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On Complementary Edge Magic Labeling of Certain Graphs

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Abstract By G(p, q) we denote a graph having p vertices and q edges, by V(G) and E(G) the vertex set and the edge – set of G respectively. But the vertices and edges are called the elements of the graph. A (p, q) – graph G is called the edge – magic if there exists a bijective function f: V(G) U E(G) → {1,2, ,p+q} such that f(u)+f(v)+f(uv)=k is a constant called the valence of f for any edge uv of G. Given an edge magic f of a graph G(p, q) the function $\overline{f}(x)$ such that $\overline{f}(x)$=p+q+1-f(x) for all elements of G is said to be complementary to f(x) or complementary edge magic labeling $\overline{f}(x)$. The purpose of this article is to search for certain graphs Km, n (m, n ≥ 1), Cn (n ≥ 3), np2, f n (fan) Bn (bwk) and nG (n ≥ 2) where G is bipartite or tripartite which have complementary edge magic strength.

Keywords Edge-Magic Labeling, Complementary Edge Magic Labeling, 1991 Mathematics Subject Classification. O5C78

1. Introduction

The subject of edge – magic labelings of graphs had its origins three decades ago in the work of Kotzing and Rosa [9,10] on what they called magic valuations of graphs (which are also commonly known as edge-magic total labeling see[4]). Interest in these labelings has been lately rekindled by a paper on the subject due to Ringel and Llado[11]. Shortly after this, Enomoto, Llado, Nakamigawa and Ringel[3] defined a more restrictive form of edge-magic labelings namely super edge magic labelings which Wallis[12] refers to as strong edge – magic labelings.

2. Objective

By G(p, q), we denote a graph having p vertices and q edge, by V(G) and E(G), the vertex set and the edge-set of G respectively. But the vertices and edges are called the elements of the graph G. A graph(p, q) is said to have an edge magic with the magic constant k (which is independent on the choice of any edge uv of G) if there exists a one one– to –one mapping f:V(G) U E(G) → {1, 2, p+q} such that f(u) + f(v) +f(uv) =k for all uv ∈ E (G). If such a labeling exists, then magic constant k is called valence of f and G is said to be edge –magic graph. Given an edge magic f of a graph G (p, q), the Function f(x) such that $\overline{f}(x)$ = p+q+1-f(x) for all elements x ∈G is said to be complementary to f(x).

Two edge magic f_1 and f_2 of G are equivalent if f_1=f_2 or f_1= $\overline{f_2}$. An edge magic f of G is said to be self complementary edge magic if f=$\overline{f}$

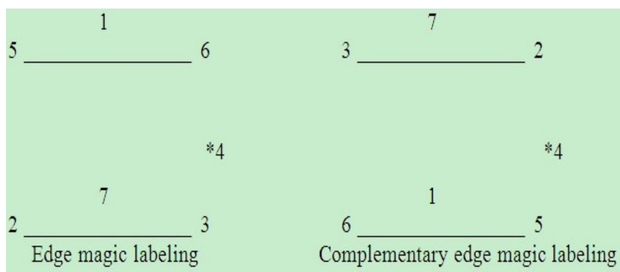

Figure 1.

The edge magic strength of a graph G is denoted by ems(G) and is defined as the minimum of all constants where the minimum is taken over all edge magic labelings of G. ems(G) = min {k: f is an edge magic labeling of G} similarly the concept of the complementary edge magic strength of a graph is introduced. The complementary edge magic strength of G is denoted by cems(G) and is defined as the minimum of all constants $\overline{k}$ where the minimum is taken over all complementary edge magic labeling of G. cems (G) = min { $\overline{k}$: $\overline{f}$ is an complementary edge magic labeling of G}.

3. Methods

In this paper, the complementary edge magic strength of some well known graphs such as $K_{m, n}$ (m,n ≥ 1), C_n (n ≥ 3),

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np_2 ($n \geq 1$), f_n, B_n and nG ($n \geq 2$) where G is bipartite or tripartite are obtained. The reader is directed to Chartered and Lesniak[2] or Hartefield and Ringel[8] for all additional terminology not provided in this paper. The following theorem is every useful for the main results.

Theorem A[1] An complementary edge – magic graph G satisfies the following equation

$$q\bar{k} = \frac{T(T+1)}{2} + \sum_{i=1}^{p}[dv_i - 1]f(v_i)$$

where $\bar{f}$ is complementary edge magic labeling of G and T=p+q.

4. Results

Main Results

In this section we proceed to study the complementary edge magic strength of $K_{m,n}$, C_n, $f_n=P_n+k_1$ and $B_n = K_{1,n} \times K_2$ and nG, where G is bipartite or tripartite.

Theorem1. A complete bipartite graph Km, n is complementary edge magic strength if m, n ≥ 1.

Proof: Let G be a complete bipartite graph $K_{m,n}$ with vertex set V partitioned into two subject V_1 and V_2 where $V_1=\{u_1, u_2, u_m\}$ and $V_2=\{v_1,v_2, v_n\}$ and

let f: VUE→ {1,2, ,m+n+mn}

Then,

$f(v_i) = m+n+mn+1-i$ $f(v_j) = mxn+n+1 - (m+1)j$

$f(a_ib_j) = -m+(m+1)j + i$

It is easy to see that f extends to a complementary edge magic labeling of $K_{m,n}$ with the magic constant $\bar{k}$.

$\bar{k} = f(v_i) + f(v_j) + f(a_ib_j)$

= [m+n+mn+1-i] + [mn+n+1 – (m+1)j] + [-m+(m+1)j +i]

= 2mn+2n+2

= 2(mn+n+1)

Complementary edge magic strength: Now by Theorem A

$$q\bar{k} = \frac{(m+n+mn)(m+n+mn+1)}{2} + (n-1)\sum_{i=1}^{n}(m+n+mn+1-i) + \sum_{j=1}^{n}[mn+n+1-(m+1)j]$$

$$[mn]\bar{k} = \frac{(m+n+mn)(m+n+mn+1)}{2} + m(n-1)(m+n+mn+1) - (n-1)\sum_{i=1}^{m} i + n(m-1)(mn+n+1) - (m^2-1)\sum_{j=1}^{n} j$$

$$= \frac{(m+n+mn+1)}{2}[m+n+mn+2m(n-1)] - \frac{m(m+1)(n-1)}{2} + n(m-1)(mn+n+1) - \frac{(m^2-1)n(n+1)}{2}$$

$$\bar{k} = 2(mn+n+1)$$

Thus scems ($K_{m,n}$) = 2(mn+n+1)

The following theorem is the complementary edge magic strength of cycle C_n for every integer n ≥ 3.

Theorem2. The cycle C_n is complementary edge magic and complementary edge magic

strength is $\frac{7n^2+2n}{2}$ where n is an integer.

Proof: Let C be the cycle with

$V(C_n) = \{v_i: 1 \leq i \leq n\}$ and $E(C_n) = \{v_i: v_{i+1} : 1 \leq i \leq n^2\}$ where i is taken modulo n (replacing o by n)

We discuss the following cases.

Case I: n is odd say n=2m+1 where m is positive integer.

Consider the function f: V(G) U E (G) → {1,2,2n}

Defined as

$$f(v_i) = \begin{cases} 2n+1-i & \text{if i odd} \\ n+1-i & \text{if i even} \end{cases}$$

and,

$$f(v_iv_{i+1}) = \begin{cases} 1+2i & i = 1,2 \quad n-1 \\ 1 & i = 2n \end{cases}$$

Since we have $\underset{E(G)}{U}$ f (V_i) = {2,4,6,10,….2n}

$\underset{V(G)}{U}$ f $(v_i v_{i+1})$ = {1,3,5.(2n-1)}

$f(v_n) + f(v_1) + f(v_nv_1) = 2n+(n+1)+1 = 3n+2$

$f(v_i)+ f(v_{i+1}) + f(v_i v_{i+1}) = (2n+1- i) + (n-i) + (1+2i)$ =3n+2 for i=1,2, n-1

Thus f is a complementary edge magic of cycle C_n with magic constant 3n+2

Complementary edge – magic string:

$$q\bar{k} = \frac{T(T+1)}{2} + \sum_{i=1}^{n}(dv_i - 1)f(v_i)$$

$$n\bar{k} = \frac{2n(2n+1)}{2} + \sum_{i=1}^{n} f(v_i)$$

$$= n(2n+1) + \frac{n}{2}\{2\times 2 + (n-1)2\}$$

$$= n(3n+2)$$

$$\bar{k} = 3n+2$$

Case 2. n ≡ o (mod 4) say n = 4l where l is the positive integer, f is defined as

$$f(v_i) = \begin{cases} 2n+1-i & i = 1,3\ldots\ldots,2l-1 \\ 2n-i & i = 2l, 2l+2\ldots\ldots 4l-2 \\ n+1-i & i = 2l+1, 2l+3\ldots\ldots 4l-3, l \geq 2 \\ n-i & i = 2,4\ldots\ldots,2l-2 \\ 2n-1, & i = 4l-1 \\ 3 & i = 4l \end{cases}$$

$$f(vi \ \ vi=1) = \begin{cases} 3+2i & i = 1,2\ldots\ldots,2l-2,2l,2l+1\ldots\ldots,4l-3 \\ 1 & i = 2l-1 \\ 2 & i = 4l-2 \\ n+1 & i = 4l-1 \\ n & i = 4l \\ 3 & i = 4l \end{cases}$$

Clearly $\underset{V(G)}{U}$ $f(v_i)$ = { 4,6,8,…..2n-4,2n} U {n+1, 2n-1}

$\underset{E(G)}{U}$ $f(v_iv_i+1)$ = { 1,3,5,…..n-1, n+3,n+5,…,2n-3} U {2, 2n-2}

Next we prove that

$f(v_i)+f(v_{i+1})+f(v_iv_{i+1}) = \bar{k}$ for all i = 1,2,..,4l

Sub case 2.1 Let i ∈ {4, 6, 8.., 2n-4,2n} U {n+1,2n-1} Then

$f(v_i)+f(v_{i+1})+f(v_iv_{i+1}) = (2n+1-i)+(n-i-1)+(3+2i) = 3(n+1)$

Sub case 2.2 Let i = 2l-1 then

$f(v_{2l-1})+f(v_{2l})+f(v_{2l-1}v_{2l})=(2n-2l+2)+(2n-2l)+1=3(n+1)$

Sub case 2.3 Let i ∈ {2l, 2l+2,…, 4l-4} Then

$f(v_i)+f(v_{i+1})+f(v_iv_{i+1}) = (2n-i)+(n-i)+3+2i=3(n+1)$

Sub case 2.4 Let i = 2l-2 Then

$f(v_{4l-2})+f(v_{4l-1})+f(v_{4l-2}v_{4l-1})=(2n+2)+(2n-1)+2=3(n+1)$

Sub case 2.5 Let I ∈ {2l+1, 2l+3,..,4l-3}, Then

$f(v_i)+f(v_{i+1})+f(v_iv_{i+1}) = (n+1-i)+(2n-1-i)+(3+2i)=3(n+1)$

Sub case 2.6 Let i ∈ {2l+1, 2l+3,…4l-3} Then

$f(v_i)+f(v_{i+1}) + f(v_iv_{i+1})= (n-i)+(2n-i)+(3+2i)=3(n+1)$

Sub case 2.7 Let i = 4l-1. Then

$f(v_{4l-1})+f(v_{4l})+f(v_{4l-1}v_{4l})=(2n-1)+(3)+(n+1)=3(n+1)$

Sub case 2.8 Let i = 4l. Then

$f(v_{4l})+f(v_1)\underline{+}f(v_{4l}v_1)= 3+(2n)+n=3(n+1)$

Therefore f is a complementary edge magic of the cycle C_n with the magic

Constant $\bar{k} = 3(n+1)$

Complementary edge magic strength:

$$q\bar{k} = \frac{2n(2n+1)}{2} + \sum_{i=1}^{n} f(v_i)$$

$$n\bar{k} = n(2n+1)+3+(n-1)/2\{2\times 4+2(n-2)\}+(2n-1)-n$$

$$\bar{k} = \frac{n(3n+1)}{n} = 3(n+1)$$

Case 3. n ≡ 2 (mod 4) that is n = 4l+2. Define the function

$$f(v_i) = \begin{cases} 2n+1/2-i/2 & \text{if } i = 1,3\ldots\ldots,2l+1 \\ 2n-i/2-i/2 & \text{if } i = 2l+3, 2l+5\ldots\ldots 4l+1 \\ 2n-6l-2 & \text{if } i = 2 \\ 2n-l-1 & \text{if } i = 2l+2 \\ 2n-2l-2, & \text{if } i = 4l+2 \\ 2n-2l-1-i/2 & \text{if } i = 4,6\ldots\ldots,2l; l \ge 2 \\ 2n-2l-i/2 & \text{if } i = 2l+4, 2l+6\ldots\ldots,4l; l \ge 2 \end{cases}$$

$$f(v_iv_{i+1}) = \begin{cases} 2n-4l-2 & \text{if } i = 1 \\ 2n-4l-1 & \text{if } i = 2 \\ 2n-8l-3 & \text{if } i = 2l+1 \\ 2n-8l-1 & \text{if } i = 2l+2 \\ 2n-6l-1 & \text{if } i = 4l+1 \\ 2n-6l-2 & \text{if } i = 4l+2 \\ 2n-8l-3+i & \text{otherwise}; l \ge 2 \end{cases}$$

Since $\bigcup_{V(G)}$ {f v_i } = { 4l + 2, 4l+3,….., 6l+1, 6l+2,6l+4,6l+5,……8l+4}

$\bigcup_{E(G)}$ {f (v_i v_{i+1})} = { 1,2,…,4l, 4l+1, 6l+3}

Next we prove that $f(v_i)+f(v_{i+1})+f(v_iv_{i+1}) = \bar{k}$ for all i = 1,2,4l+2

Subcase 3.1 Let i = 1 Then

$f(v_i)+f(v_{i+1})+f(v_iv_{i+1})= 2n+(2n-6l-2)+(2n-4l-2)$
$=2(3n-5l-2)=(7n+2)/2$

Sub case 3.2 Let i = {3,5,..,2l-1} Then

$f(v_i)+f(v_{i+1})+f(v_iv_{i+1})= (2n+1/2 – i/2)$
$+ \{2n-2l-1-(i+1)/2\}+(2n-8l-3-i)$
$= 2(3n-5l-2)= (7n+2)/2$

Sub case 3.3 Let i = 2l+1. Then

$f(v_{2l+1})+ f(v_{2l+2})+f(v_{2l+1}v_{2l+2})$
$= (2n-1)(2n-l-1)+(2n-8l-3)=3(2n-5l-2)=(7n+2)/2$

Sub case 3.4 Let i = {2l+3, 2l+5,…, 4l-1} Then

$f(v_i)+f(v_{i+1})+f(v_iv_{i+1})= (2n-1/2-i/2)$
$+\{2n-2l-1-(i-1)/2\}+(2n-8l-3+i)$
$= 6n-10l-4 = 2(3n-5l-2) = (7n+2)/2$

Sub case 3.5 Let i = 4l+1. Then

$f(v_{4l+1})+f(v_{4l+2})+f(v_{4l+1},v_{4l+2})= (2n-1/2-(4l+1)/2)$
$+(2n-2l-2)+(2n-6l-1)= 6n-10l-4 = 2(3n-5l-2)$
$= (7n+2)/2$

Sub case 3.6 Let i = 2. Then

$f(v_2)+f(v_2)+f(v_2v_3)= (2n-6l-2)+(2n-1)+(2n-4l+1)$
$= 6n-10l-4 = 2(3n-5l-2) =(7n+2)/2$

Sub case 3.7 Let i = 2k+2. Then

$f(v_{2l+2}) + f(v_{2l+3}) + f(v_{2l+2}v_{2l+3}) = (2n-l-1)$
$+(2n-l-1)+(2n-8l-1)=2(3n-5l-2)= (7n+2)/2$

Sub case 3.8 Let i = 4l+2. Then

$f(v_{4l+2})+f(v_1)+f(v_{4l+2}v_1)= (2n-2l-2)+(2n+1)+(2n-6l-2)$
$=2(3n-5l-2) =(7n+2)/2$

Sub case 3.9 Let i ∈ {4,6,…2k}. Then

$f(v_i)+f(v_{i+1})+f(v_iv_{i+1})= (2n-2l-1-i/2)$
$+(2n-i/2)+(2n-8l-3+i)= 2(3n-5l-2)= (7n+2)/2$

Sub case 3.10 Let i ∈ {2l+4, 2l+6,….4l}. Then

$f(v_i)+f(v_{i+1})+f(v_iv_{i+1})= (2n-2l-i/2)$
$+(2n-1-i/2)+(2n-8l-3+i)=2(3n-5l-2)=(7n+2)/2$

Thus f is a complementary edge magic function of the cycle C_{4l+2} with the magic constant $\bar{k}$=2(3n-5l-2)= (7n+2)/2

Complementary edge magic strength:

$q\bar{k}$ = {2n(2n+1)}/2 + ∑f (v_i)
= {2n(2n+1)}/2+[(n+2)/2+ (n-1)/2 x{2 (n+2)+1 (n-1)}]
$n\bar{k}$ == $7n^2$+ 2n/2
$\bar{k}$ = 7n+2/2.

Theorem 3. Let G = nP_2 be a regular graph of degree one. Then G is a Complementary edge magic strength if and Only if n is odd.

Proof: Since (3n+1)/2 must be necessarily an integer. Hence it follows that n must be odd say n= 2l+1

Let u_1, u_2, u_n and v_1, v_2,v_n; u_1v_1, u_2v_2 u_nv_n be vertices and edges of G respectively. Define f by

$$f(u_i) = 3n+1-i, f(v_i)\begin{cases} 3n-5l-1-i & i = 1,2\ldots\ldots,l+l \\ 3n-3l-i & i = 1+2, l+3\ldots\ldots,2l+1 \end{cases}$$

And

$$f(u_iv_i) = \begin{cases} 3n-4l-3+2i & i = 1,2\ldots\ldots,l+1 \\ 3n-6l-4+2i & i = 1+2, l+3\ldots\ldots,2l+1 \end{cases}$$

We have $\bigcup_{i=1}$ {f u_i } = {2n+1, 2n+2, …..3n} and

$\bigcup_{i=1}$ {f (u_iv_i)} = {n+1,n+2,….., 2n}

Now we show that

$\bar{k}$ = f (u_i)+f (v_i)+f ($u_i$vi)= (3n+1-i) + (3n-3l-i)
+(3n-6l-4+2i)= 9n-9l-3 = 3{(9n+1)}/2

Complementary edge magic strength:

$q\bar{k}$ = {3n (3n+1)}/2 Therefore, $\bar{K}$ = {3(3n+1)}/2

Theorem 4. The fan $f_n = P_n+K_1$ is a complementary edge magic for any positive integer n

Proof . Let V (f_n) = {u} U { v_i :1≤ i ≤ n }

And E (f_n) = { uv_i : 1≤i≤n} U {v_iv_{i+1} : 1≤i≤n-1}

Now define the function f: V (G) U E(G)→{1,2,…..,3n} as follows

$$f(x)=\begin{cases} 3n & if\ x=u \\ \{12n+3+5(-1)^i-6i\}/4 & if\ x=v_i \text{ and } 1\le i\le n \\ \{-3-5(-1)^i+6i\}/4 & if\ x=ux_i \text{ and } 1\le i\le n \\ 3i & if\ x=v_i\,v_{i+1} \text{ and } 1\le i\le n-1 \end{cases}$$

Observe that f(x)+f(y)+f(xy)=6n for every edge xy ∈ E (fn)

Also notice that alternative type labeling is as follows:

{f (v_{2i+1}) : o ≤ i ≤ |_(n-1)/2_|

= {3(n-i)+1 : 1 ≤ i ≤ |_(n+1)/2_|}

{f (uv_{2i}) : 1≤ i ≤ ┌ (n-1)/2 ┐

= {3(n-i)+1 :1+|_(n+1)/2_| ≤ i ≤ n}

{f (v_i v_{i+1}): 1≤i ≤ n-1

= {3(n-i)+1 : 1+|_(n+1)/2_| ≤ i ≤ n}

{f (v_{2i}): 1≤ i ≤ ┌ (n-1)/2 ┐

= {3(n-i)-1: o ≤ i ≤ |_(n-2)/2_|}

{f (uv_{2i+1}): 0 ≤ i ≤ |_(n-1)/2] _|

= {3(n-i)-1: 1+|_(n-2)/2_| ≤ i ≤ n-1}

And f (u) = 3n. Thus all integers 1,2,…,3n are used exactly once. Hence f is an complementary edge magic labeling of f_n with magic constant (valence) 6n.

Complementary edge magic strength :

$q\bar{k}$ = {3n(3n+1)}/ 2 +2∑{12n+3+5(-1)^i-6i }/ 4

+ 3n(n-1)-(3n-2)- {6n+3+5(-1)^n }/ 4

=1/2[9n^2+3n+12n^2+3n+∑5(-1)^i-6n(n+1)/2

+6n^2-15n+5/2-5/2(-1)^n]

(2n-1) $\bar{k}$ =1/2[24n^2-12n+∑5(-1)^i+5/2-5/2(-1)^n]

$\bar{k}$ =1/2[24n^2-12n]/2n-1 = 6n if n is odd or even

Theorem 5. The book $B_n=K_1,n^x\ K_2$ is complementary edge magic strength for any positive integer n

Proof. Let B_n be the book defined as follows:

V (B_n) = {uv}U{u_iv_i :1≤ i ≤ n} and

E (B_n) = {uv}U{uu_i, vv_i, u_iv_i : 1 ≤ i≤n}

Now consider the function f :

V(B_n)UE(B_n)→ {1,2,..,5n+3}

Where

$$f(x)=\begin{cases} 5n+3 & if\ x=u \\ 1 & if\ x=v \\ 3n+2 & if\ x=uv \\ 3n+2-i & if\ x=u_i \text{ and } 1\le i\le n \\ 3n+2+2i & if\ x=v_i \text{ and } 1\le i\le n \\ 1+i & if\ x=uu_i \text{ and } 1\le i\le n \\ 2n+2-i & if\ x=uv_i \text{ and } 1\le i\le n \\ 5n+3-2i & if\ x=vv_i \text{ and } 1\le i\le n \end{cases}$$

5. Conclusions

Finally observe that f is complementary edge magic labeling of B_n having valence 8n+6

Complementary edge magic strength

$q\bar{k}$ = (5n+3)(5n+4)/2+n(5n+3)

+n(1)+∑(3n+2-i)+∑(3n+2+2i)

= (25n^2+35n+12)/2+5n^2+4n+∑(6n+4+i)

(3n+1) $\bar{k}$ = (35n^2+43n+12) / 2 + n(6n+4)+ {n(n+1)}/2

$\bar{k}$ = {48n^2+52n+12}/{2(3n+1)} = 8n+6

This section has a tool that allows us to generate infinite classes of disconnected complementary edge magic n- partite graphs with relative case where n = 2,or 3. Previously no such a technique was available for graphs within those classes sec[6] except in[5].

Theorem 6. If G is a complementary edge magic bipartite or tripartite graph and n is odd then nG (n ≥ 2) is complementary edge magic.

Proof. Assume that n≥3. If (p,q) graph G is a complementary edge magic bipartite or tripartite graph with partite sets A, B, C [Let C=Φ if G is bipartite] then let

E(G) = AB U AC U BC where the juxta-position of two partite sets denotes the set of edges between those two sets.

Let f :V (G)UE(G)→{1,2, ,p+q} to be an arbitrary complementary edge magic labeling of G. Then X ≡ nG to be the graph with vertex set

V(X)= $\bigcup_{i=1}^{n}$ ($A_iB_iUA_iC_iUB_iC_i$) where x_i ∈ Y_i for 1 ≤ i ≤ n

if and only if x ∈ Y (Y is one of the sets A,B,C,AB,AC,BC)

Consider the labeling g : V(X)UE(X)→{1,2,…..n (p+q)} such that

$$f(x)=\begin{cases} nf(x)+1-i & if\ x\in CUAB & 1\le i\le n \\ nf(x)-n+2i & if\ x\in AUBC & 1\le i\le (n-1)/2 \\ nf(x)-2n+2i & if\ x\in AUBC & (n+1)/2\le i\le n \\ nf(x)-n/2+1/2+i & if\ x\in BUAC & 1\le i\le (n-1)/2 \\ nf(x)+n/2+1/2-i & if\ x\in BUAC & (n+1)/2\le i\le n \end{cases}$$

Clearly g is a complementary edge magic labeling of X with valence

g(u)+g(v)+g(uv)=2n(p+q)-3(n-1)/2 for each edge uv ∈ E(X) where k_f is the valence of f.

Next to observe that g(V(H)UE(H))={1,2,n(p+q) is spread by the function g to the entirety of its range.

In the above theorem n can not be even for Kotzig and Rosa [9] have shown that the forest nP_2 is edge magic if and only n is odd

Immediately the corollary follows from above Theorem

Corollary: If n is odd and m>1 then 2-regular graph nC_{2m} is complementary edge magic.

Poof. Kotzig and Rosa have shown that all cycles are edge magic. But an alternative labeling of even cycles can be found in[7].

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