Modified Ratio Estimators Using Known Median and Co-Efficent of Kurtosis

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Abstract   The present paper deals with two modified ratio estimators for estimation of population mean of the study variable using the linear combination of the known population values of the Median and the Co-efficient of Kurtosis of the auxiliary variable. The biases and the mean squared errors of the proposed estimators are derived and are compared with that of existing modified ratio estimators for certain natural populations. Further we have also derived the conditions for which the proposed estimators perform better than the existing modified ratio estimators. From the numerical study it is also observed that the proposed modified ratio estimators perform better than the existing modified ratio estimators.

Keywords   Auxiliary Variable, Bias, Modified Ratio Estimators, Simple Random Sampling

1. Introduction

Consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of N distinct and identifiable units. Let Y is a study variable with value Y_i measured on $U_i, i = 1,2,3, ..., N$ giving a vector $Y = \{Y_1, Y_2, ..., Y_N\}$. The problem is to estimate the population mean $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ with some desirable properties on the basis of a random sample selected from the population U. The simplest estimator of population mean is the sample mean obtained by using simple random sampling without replacement, when there is no additional information on the auxiliary variable available. Sometimes in sample surveys, along with the study variable Y, information on auxiliary variable X, correlated with Y is also collected. This information on auxiliary variable X, may be utilized to obtain a more efficient estimator of the population mean. Ratio method of estimation, using auxiliary information is an attempt made in this direction. Before discussing further about the modified ratio estimators and the proposed modified ratio estimators the notations to be used are described.

- N − Population size
- n − Sample size
- $f = n/N$, Sampling fraction
- Y − Study variable
- X − Auxiliary variable
- $\overline{X}, \overline{Y}$ − Population means
- $\overline{x}, \overline{y}$ − Sample means
- S_X, S_y − Population standard deviations
- C_X, C_v − Co-efficient of variations
- ρ − Co-efficient of correlation
- $\beta_1 = \frac{N \sum_{i=1}^{N}(X_i - \overline{X})^3}{(N-1)(N-2)S^3}$, Co-efficient of skewness of the auxiliary variable
- $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i - \overline{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$, Co-efficient of kurtosis of the auxiliary variable
- M_d −Median of the auxiliary variable
- $B(.)$ − Bias of the estimator
- $MSE(.)$ − Mean squared error of the estimator
- $\widehat{\overline{Y}}_i(\widehat{\overline{Y}}_{pi})$ − Existing (proposed) modified ratio estimator of $\overline{Y}$

The Ratio estimator for estimating the population mean $\overline{Y}$ of the study variable Y is defined as

$$\widehat{\overline{Y}}_R = \frac{\overline{y}}{\overline{x}}\overline{X} = \widehat{R}\,\overline{X} \qquad (1)$$

where $\widehat{R} = \frac{\overline{y}}{\overline{x}} = \frac{y}{x}$ is the estimate of $R = \frac{\overline{Y}}{\overline{X}} = \frac{Y}{X}$

The Ratio estimator given in (1) is more precise than the simple random sample mean, when there exists a positive correlation between X and Y. Sen [8] has presented the historical developments of the ratio method of estimation starting from the year 1662. Hence the readers, who are interested in knowing more details on the chronological developments of the ratio methods of estimation, are referred to Sen [8] and the references cited therein. Further improvements are also achieved on the classical ratio estimator by introducing a large number of modified ratio estimators with the use of known parameters like, Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness, Population Correlation Coefficient and Median. It is to be noted that "the existing modified ratio estimators"

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means the list of modified ratio estimators to be considered in this paper unless otherwise stated. It does not mean to the entire list of modified ratio estimators available in the literature. For a more detailed discussion on the ratio estimator and its modifications one may refer to Cochran[1], Kadilar and Cingi[2,3], Koyuncu and Kadilar[4], Murthy[5], Prasad[6], Rao[7], Singh[10], Singh and Tailor[11,12], Singh et.al[13], Sisodia and Dwivedi[14], Srivastava [15,16], Subramani and Kumarapandiyan[17,18], Upadhyaya and Singh[19], Walsh [20], Yan and Tian[21] and the references cited there in.

The lists of modified ratio estimators together with their biases, mean squared errors and constants available in the literature are classified into two classes namely Class 1, Class 2 and are given respectively in Table 1 and Table 2 respectively.

The modified ratio estimators given in Table 1 and Table 2 are biased but have minimum mean squared errors compared to the classical ratio estimator. The list of estimators given in Table 1 and Table 2 uses the known values of the parameters like $\bar{X}, C_x, \beta_1, \beta_2, \rho, M_d$ and their linear combinations. However, it seems, no attempt is made to use the linear combination of known values of the Median and Co-efficient of Kurtosis of the auxiliary variable to improve the ratio estimator. The points discussed above have motivated us to introduce two modified ratio estimators using the linear combination of the known values of Median and Co-efficient of Kurtosis of the auxiliary variable. It is observed that the proposed estimators perform better than the existing modified ratio estimators listed in Table 1 and Table 2.

2. Proposed Modified Ratio Estimators

In this section, we have suggested two modified ratio estimators using the linear combination of Median and Co-efficient of Kurtosis of the auxiliary variable. The proposed modified ratio estimators for estimating the population mean $\bar{Y}$ together with the first degree of approximation, the biases and mean squared errors and the constants are given below:

$$\widehat{\bar{Y}}_{p1} = \bar{y}\left[\frac{\bar{X}\beta_2 + M_d}{\bar{x}\beta_2 + M_d}\right]$$

$$B\left(\widehat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\bar{Y}(\theta_{p1}^2 C_x^2 - \theta_{p1} C_x C_y \rho)$$

$$\text{MSE}\left(\widehat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{p1}^2 C_x^2 - 2\theta_{p1} C_x C_y \rho)$$

$$\text{where } \theta_{p1} = \frac{\bar{X}\beta_2}{\bar{X}\beta_2 + M_d} \qquad (2)$$

$$\widehat{\bar{Y}}_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + M_d)}(\bar{X}\beta_2 + M_d)$$

$$B\left(\widehat{\bar{Y}}_{p2}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p2}^2$$

$$\text{MSE}\left(\widehat{\bar{Y}}_{p2}\right) = \frac{(1-f)}{n}\left(R_{p2}^2 S_x^2 + S_y^2(1-\rho^2)\right)$$

$$\text{where } R_{p2} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + M_d} \qquad (3)$$

3. Efficiency Comparison

For want of space; for the sake of convenience to the readers and for the ease of comparisons, the modified ratio estimators given in Table 1, Table 2 are represented into two classes as given below. Further it is to be noted that the proposed estimator $\widehat{\bar{Y}}_{p1}$ is compared with that of the modified ratio estimators listed in Class 1 whereas the proposed estimator $\widehat{\bar{Y}}_{p2}$ is compared with that of the modified ratio estimators listed in Class 2.

Table 1. Existing modified ratio estimators (Class 1) with their biases, mean squared errors and their constants

Estimator	Bias - $B(.)$	Mean squared error $MSE(.)$	Constant θ_i
$\widehat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X}C_x + \beta_2}{\bar{x}C_x + \beta_2}\right]$ Upadhyaya and Singh[19]	$\frac{(1-f)}{n}\bar{Y}(\theta_1^2 C_x^2 - \theta_1 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho)$	$\theta_1 = \frac{\bar{X}C_x}{\bar{X}C_x + \beta_2}$
$\widehat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X}\beta_2 + C_x}{\bar{x}\beta_2 + C_x}\right]$ Upadhyaya and Singh[19]	$\frac{(1-f)}{n}\bar{Y}(\theta_2^2 C_x^2 - \theta_2 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho)$	$\theta_2 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2 + C_x}$
$\widehat{\bar{Y}}_3 = \bar{y}\left[\frac{\bar{X}\beta_2 + \beta_1}{\bar{x}\beta_2 + \beta_1}\right]$ Yan and Tian[21]	$\frac{(1-f)}{n}\bar{Y}(\theta_3^2 C_x^2 - \theta_3 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_x C_y \rho)$	$\theta_3 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2 + \beta_1}$
$\widehat{\bar{Y}}_4 = \bar{y}\left[\frac{\bar{X}\beta_1 + \beta_2}{\bar{x}\beta_1 + \beta_2}\right]$ Yan and Tian[21]	$\frac{(1-f)}{n}\bar{Y}(\theta_4^2 C_x^2 - \theta_4 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_x C_y \rho)$	$\theta_4 = \frac{\bar{X}\beta_1}{\bar{X}\beta_1 + \beta_2}$

Table 2. Existing modified ratio estimators **(Class 2)** with their biases, mean squared errors and their constants

Estimator	Bias-$B(.)$	Mean squared error $MSE(.)$	Constant R_i
$\widehat{\overline{Y}}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_5^2$	$\frac{(1-f)}{n}\left(R_5^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_5 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x}$
$\widehat{\overline{Y}}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2)$ Kadilar and Cingi[2]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_6^2$	$\frac{(1-f)}{n}\left(R_6^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_6 = \frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}$
$\widehat{\overline{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+\beta_2)}(\bar{X}\beta_1+\beta_2)$ Yan and Tian[21]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_7^2$	$\frac{(1-f)}{n}\left(R_7^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_7 = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1+\beta_2}$
$\widehat{\overline{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_8^2$	$\frac{(1-f)}{n}\left(R_8^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_8 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho}$
$\widehat{\overline{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2)$ Kadilar and Cingi[3]	$\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_9^2$	$\frac{(1-f)}{n}\left(R_9^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_9 = \frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$

Class 1: The biases, the mean squared errors and the constants of the modified ratio estimators $\widehat{\overline{Y}}_1$ to $\widehat{\overline{Y}}_4$ listed in the Table 1 are represented in a single class (say, Class 1), which will be very much useful for comparing with that of proposed modified ratio estimators and are given below:

$$B\left(\widehat{\overline{Y}}_i\right) = \frac{(1-f)}{n}\bar{Y}\left(\theta_i^2 C_x^2 - \theta_i C_x C_y \rho\right)$$

$$MSE\left(\widehat{\overline{Y}}_i\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i C_x C_y \rho\right); i = 1,2,3,4$$

where $\theta_1 = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_2}, \theta_2 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+C_x}, \theta_3 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+\beta_1}$ and $\theta_4 = \frac{\bar{X}\beta_1}{\bar{X}\beta_1+\beta_2}$ (4)

Class 2: The biases, the mean squared errors and the constants of the remaining 5 modified ratio estimators $\widehat{\overline{Y}}_5$ to $\widehat{\overline{Y}}_9$ listed in the Table 2 are represented in a single class (say, Class 2), which will be very much useful for comparing with that of proposed modified ratio estimators and are given below:

$$B\left(\widehat{\overline{Y}}_i\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_i^2$$

$$MSE\left(\widehat{\overline{Y}}_i\right) = \frac{(1-f)}{n}\left(R_i^2 S_x^2 + S_y^2(1-\rho^2)\right); i = 5,6,7,8,9$$

where $R_5 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x}, R_6 = \frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}, R_7 = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1+\beta_2}, R_8 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho}$ and $R_9 = \frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$ (5)

As derived earlier in section 2, the biases, the mean squared errors and the constants of two proposed modified ratio estimators are given below:

$$B\left(\widehat{\overline{Y}}_{p1}\right) = \frac{(1-f)}{n}\bar{Y}\left(\theta_{p1}^2 C_x^2 - \theta_{p1} C_x C_y \rho\right)$$

$$MSE\left(\widehat{\overline{Y}}_{p1}\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_{p1}^2 C_x^2 - 2\theta_{p1} C_x C_y \rho\right)$$

$$\text{where } \theta_{p1} = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+M_d} \quad (6)$$

$$B\left(\widehat{\overline{Y}}_{p2}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p2}^2$$

$$MSE\left(\widehat{\overline{Y}}_{p2}\right) = \frac{(1-f)}{n}\left(R_{p2}^2 S_x^2 + S_y^2(1-\rho^2)\right) \text{ where } R_{p2} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+M_d} \quad (7)$$

From the expressions given in (4) and (6) we have derived the conditions for which the proposed estimator $\widehat{\overline{Y}}_{p1}$ is more efficient than the existing modified ratio estimators given in Class 1, $\widehat{\overline{Y}}_i$; $i = 1, 2, 3, 4$ and are given below:

$$MSE\left(\widehat{\overline{Y}}_{p1}\right) < MSE\left(\widehat{\overline{Y}}_i\right) \text{ if } \rho < \frac{(\theta_{p1}+\theta_i)}{2}\frac{C_x}{C_y} \text{ ; } i = 1, 2, 3, 4 \quad (8)$$

From the expressions given in (5) and (7) we have derived the conditions for which the proposed estimator $\widehat{\overline{Y}}_{p2}$ is more efficient than the existing modified ratio estimators given in Class 2, $\widehat{\overline{Y}}_i$; $i = 5, 6, 7, 8$ and 9 and are given below:

$$MSE\left(\widehat{\overline{Y}}_{p2}\right) < MSE\left(\widehat{\overline{Y}}_i\right) \text{ if } R_{p2} < R_i \text{ ; } i = 5, 6, 7, 8 \text{ and } 9 \quad (9)$$

4. Numerical Study

The performances of the proposed modified ratio estimators are assessed with that of existing modified ratio estimators listed in Table 1 and Table 2 for certain natural populations. In this connection, we have considered four natural populations for the assessment of the performances of the proposed modified ratio estimators with that of existing modified ratio estimators. The population 1 and 2 are taken from Singh and Chaudhary[9] given in page 177, population 3 is taken from Singh and Chaudhary[9] given in page 141 and population 4 is taken from Cochran[1] given in page 152. The population parameters and the constants computed from the above populations are given below:

The constants of the existing and proposed modified ratio estimators for the above populations are given in the Table 4 and Table 5:

The biases of the existing and proposed modified ratio estimators for the above populations are given in the Table 6 and Table 7:

The mean squared errors of the existing and proposed modified ratio estimators for the above populations are given in the Table 8 and Table 9:

From the values of Table 6 and Table 7, it is observed that

the bias of the proposed modified ratio estimator $\widehat{\overline{Y}}_{p1}$ is less than the biases of the existing modified ratio estimators $\widehat{\overline{Y}}_i$; i = 1, 2, 3, 4 given in Class 1 and the bias of the proposed modified ratio estimator $\widehat{\overline{Y}}_{p2}$ is less than the biases of the existing modified ratio estimators $\widehat{\overline{Y}}_i$; i = 5, 6, 7, 8 and 9 given in Class 2. Similarly from the values of Table 8 and Table 9, it is observed that the mean squared error of the proposed modified ratio estimator $\widehat{\overline{Y}}_{p1}$ is less than the mean squared errors of the existing modified ratio estimators $\widehat{\overline{Y}}_i$; i = 1, 2, 3, 4 given in Class 1 and the mean squared error of the proposed modified ratio estimator $\widehat{\overline{Y}}_{p2}$ is less than the mean squared errors of the existing modified ratio estima- tors $\widehat{\overline{Y}}_i$; i = 5, 6, 7, 8 and 9 given in Class 2.

Table 3. Parameters and Constants of the Populations

Parameters	Population 1	Population 2	Population 3	Population 4
N	34	34	22	49
n	20	20	5	20
$\overline{Y}$	856.4117	856.4117	22.6209	116.1633
$\overline{X}$	208.8823	199.4412	1467.5455	98.6735
ρ	0.4491	0.4453	0.9022	0.6904
S_y	733.1407	733.1407	33.0469	98.8286
C_y	0.8561	0.8561	1.4609	0.8508
S_x	150.5059	150.2150	2562.1449	102.9709
C_x	0.7205	0.7532	1.7459	1.0436
β_2	0.0978	1.0445	13.3694	5.9878
β_1	0.9782	1.1823	3.3914	2.4224
M_d	150.0000	142.5000	534.5000	64.0000

Table 4. The constants of the (Class 1) existing and proposed modified ratio estimators

Estimator	Constants θ_i			
	Population 1	Population 2	Population 3	Population 4
$\widehat{\overline{Y}}_1$ Upadhyaya and Singh[18]	0.9994	0.9931	0.9948	0.9450
$\widehat{\overline{Y}}_2$ Upadhyaya and Singh[18]	0.9658	0.9964	0.9999	0.9982
$\widehat{\overline{Y}}_3$ Yan and Tian[21]	0.9542	0.9944	0.9998	0.9959
$\widehat{\overline{Y}}_4$ Yan and Tian[21]	0.9995	0.9956	0.9973	0.9756
$\widehat{\overline{Y}}_{p1}$ (Proposed estimator)*	0.1195*	0.5938*	0.9735*	0.9023*

Table 5. The constants of the (Class 2) existing and proposed modified ratio estimators

Estimator	Constants R_i			
	Population 1	Population 2	Population 3	Population 4
$\widehat{\overline{Y}}_5$ Kadilar and Cingi[2]	3.9598	4.2786	0.0154	1.1752
$\widehat{\overline{Y}}_6$ Kadilar and Cingi[2]	4.0973	4.2644	0.0153	1.1126
$\widehat{\overline{Y}}_7$ Yan and Tian[21]	4.0980	4.2751	0.0154	1.1485
$\widehat{\overline{Y}}_8$ Kadilar and Cingi[3]	4.0115	4.2849	0.0154	1.1759
$\widehat{\overline{Y}}_9$ Kadilar and Cingi[3]	4.0957	4.2441	0.0153	1.0821
$\widehat{\overline{Y}}_{p2}$ (Proposed estimator)*	0.4899*	2.5499*	0.0150*	1.0622*

Table 6. The biases of the (Class 1) existing and proposed modified ratio estimators

Estimator	Bias $B(.)$			
	Population 1	Population 2	Population 3	Population 4
$\widehat{\overline{Y}}_1$ Upadhyaya and Singh[19]	4.2607	4.8369	2.5432	1.3519
$\widehat{\overline{Y}}_2$ Upadhyaya and Singh[19]	3.8212	4.8860	2.6106	1.6268
$\widehat{\overline{Y}}_3$ Yan and Tian[21]	3.6732	4.8556	2.6095	1.6144
$\widehat{\overline{Y}}_4$ Yan and Tian[21]	4.2629	4.8739	2.5763	1.5070
$\widehat{\overline{Y}}_{p1}$ (Proposed estimator)*	0.4529*	0.5207*	2.2674*	1.1462*

Table 7. The biases of the (Class 2) existing and proposed modified ratio estimators

Estimator	Bias $B(.)$			
	Population 1	Population 2	Population 3	Population 4
$\widehat{\overline{Y}}_5$ Kadilar and Cingi[2]	8.5387	9.9303	10.6540	3.7302
$\widehat{\overline{Y}}_6$ Kadilar and Cingi[2]	9.1421	9.8646	10.5456	3.3433
$\widehat{\overline{Y}}_7$ Yan and Tian[21]	9.1452	9.9143	10.5989	3.5627
$\widehat{\overline{Y}}_8$ Kadilar and Cingi[3]	8.7629	9.9597	10.6549	3.7347
$\widehat{\overline{Y}}_9$ Kadilar and Cingi[3]	9.1349	9.7711	10.4439	3.1630
$\widehat{\overline{Y}}_{p2}$ (Proposed estimator)*	0.1307*	3.5269*	10.0982*	3.0475*

Table 8. The mean squared errors of the (Class 1) existing and proposed modified ratio estimators

Estimator	Mean Squared Error **MSE**(.)			
	Population 1	Population 2	Population 3	Population 4
$\widehat{\overline{Y}}_1$ Upadhyaya and Singh[18]	10534.5417	10902.7384	45.2894	214.7486
$\widehat{\overline{Y}}_2$ Upadhyaya and Singh[18]	10298.4432	10930.3879	45.8857	233.6573
$\widehat{\overline{Y}}_3$ Yan and Tian[21]	10220.4736	10913.2804	45.8758	232.7813
$\widehat{\overline{Y}}_4$ Yan and Tian[21]	10535.7860	10923.6103	45.5814	225.2956
$\widehat{\overline{Y}}_{p1}$ (Proposed estimator)*	10178.2990*	8937.4062*	42.9321*	201.3263*

Table 9. The mean squared errors of the (Class 2) existing and proposed modified ratio estimators

Estimator	Mean Squared Error **MSE**(.)			
	Population 4	Population 2	Population 3	Population 4
$\widehat{\overline{Y}}_5$ Kadilar and Cingi[2]	16146.6142	17376.0389	272.4185	584.5606
$\widehat{\overline{Y}}_6$ Kadilar and Cingi[2]	16663.3064	17319.7468	269.9654	539.6120
$\widehat{\overline{Y}}_7$ Yan and Tian[21]	16665.9758	17362.2582	271.1716	565.0981
$\widehat{\overline{Y}}_8$ Kadilar and Cingi[3]	16338.6465	17401.1397	272.4393	585.0781
$\widehat{\overline{Y}}_9$ Kadilar and Cingi[3]	16657.1867	17239.6579	267.6660	518.6688
$\widehat{\overline{Y}}_{p2}$ (Proposed estimator)*	8945.8872*	11892.0742*	259.8459*	505.2469*

5. Conclusions

In this paper we have proposed two modified ratio estimators using linear combination of Median and Co-efficient of Kurtosis of the auxiliary variable. The biases and mean squared errors of the proposed estimators are obtained and compared with that of existing modified ratio estimators. Further we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators. We have also assessed the performances of the proposed estimators for some known populations. It is observed that the biases and mean squared errors of the proposed estimators are less than the biases and mean squared errors of the existing modified ratio estimators for certain known populations. Hence we strongly recommend that the proposed modified estimators may be preferred over the existing modified ratio estimators for the use of practical applications.

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