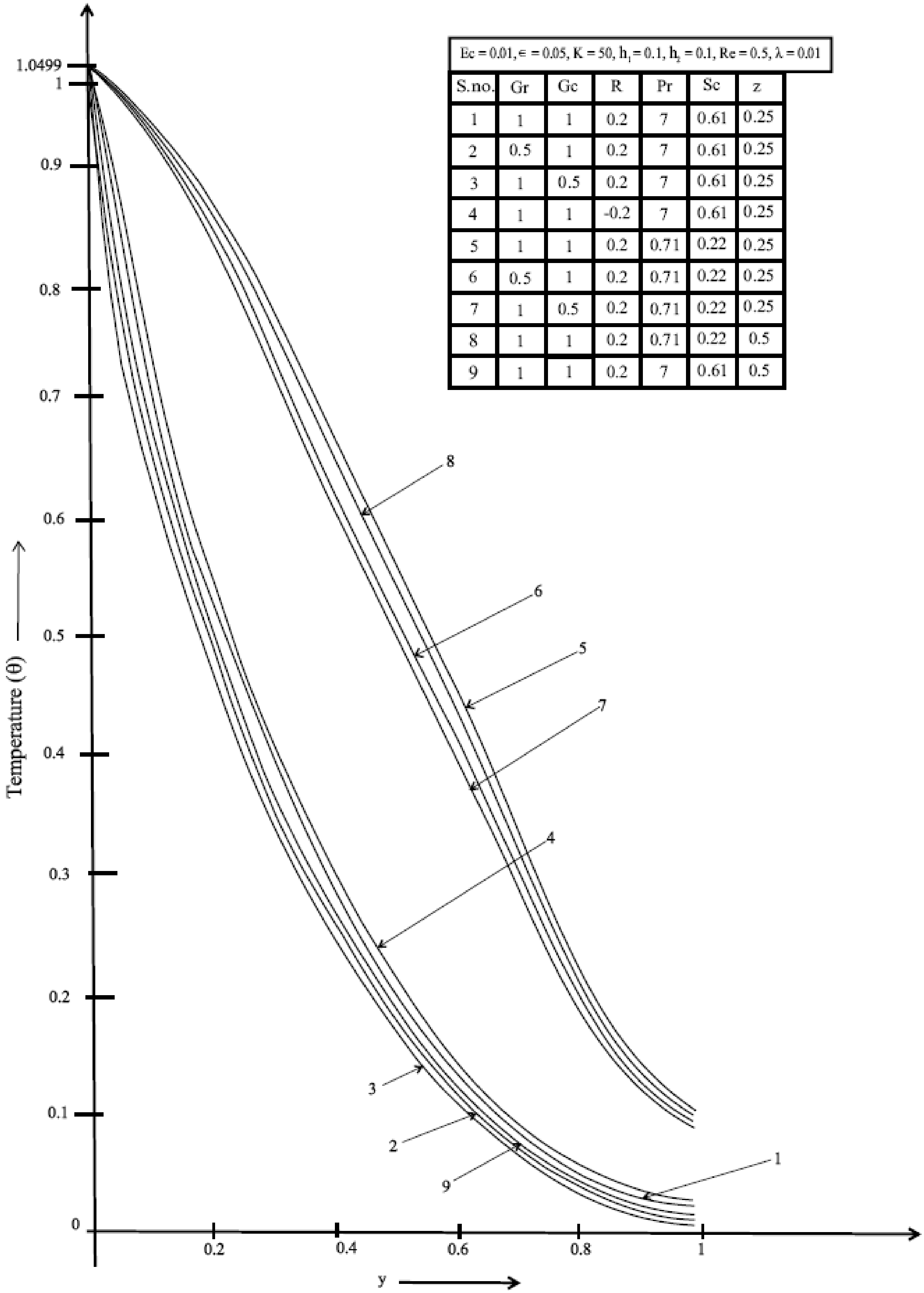


Figure 5. Temperature profiles plotted against y for different values of Gr, Gc, R, Pr, Sc and z

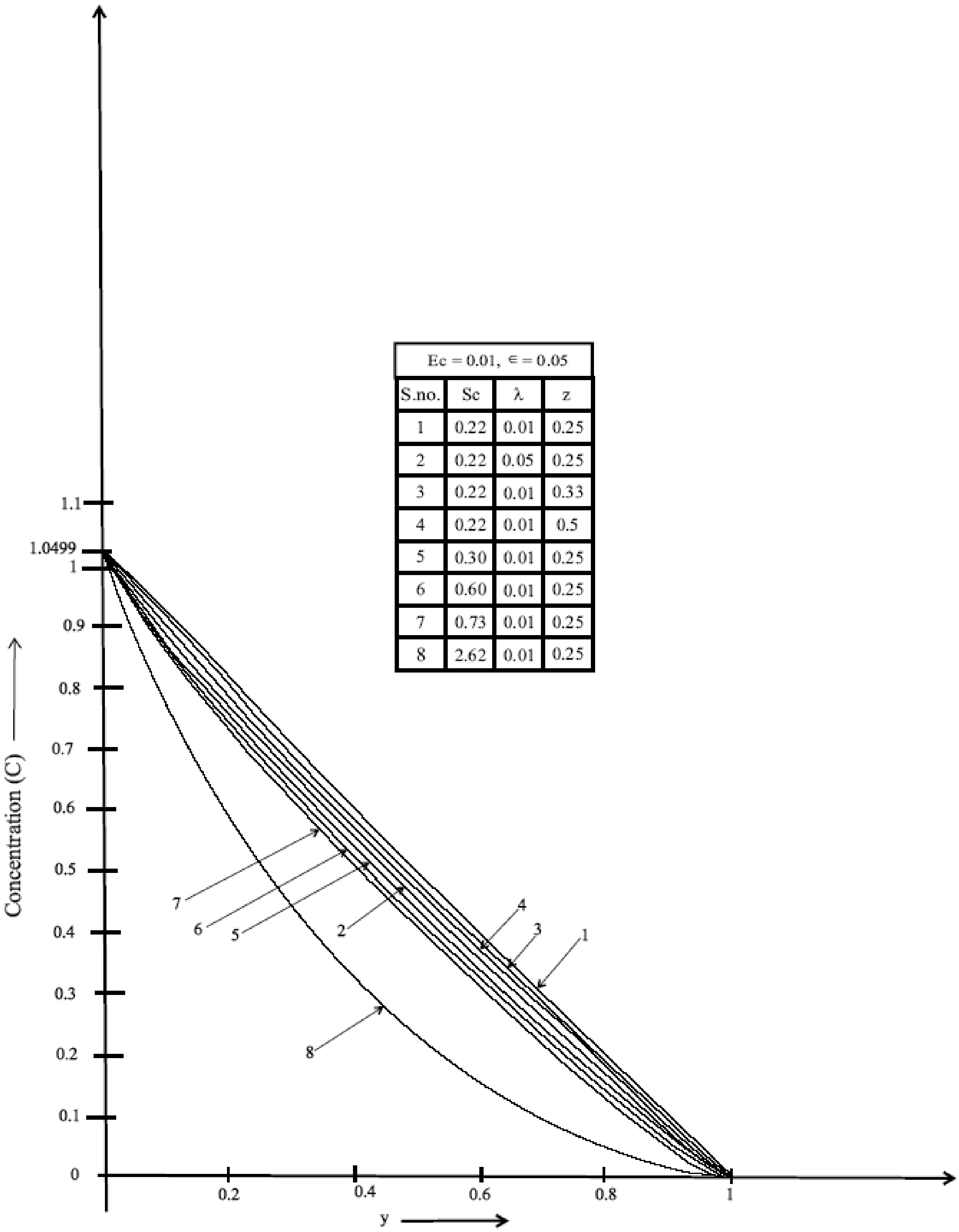
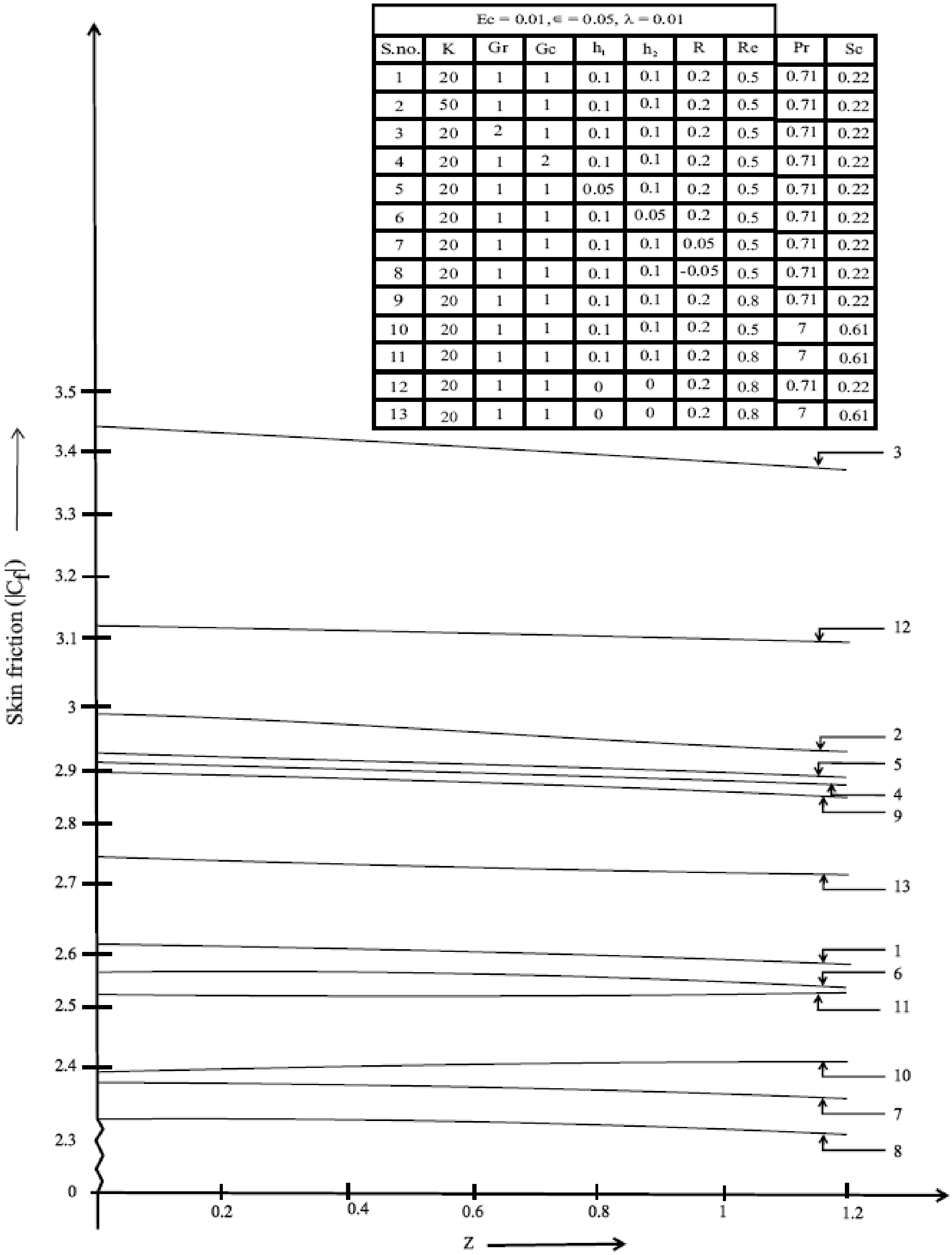


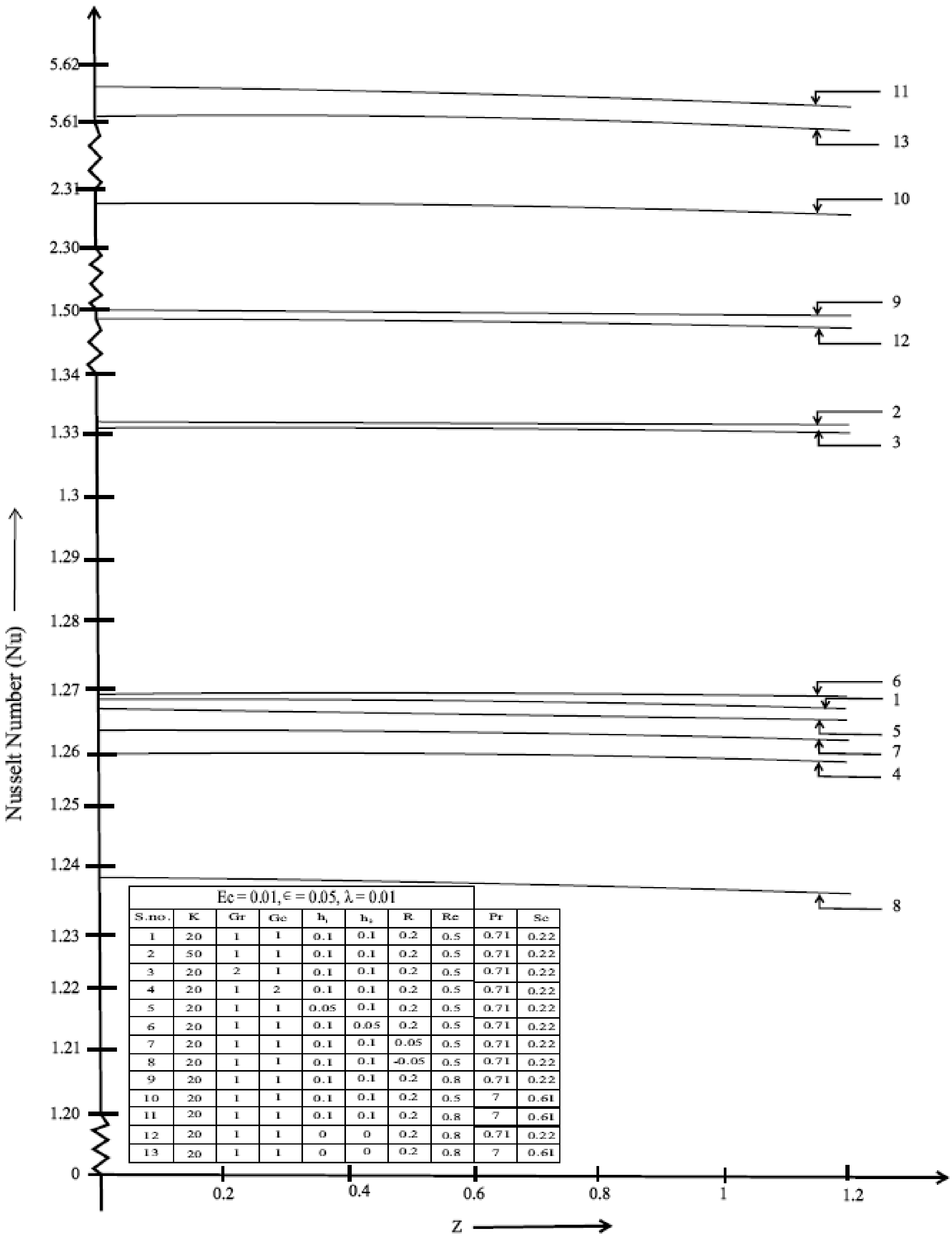
Figure 6. Concentration profiles plotted against y for different values of Sc, λ and z

In figure 2 and 3, velocity profiles are plotted against y fixing $\lambda = 0.01$. We observe that decreasing h_1, h_2 and Re increases the velocity. On the other hand velocity decreases on decreasing K, Gr, Gc and z. Physically, we can say that when we decrease the permeability parameter (K) the medium becomes less porous and hence the velocity decreases also when we decrease the slip velocity on the boundaries the flow will increase hence increasing the velocity. For negative of radiation i.e. absorption velocity increases slightly. In general we notice that velocity is less for air (Pr = 0.71, Sc = 0.22) than for water (Pr = 7, Sc = 0.61). We specially notice the case of free flow (K= ∞) along with no slip ($h_1 = 0, h_2 = 0$) velocity increases for water ($Pr = 7$, $Sc = 0.61$) and for air ($Pr = 0.71$, $Sc = 0.22$) velocity increases near the plate $y = 0$ then drops in the middle of the channel and rises again at the plate $y = 1$ as compared to the case with slip at the boundary and medium being porous. The velocity becomes negative near the plate $y = 1$, this is due to the fact that the plate at $y = 1$ moves in negative direction i.e. down wards.



Ec = 0.01, ∈ = 0.05, λ = 0.01									
S.no.	K	Gr	Gc	h_1	h_2	R	Re	Pr	Sc
1	20	1	1	0.1	0.1	0.2	0.5	0.71	0.22
2	50	1	1	0.1	0.1	0.2	0.5	0.71	0.22
3	20	2	1	0.1	0.1	0.2	0.5	0.71	0.22
4	20	1	2	0.1	0.1	0.2	0.5	0.71	0.22
5	20	1	1	0.05	0.1	0.2	0.5	0.71	0.22
6	20	1	1	0.1	0.05	0.2	0.5	0.71	0.22
7	20	1	1	0.1	0.1	0.05	0.5	0.71	0.22
8	20	1	1	0.1	0.1	-0.05	0.5	0.71	0.22
9	20	1	1	0.1	0.1	0.2	0.8	0.71	0.22
10	20	1	1	0.1	0.1	0.2	0.5	7	0.61
11	20	1	1	0.1	0.1	0.2	0.8	7	0.61
12	20	1	1	0	0	0.2	0.8	0.71	0.22
13	20	1	1	0	0	0.2	0.8	7	0.61

Figure 7. Skin friction plotted against z for different values of K, Gr, Gc, h_1, h_2, R, Re, Pr and Sc



Ec = 0.01, ∈ = 0.05, λ = 0.01									
S.no.	K	Gr	Gc	h_1	h_2	R	Re	Pr	Sc
1	20	1	1	0.1	0.1	0.2	0.5	0.71	0.22
2	50	1	1	0.1	0.1	0.2	0.5	0.71	0.22
3	20	2	1	0.1	0.1	0.2	0.5	0.71	0.22
4	20	1	2	0.1	0.1	0.2	0.5	0.71	0.22
5	20	1	1	0.05	0.1	0.2	0.5	0.71	0.22
6	20	1	1	0.1	0.05	0.2	0.5	0.71	0.22
7	20	1	1	0.1	0.1	0.05	0.5	0.71	0.22
8	20	1	1	0.1	0.1	-0.05	0.5	0.71	0.22
9	20	1	1	0.1	0.1	0.2	0.8	0.71	0.22
10	20	1	1	0.1	0.1	0.2	0.5	7	0.61
11	20	1	1	0.1	0.1	0.2	0.8	7	0.61
12	20	1	1	0	0	0.2	0.8	0.71	0.22
13	20	1	1	0	0	0.2	0.8	7	0.61

Figure 8. Nusselt number plotted against z for different values of K, Gr, Gc, h1, h2, R, Re, Pr and Sc

Temperature profiles are plotted against y, fixing $\lambda = 0.01$, in figures 4 and 5, we notice that decreasing K, h_1, Re and z increases the temperature where as decreasing h_2, Gr and Gc tends the temperature to fall. For negative of radiation i.e. absorption temperature increases which is obvious as when it absorbs energy temperature increase. We observe that temperature is higher for air (Pr = 0.71, Sc = 0.22) than for water (Pr = 7, Sc = 0.61) also we see that for Re and z results are same in both air and water. For the case of free flow ($K = \infty$), and no slip ($h_1 = 0$, $h_2 = 0$) temperature decreases near the plate y = 0 but increases gradually.

Figure 6, shows concentration profiles plotted against y. We notice that concentration drops on increasing λ, z and Sc. Concentration is highest for air (Sc = 0.22) and least for propyl benzene (Sc = 2.62).

$|C_f|$ is plotted against z in figure 7, fixing $\lambda = 0.01$. We observe that for air (Pr = 0.71 and Sc = 0.22), increasing K, Gr, Gc, h_2 and R increases the skin friction ($|C_f|$) where as decreasing h_1 increases the skin friction ($|C_f|$). Rise in Re tends the skin friction to rise for both air (Pr = 0.71, Sc = 0.22) and water (Pr = 7, Sc = 0.61). We observe here that skin friction ($|C_f|$) is lower for water than for air physically this is because air is lighter than water. Physically we can see that since skin friction is lower for water hence temperature is low for water because less friction will lead to smaller rise in temperature also on the other hand lower skin friction for water implies high velocity for water.

In figure 8, the rate of heat transfer is plotted against z for air (Pr = 0.71, Sc = 0.22), fixing $\lambda = 0.01$. We observe that increasing K, Gr, h_1 and R increases the Nusselt number but on the other hand increasing Gc and h_2 decreases the Nusselt number. Rise in Re rises the Nusselt number for both air (Pr = 0.71, Sc = 0.22) and water (Pr = 7, Sc = 0.61).Here we observe that the rate of heat transfer is higher for water than for air, this is due to the fact that gap between the particles in air is more as compare to water.

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