

Mathematical Modeling of Filtration from Canals and Sprinklers of Irrigation Systems

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Abstract We consider several schemes of seepage flows from canals and sprinklers of irrigation systems through the soil layer, underlain good underlying permeable confined aquifer or water-resistant base. For their study and formulated using the method of P.Y. Polubarinova-Kochina solved multivariable mixed boundary value problems of the theory of analytic functions. On the basis of these models are developed algorithms for calculating the size of the saturated zone in situations when the filter has to evaluate the combined effect of the painting movement of such important factors as seepage back pressure from the underlying confined aquifer or an impermeable base, cross-sectional shape and channel the power supply level of water in it, capillarity of the soil and evaporation from the free surface of groundwater. The results of calculations for all the schemes are compared with the same filtration filter parameters depending on the shape of the channel as a power source (canal or irrigation), and the type of foundation soil layer (silnopronitsaemy confined aquifer or aquitard).

Keywords Filtering, Channel, Irrigation, Soil Capillarity, Pressure, Evaporation, Infiltration, Free Surface, The Complex Flow Velocity Method Polubarinova-Cochina, Conformal Mapping

1. Introduction

It is known (see, eg, reviews[1-4]) that the mathematical modeling of filtration flows from canals and sprinklers of irrigation systems used a variety of research methods applied analytic functions introduced in[5-9], the method of conformal mappings of the rectilinear polygons based on the application of the Schwarz-Christoffel formula[10-13] and functions Riesenkauf and Verigin[14], the method of P.Y. Polubarinova-Cochina, based on the application of the analytic theory of linear differential equations of Fuchs class[15-18], the theory of inverse boundary value problems[19,20], etc. Despite the importance of these research studies in this direction so far alienated and not fully reflect the specific movements of the canals and sprinklers, which is characterized by the presence of such important factors as seepage back pressure from the underlying permeable aquifer well containing a pressurized underground (or artesian) water or water-resistant base, soil capillarity and evaporation (or infiltration) at the free surface of groundwater. All marked work associated with the presence of one or two of the three factors that significantly affect the simulated processes. Accounting for the simultaneous influence of these three characteristics of the filter has not become a

broad domain of exact analytical solutions.

In contrast to these studies in this paper, which is a continuation and development[21-25], both case of motion of the water of the canals and sprinklers are considered together, which apparently has not yet appeared in the literature. In this case it is shown that the solution for sprinklers obtained by solving the filtration problem for channels with a limit associated with the conformal mapping parameters contained in the required dependencies. Provides a uniform method of solving problems, which allows to consider four different schemes of the flow and take into account in the study at once all three filtration characteristics - capillarity, evaporation, and props - that is of considerable practical interest because it allows us to estimate the combined influence of these factors on the phenomenon. For this purpose, the method of P.Y. Polubarinova-Kochina[1-4], based on the use of conformal mapping of circular polygons. With the help of developed[26-29] methods of solving the corresponding linear differential equations of the Fuchs class, which are very common problems of underground hydro-mechanics solve mixed boundary value problem of multi-parameter theory of analytic functions. On the basis of the obtained exact analytical relationships and numerical calculations are carried out by hydrodynamic analysis of the influence of physical parameters of the schemes of flow on filtration characteristics. We give a comparison of the results of calculations for these schemes under the same filter characteristics depending on the shape of a channel-power sources of water (or irrigation canal) and the type of under-

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lying soil base (well permeable confined aquifer or aquitard).

2. Filtration of the Channels in the Presence of the Basis of Well-Permeable Horizon Containing Pressurized Groundwater (Scheme 1)

In fig. 1 shows a right-symmetric picture steady plane flow of an incompressible fluid by Darcy's law with known filtration coefficient $\kappa = \text{const}$ of channel width l with a shallow depth of water at uniform evaporation from the free surface of CE intensity ε ($0 < \varepsilon < 1$), divided by the coefficient of filtration. At some depth from the bottom of the channel T is silnpronitsaemy pressure horizon FE, piezometric water level in which the $H_0 + h_k$ (h_k – static height of capillary rise of groundwater[1,2]) above the horizontal interface between it and a layer of soil, which studies the movement. The channel width l , thickness of the soil layer T , the pressure (above the roof) in the underlying permeable horizon is well along H_0 with the intensity of evaporation ε and high vacuum hk , due to capillary forces in the soil are given.

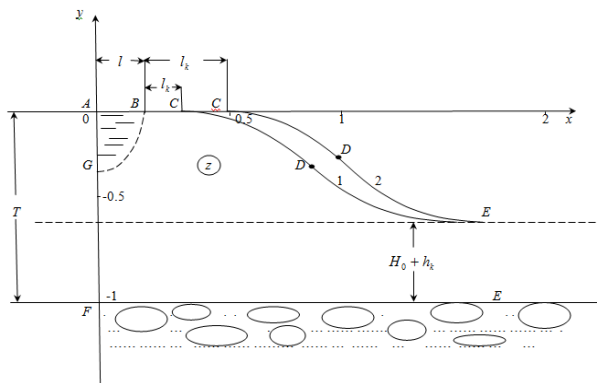


Figure 1. The flow pattern with curved depression in filtering out of the canal (Figure 1, curve 1) and sprinkler (Fig. 2, curve 2) calculated at $h_k = 0.3$, $\varepsilon = 0.05$, $l = 0.2$, $T = 1.0$ and $H_0 = 0.1$

If we introduce the complex potential of motion $\omega = \varphi + i\psi$ (φ – velocity potential, ψ – stream function) and complex coordinate $z = x + iy$, classified κT , and T , then the problem lies in determining the position of the depression curve, CE and, hence, the width l_k of the capillary spreading of water filtration and BC flow Q with the following boundary conditions:

$$AB: y = 0, \phi = 0; \quad BC: y = 0, \psi = Q; \quad AF: x = 0, \psi = 0; \quad (1)$$

$$FE: y = -T, \phi = T - H_0 = H; \quad CE: \phi = -y + h_k, \\ \psi = Q - \varepsilon(x - l - l_k).$$

We introduce an auxiliary parametric variable t , the function $z(t)$, conformally maps the half-strip $\text{Re}t > 0$, $0 < \text{Im}t < 0.5\pi$ in the t plane, the flow region in accordance points $t_E = 0$, $t_F = if$, $t_A = ia$, $t_B = i0.5\pi$, $t_C = \infty$ and derivatives $d\omega/dt$ и dz/dt .

In fig. 2, a shows a complex velocity field w , corresponding to the boundary conditions (1), which is a circular quadrangle with a cut to the top of D, and the angle $\pi\nu = \text{arctg}\sqrt{\varepsilon}$ at the vertex C. Function effecting the conformal mapping of t on the field w has the form

$$w = -\sqrt{\varepsilon}i \frac{\sqrt{\varepsilon}Y_1 + iY_2}{Y_1 - \sqrt{\varepsilon}iY_2}, \\ Y_1 = \frac{\text{ch}t\text{ch}\nu t + \text{Csh}t\text{sh}\nu t}{\text{ch}^{1+\nu}t}, \quad Y_2 = \frac{\text{ch}t\text{sh}\nu t + \text{Csh}t\text{ch}\nu t}{\text{ch}^{1+\nu}t}. \quad (2)$$

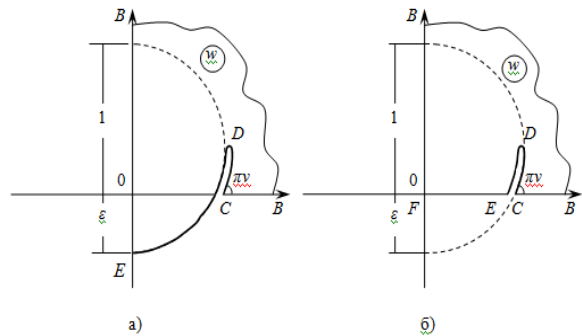


Figure 2. On integrated circuits W for speeds 1 and 2 (a) and figure 3, 4 (b)

Here $Y_{1,2}$ – are linearly independent integrals of the corresponding linear differential equation of Fuchs class[26-28], C – a suitable real constant conformal mapping.

Applying the method of P.Y. Polubarinova-Cochina and taking into account the relation (2), as well as the fact that $w = d\omega/dz$, we find a parametric solution of (1) as

$$\frac{dz}{dt} = -\frac{M}{\sqrt{\varepsilon}} \frac{(Y_1 - \sqrt{\varepsilon}iY_2)\text{ch}^{1+\nu}t}{\Delta}, \quad \frac{d\omega}{dt} = Mi \frac{(\sqrt{\varepsilon}Y_1 + iY_2)\text{ch}^{1+\nu}t}{\Delta}, \\ \Delta = \text{sht}\sqrt{(\text{sh}^2t + A^2)(\text{sh}^2t + F^2)}. \quad (3)$$

Here $M > 0$ – a large-scale simulation of a constant, $a = \arcsin A$, $b = \arcsin F$, a and f ($0 < f < a \leq 0.5\pi$) – unknown ordinates of the points A and F in the plane t [21-23]. Integrating the representation (3) along the contour of the auxiliary field t , we can obtain expressions for the physical characteristics of the model

$$-\int_f^a \Phi_{AF} dt = H, \quad \int_a^{0.5\pi} X_{AB} dt = l, \quad \frac{1}{\cos(0.5\pi\nu)} \int_0^\infty \Phi_{BC} dt = h_k, \\ \int_f^a Y_{AF} dt = T, \quad (4)$$

are used to determine the unknown parameters of conformal mappings a , f , C and M . After finding these constants are calculated and the required filtration characteristics l_k and Q coordinates of points of depression

$$l_k = \frac{1}{\sin(0.5\pi\nu)} \int_0^\infty X_{BC} dt, \quad Q = - \int_a^{0.5\pi} \Psi_{AB} dt, \quad (5)$$

$$x_{CE}(t) = l + l_k + \int_t^\infty X_{CE} dt, \quad y_{CE}(t) = - \int_t^\infty Y_{CE} dt, \quad 0 \leq t \leq \infty. \quad (6)$$

In (4) – (6) the integrands – right-hand sides of expression (3) in the corresponding sections of the contour plane t .

Private and limiting cases of figure 1 are described in detail in[23].

3. Filtration of the Irrigators in the Presence of the Basis of Well-Permeable Horizon Containing Pressurized Groundwater (Scheme 2)

In fig. 1 shows the pattern of motion of the sprinkler borozdovogo type with a radius l which is filled with water up to ground level, taken as a horizontal plane. During the initial review sprinkler replace a point source located at a point A . Then the solution for the source is obtained by solving the problem of filtration for the channel (3) with the values of conformal mapping $a = 0.5\pi$, $A = 1$, is at the confluence of the points A and B in the plane of flow.

Further, according to[1,2,6,7], we take one of the lines of equal pressures, for example BG (shown in fig. 1 smooth curve) for the cross-section of the sprinklers with radius l , and let her $\phi = 0$. Under these conditions along portions of the border region z will continue to run the boundary conditions (1), and the complex velocity field w (fig. 2, a) and the unknown function (3) (6) retain their form. The formulas (4), (5) for the quantities l and Q replaced by the following:

$$\frac{1}{\sin(0.5\pi\nu)} \int_0^b X_{AB} dt = l, \quad Q = \pi \left(\int_0^\infty \Psi_{CE} dt - \int_0^f \Psi_{FE} dt \right), \quad (7)$$

where this time the unknown parameter b ($0 < b < \infty$) the abscissa of point B in the plane of the auxiliary parametric variable $t_B = b + i0.5\pi$. Private and limiting cases of the model 2 in detail in[25].

4. Comparison of Numerical Results for the Channels (Scheme 1) and Sprinklers (Scheme 2)

In fig. 1 shows the curves of depression during the filtration of the channels (left curve CE) and sprinklers (right curve CE), calculated with $\varepsilon = 0.05$, $T = 1.0$, $H_0 = 0.1$, $h_k = 0.3$ and $l = 0.2$ (baseline). The results of calculations of the influence of physical parameters ε , T , H_0 , h_k and l the width of the capillary spreading of the liquid flow rate l_k and filtration Q are shown in table. 1 and 2 (upper line corresponds to the sprinklers, the lower – channel). In each of the blocks of the tables, which are separated by double vertical lines, varies in the allowable range is one of the above parameters, and values of other fixed base values. In fig. 3 shows a plot of the size l_k and flow Q channels (curves 1) and sprinklers (curves 2) on the parameters ε , T , H_0 , h_k и l .

Analysis of the data tables and graphs to the following conclusions.

The increase in channel width and radius of the sprinkler, the pressure in the underlying aquifer and the height of the vacuum caused by capillary forces in the soil and reduce evaporation and capacity of the soil layer lead to increase the width of the capillary spreading of the water. Reduce evaporation and increase the backwater from the waters of the underlying horizon is accompanied by increase in the magnitude l_k and, conversely, decrease the filtration rate Q . A similar behavior is observed l_k and Q the values and decreasing capacity of the soil layer T . Thus, with respect to the filtration of the canals and sprinklers and evaporation capacity of the soil layer plays the same role as props, as the quantity H_0 .

Table 1. The calculation results l_k and Q in the variation ε , T and H_0 for schemes 1 and 2

ε	l_k	Q	T	l_k	Q	H_0	l_k	Q
0.001	0.263	0.671	0.6	0.333	0.569	0	0.236	0.736
	0.149	0.471		0.217	0.378		0.133	0.513
0.01	0.262	0.673	1.0	0.255	0.680	0.01	0.238	0.731
	0.147	0.479		0.143	0.483		0.134	0.510
0.25	0.229	0.712	1.4	0.235	0.741	0.20	0.285	0.615
	0.129	0.502		0.128	0.530		0.160	0.446
0.50	0.208	0.746	1.7	0.227	0.770	0.45	0.525	0.428
	0.117	0.520		0.124	0.547		0.292	0.323
0.80	0.190	0.779	1.8	0.226	0.778	0.49	0.660	0.403
	0.110	0.538		0.122	0.547		0.350	0.299

Table 2. The results of calculations of the width of the capillary spreading l_k and Q the variation h_k and l for circuits 1 and 2

h_k	l_k	Q	r	l_k	Q
0.18	0.158	0.624	0.04	0.152	0.336
	0.079	0.419		0.106	0.264
0.20	0.173	0.637	0.10	0.205	0.485
	0.089	0.433		0.127	0.364
0.40	0.367	0.696	0.20	0.255	0.680
	0.217	0.510		0.143	0.483
0.45	0.451	0.697	0.30	0.297	0.854
	0.257	0.518		0.151	0.582
0.50	0.592	0.698	0.40	0.370	1.013
	0.334	0.521		0.158	0.668

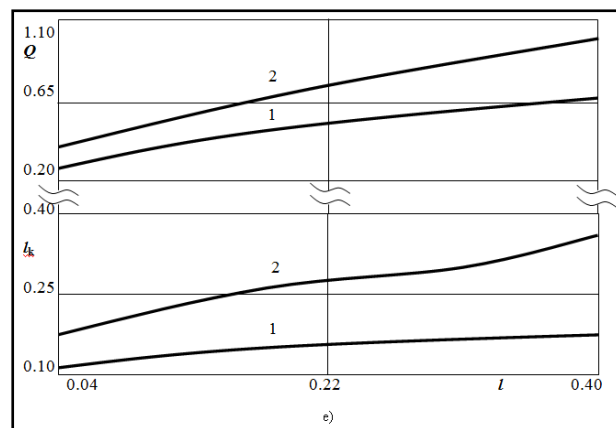
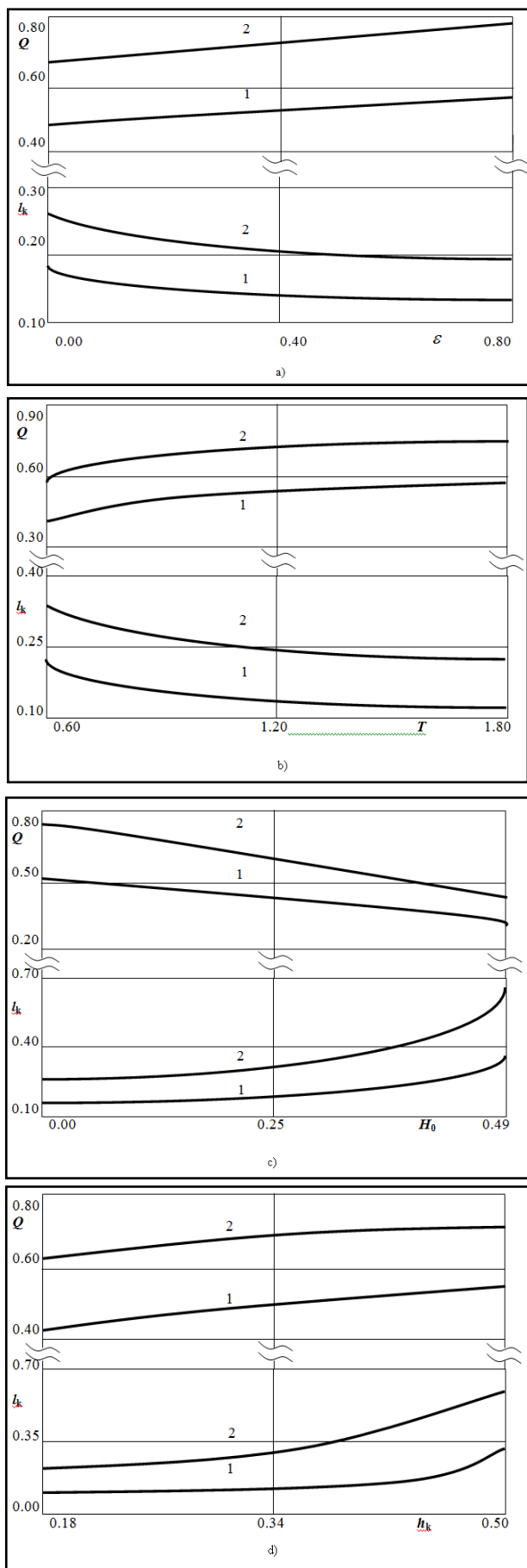


Figure 3. Depending on the value l_k and Q for all channels (1) and sprinklers (2) from ε (a) at constant $T=1.0$, $H_0=0.1$, $h_k=0.3$, $l=0.2$; of T (b) with constant $\varepsilon=0.05$, $H_0=0.1$, $h_k=0.3$, $l=0.2$; of H_0 (c) with constant $\varepsilon=0.05$, $T=1.0$, $h_k=0.3$, $l=0.2$; of h_k (d) with constant $\varepsilon=0.05$, $T=1.0$, $H_0=0.1$, $l=0.2$; of l (e) at constant $\varepsilon=0.05$, $T=1.0$, $H_0=0.1$, $h_k=0.3$

Attention is drawn to the same qualitative dependence of the width of the capillary spreading of the water l_k and the seepage flow Q on the parameters h_k and l : with increasing height of capillary rise of water and the channel width or radius of irrigators l_k and Q the value increases.

The effect of evaporation on the filtration characteristics can be traced back to the left block of table. 1: The variation of the parameter ε corresponds to the decrease l_k of 1.4 times, while Q consumption changes by only 12-16%.

The average block table. 1 allows us to determine the nature of the influence of layer thickness T : the shallow horizon silnopronitsaemogo spreading width of the capillary water may exceed the height of the capillary vacuum. Thus, when $T=0.6$ we have $l_k=0.333$ and therefore $l_k/h_k=1.111$. With the increase T in this ratio becomes even greater. The presence of the underlying horizon practically affect the values of l_k and Q at $T < 1.8$. At higher values T of the deviation of the values l_k and Q does not exceed 1%.

We have said above is qualitatively the opposite nature of variation of the unknown quantities, l_k and Q by varying the values of T and H_0 . Sections of the table 1 relating to the parameter H_0 indicates that, on the contrary, at relatively large values of the overpressure ratio l_k/h_k can also be very significant. For example, when $h_k=0.3$ for $H_0=0.49$ we get $l_k=0.66$, so that the width of the capillary spreading of the water l_k exceeds the height of capillary rise of liquids up to 120%.

However, the greatest influence on l_k the capillarity of the soil has. From table 2 shows that the parameter l_k changes the width of the capillary spreading of the water l_k is increased by 275.4%. In this case, for example, for the value $h_k=0.5$ we have $l_k/h_k=1.19$, with increasing h_k values of this ratio increases, with the largest difference is achieved for values $h_k \approx H$ and may approach 200%.

Thus, the capillarity of the soil increases as the filtration

Table 4. The calculation results L and Q in the variation H , ε , l and T for scheme 4

H	L	Q	ε	L	Q	l	L	Q	T	L	Q
0.17	2.400	0.630	0.10	4.854	0.425	0.17	2.273	0.736	0.62	1.653	0.369
0.38	2.643	0.725	0.22	3.411	0.618	0.38	2.547	0.759	1.39	2.793	0.767
0.60	2.793	0.767	0.35	2.793	0.767	0.60	2.793	0.767	2.16	3.701	1.085
0.81	2.881	0.798	0.47	2.469	0.878	0.81	3.013	0.771	2.93	4.402	1.331
1.02	2.933	0.817	0.60	2.234	0.981	1.02	3.227	0.772	3.70	4.951	1.523

The analysis shows that in both schemes 3 and 4, reducing evaporation and increasing the capacity of the soil layer and the width of the channel leading to an increase of the saturated zone. Evaporation continues to have a very significant impact on the spreading width of the water: according to the table 4 with decreasing width parameter increases by 2.2 times. As previously in figure 3 the dependence L of the channel width l is close to linear, and depending on the quantities L and Q the water level H are qualitatively similar if we bear in mind that (9).

With regard to flow Q , in both schemes, with an increase in performance ε , l and T consumption increases, with the largest influence on the reservoir Q has a capacity T : from table 4 shows that the variation of the parameter T is accompanied by an increase of 313%, ie practically the same as in figure 3.

However, in comparison with the scheme 3 biggest changes by varying the layer thickness is undergoing now spreading width of the water: with increasing T width L parameter increased by almost 200%.

The data in table 4 that the parameter H leads to a very small (in the range of 20-30%) deviations of the width of the spreading of water L and the seepage flow, so that the influence of water level in the canals of little practical effect on the flow pattern.

7. Conclusions

Developed in the computational scheme of filtering channels and sprinklers of irrigation systems in soils underlain silnopronitsaemym confined aquifer or water-resistant base, allow us to make a comparative assessment of the role of cross-sectional shape channel power supply and the water level in it, backwater from the waters of the underlying aquifer or confining layer, soil capillarity and evaporation or seepage at the free surface of the ground water flow pattern.

In particular, we establish the following:

– compared with the filtration of the sprinklers for irrigators width of the capillary spreading of the liquid and the magnitude of the seepage flow can exceed the corresponding characteristics of more than 50%;

– the greatest influence on the width of the capillary spreading of water and filtration rate has capillarity of soil, which improves the efficiency of canals and irrigation canals;

– influence the level of water in the channels has practically no effect on the size of the zone of saturation and value of the seepage flow.

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