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On Thermosolutal-Convective Instability in Walters B' Heterogeneous Viscoelastic Fluid Layer through Porous Medium

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Abstract The thermosolutal convection in Walters B' heterogeneous viscoelastic fluid through Brinkman porous medium is considered. Following the linearized stability theory, Boussinesq approximation and normal mode analysis, the dispersion relation is obtained. Discussion of different modes revealed that the principle of exchange of stabilities is not valid for the problem. Further, it is found that oscillatory modes exist under certain conditions and non-oscillatory modes are unstable.

Keywords Thermosolutal Convection, Heterogeneous Walters B' Viscoelastic Fluid, Porous Medium, Linear Stability Theory, Normal Mode Technique

1. Introduction

The theoretical and experimental results on thermal convection in a fluid layer, in the absence and presence of rotation and magnetic field have been given by Chandrasekhar[1]. Thermal convection is the most convective instability when crystals are produced from single element like silicon. However, gallium arsenide and other semi-conductors which require crystals made from compounds of elements are beginning to take on a prominent position in modern technologies. Hence, at present there is strong industrial demand for understanding the additional effects that can occur in the solidification of a mixture, which do not take place in one component system. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis[2]. The buoyancy force can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. Double-diffusive convection problems arise in oceanography (salt fingers occur in the ocean when hot saline water overlies cooler fresher water which believed to play an important role in the mixing of properties in several regions of the ocean), limnology and engineering. The migration of moisture in fibrous insulation, bio/chemical contaminants transport in environment, underground disposal of nuclear wastes, magmas, groundwater, high quality crystal production and production of pure medication are some examples where double-diffusive convection is involved. Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz,[3]) and some Antarctic lakes (Shirtcliffe,[4]). The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamicists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. Such flows has shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs; in chemical engineering for filtration and purification processes and in the field of agriculture engineering to study the underground water resources, seepage of water in river beds. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and

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astrophysics. The study of thermosolutal convection in fluid saturated porous media has diverse practical applications, including that related to the materials processing technology, in particular, the melting and solidification of binary alloys. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust (Lister,[5]). Generally it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice- versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context (McDonnel,[6]).

In all the above studies, the fluid has been considered to be Newtonian. Since viscoelastic fluids play an important role in polymers and electrochemical industry, the studies of waves and stability in different viscoelastic fluid dynamical configuration has been carried out by several researchers in the past. The stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below has been investigated by Vest and Arpaci[7]. The nature of instability and some factors may have different effects, on viscoelastic fluids as compared to the Newtonian fluids. For example, Bhatia and Steiner[8] have considered the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that rotation has a destabilizing effect in contrast to the stabilizing effect on Newtonian fluid. In another study, Sharma and Sharma[9] have considered the thermal instability of a rotating Maxwell fluid through porous medium and found that, for stationary convection, the rotation has stabilizing effect whereas the permeability of the medium has both stabilizing as well as destabilizing effect, depending on the magnitude of rotation. In another study, Sharma[10] has studied the stability of a layer of an electrically conducting Oldroyd fluid[11] in the presence of a magnetic field and has found that the magnetic field has a stabilizing influence.

There are many elastico-viscous fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such class of viscoelastic fluids is Walters B′ fluid[12] having relevance and importance in geophysical fluid dynamics, chemical technology, and petroleum industry. It is well known that the Walters B' fluid [12] is characterized by the constitutive equations

$$S = -pI + \mu A_1 - \mu' A_2 + \mu^{ii} A_1^2 + \mu^{iii} A_2^2 + \mu^{iv}(A_1 A_2 + A_2 A_1) + \mu^{v}(A_1^2 A_2 + A_2 A_1^2) + \mu^{vi}(A_1 A_2^2 + A_2^2 + A_1) + \mu^{vii}(A_1^2 A_2^2 + A_2^2 A_1^2)$$

where S is the Cauchy stress tensor, 'p' is an arbitrary hydrostatic pressure, I is the unit tensor and μ's are polynomial functions of the traces of the various tensors occurring in the representation, matrices 'A1' and 'A2' are defined by

$$[A_1]_{ij} = (q_{i,j} + q_{j,i}) \qquad and$$

$$[A_2]_{ij} = \frac{\partial [A_1]_{ij}}{\partial t} + q_p [A_1]_{ij,p} + [A_1]_{ip} q_{p,j} + [A_1]_{pj} q_{p,i}$$

'q_p' being velocity vector.

On neglecting the squares and products of 'A_2', we have

$$S = -pI + \mu A_1 - \mu' A_2 + \mu^{ii} A_1^2 \qquad ,$$

where μ, μ^i *and* μ^{ii} are three material constants. It is customary to call μ , the coefficient of ordinary viscosity, μ' the coefficient of viscoelasticity *and* μ^{ii} , the coefficient of cross-viscosity. The μ, μ^i *and* μ^{ii} are general functions of temperature and material properties. For many fluids such as aqueous solution of polycrylamid and poly-isobutylene, μ, μ^i *and* μ^{ii} may be taken as constants. Such and other polymers are used in the manufacture of parts of spacecrafts, aeroplane parts, tyres, belt conveyers, ropes, cushions, seats, foams, plastics, engineering equipments, adhesives, contact lens etc. Recently, polymers are also used in agriculture, communication appliances and in biomedical applications. Walters'[13] reported that the mixture of polymethyl methacrylate and pyridine at 25 ℃ containing 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters B′ viscoelastic fluid. Polymers are used in the manufacture of spacecrafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastics engineering equipments, contact lens, etc. Walters B′ viscoelastic fluid forms the basis for the manufacture of many such important and useful products. The flow through a porous medium has been of considerable interest in recent years, particularly among geophysical fluid dynamicists. The gross effect when the fluid slowly percolates through the pores of the rock is given by Darcy's law. As a result, the usual viscous term in the equations of fluid motion is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu - \mu' \frac{\partial}{\partial t}\right)\vec{q}\right]$ for the Walters B' viscoelastic fluid, where μ and μ′ are the viscosity and viscoelasticity of the fluid, k_1 is the medium permeability and $\vec{q}$ is the Darcian (filter) velocity of the fluid. Chakraborty and Sengupta[14] have studied the flow of unsteady viscoelastic (Walters B′ liquid) conducting fluid through two porous concentric non-conducting infinite circular cylinders rotating with different angular velocities in the presence of uniform axial magnetic field. Sharma and Kumar[15] studied the stability of the plane interface separating two viscoelastic (Walters B′) superposed fluids of uniform densities. In another study, Sharma and Kumar[16] studied Rayleigh-Taylor instability of superposed conducting Walters B′ viscoelastic fluids in hydromagnetics. Kumar[17] has considered the thermal instability of a layer of Walters B′ viscoelastic fluid acted on by a uniform rotation and found that for stationary convection, rotation has a stabilizing effect. Kumar et al.[18] have considered the stability of plane interface separating the Walters B′ viscoelastic superposed fluids of uniform densities in the presence of suspended particles.

Keeping in mind the importance in various fields particularly in the soil sciences, ground water hydrology, geophysics, astrophysics and bio-mechanics, the thermosolutal convection of a Walters B' viscoelastic incompressible and heterogeneous fluid layer saturated with porous medium, where density is $\rho_0 f(z)$, ρ_0 being a positive constant having the dimension of density, and $f(z)$ is a monotonic function of the vertical coordinate z, with $f(0)=1$ has been considered in the present paper.

2. Formulation of the Problem and Basic Equations

Let us consider an infinite horizontal layer of incompressible and heterogeneous Walters B' viscoelastic fluid of thickness 'd ', in porous medium of porosity ε and medium permeability k_1, bounded by the planes $z=0$ and $z=d$. Let z-axis be vertically upwards. The interstitial fluid (which is the fluid in pores) of variable density is viscous, incompressible and heterogeneous. The initial inhomogenenity in the fluid is assumed to be of the form $\rho_0 f(z)$, where ρ_0 is the density at the lower boundary and $f(z)$ be the function of vertical co-ordinate z such that $f(0)=1$. The fluid layer is infinite in horizontal direction and is heated and soluted from below leading to an adverse temperature gradient $\beta = (T_0 - T_1)/d$ and a uniform solute gradient $\beta' = (S_0 - S_1)/d$ where T_0 and T_1 are the constant temperatures of the lower and upper boundaries with $T_0 > T_1$ and also S_0 and S_1 are the constant solute concentrations of the lower and upper surfaces with $S_0 > S_1$. The effective density is the superposition of the inhomogeneity described by (a) $\rho = \rho_0 f(z)$, and (b) $\rho = \rho_0[1 + \alpha(T_0 - T) - \alpha'(S_0 - S)]$ which is caused by temperature gradient and solute gradients. This leads to the effective density

$$\rho = \rho_0[f(z) + \alpha(T_0 - T) - \alpha'(S_0 - S)], \quad (1)$$

where α and α' are the thermal and solute expansion coefficients.

The relevant Brinkman-Oberbeck-Boussinesq equations describing our problem are:

$$\rho_0 \frac{D\vec{q}}{Dt} = -grad\ p + \rho\vec{g} + \left(\mu - \mu'\frac{\partial}{\partial t}\right)\left[\nabla^2\vec{q} - \frac{1}{k_1}\vec{q}\right], \quad (2)$$

$$div\ \vec{q} = 0, \quad (3)$$

$$\frac{\partial \rho}{\partial t} + (\vec{q}.\nabla)\rho = 0, \quad (4)$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = K\nabla^2 T, \quad (5)$$

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = K'\nabla^2 S, \quad (6)$$

where $\vec{q}$, μ, μ', ρ and p are the velocity, coefficient of viscosity, viscoelastcity, density and pressure of the fluid, T the temperature, S the solute concentration, $\vec{g}(0, 0, -g)$ is the acceleration due to gravity, K and K' are the thermal and solute diffusivities and k_1 is the intrinsic permeability of the medium ($k_1 \to \infty$ corresponds to non-porous medium).

Here in writing equations (2)-(6), porosity ε $(0 < \varepsilon < 1$ and $\varepsilon \to 1$ corresponds to non-porous medium) corrections have not been included for avoiding the involvement of too many constants. In fact it does not affect the essence of discussions of the results. Strictly speaking, a constant factor $E[= \varepsilon(1-\varepsilon)\rho_S C_S / \rho_0 C]$ multiplies in the first term of equation (4) and a term $\frac{1}{\varepsilon}$ multiplies in the velocity term except in the Darcy's resistance term $\left(-\frac{\mu}{k_1}\vec{q}\right)$. Here ρ_S and C_S are respectively the density and heat capacity of the solid material which forms the porous matrix and C is the heat capacity of the liquid. The thermal diffusivity K is defined as $K = \frac{\lambda^*}{\rho_0 C}$ where $\lambda^* = \varepsilon\lambda + (1-\varepsilon)\lambda_S$ is the effective thermal conductivity and λ and λ_S are the thermal conductivities of the fluid and solid respectively. The solute diffusivity K' is defined analogously. Also a factor E' analogous to E is multiplied in the first term of equation (5).

The initial state whose stability is to be examined is characterized by

$$\vec{q} = 0, T = T_0 - \beta z, S = S_0 - \beta' z,$$

$$\rho = \rho_0[f(z) + \alpha\beta z - \alpha'\beta' z], \ p = p_0 - \int_0^z g\rho\, dz, \quad (7)$$

where p_0 is the pressure at $\rho = \rho_0$.

Let the system be slightly disturbed and as a result of this small perturbation, the various physical quantities undergo a change

$$\vec{q} \to \vec{0} + \delta\vec{q}, T \to T + \theta, S \to S + \gamma, p \to p + \delta p$$
$$and\ \rho \to \rho_0 \begin{bmatrix} f(z) + \alpha(T_0 - T - \theta) - \\ \alpha'(S_0 - S - \gamma) \end{bmatrix} + \delta\rho \qquad (8)$$

Substituting (8) in equations (1)-(6) and linearizing them by neglecting second and higher terms and retaining only relevant terms appropriate to physical conditions, we obtain the linearized perturbations equations in component form as

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p - \left(\mu - \mu' \frac{\partial}{\partial t}\right)\left[\frac{1}{k_1}u - \nabla^2 u\right], \qquad (9)$$

$$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p - \left(\mu - \mu' \frac{\partial}{\partial t}\right)\left[\frac{1}{k_1}v - \nabla^2 v\right], \qquad (10)$$

$$\rho_0 \frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p - g(\delta\rho - \alpha\rho_0\theta + \alpha'\rho_0\gamma) - \left(\mu - \mu' \frac{\partial}{\partial t}\right)\left[\frac{1}{k_1}w - \nabla^2 w\right], \qquad (11)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \qquad (12)$$

$$\frac{\partial}{\partial x}\delta\rho + \rho_0 w \frac{df}{dz} = 0, \qquad (13)$$

$$\frac{\partial \theta}{\partial t} - \beta w = K\nabla^2\theta, \qquad (14)$$

$$\frac{\partial \gamma}{\partial t} - \beta' w = K'\nabla^2\gamma, \qquad (15)$$

$$\text{where } \delta\vec{q} = (u, v, w). \qquad (16)$$

3. Analysis in Terms of Normal Modes

The analysis of an arbitrary disturbance is carried out in terms of normal modes following Chandrasekhar[1]. The stability of each of the modes is discussed separately. We seek solutions of the equations (9)-(15) whose dependence on space-time coordinates are of the form

$$[u, v, w, \theta, \gamma, \delta p, \delta\rho] = [U(z), V(z), W(z), \Theta(z), \Gamma(z), L(z), Y(z)] \exp[ik_x x + ik_y y + nt], \qquad (17)$$

where k_x and k_y are the horizontal wave numbers and n is the frequency of the harmonic disturbances. Also

$$k = \sqrt{(k_x^2 + k_y^2)}, \qquad (18)$$

gives the wave number of the perturbation propagation.

Using expression (17), equations (9)-(15), on simplification, give

$$-k^2 L = \rho_0 \begin{bmatrix} \left(n + \frac{\nu - \nu' n}{k_1}\right) - \\ (\nu - \nu' n)(D^2 - k^2) \end{bmatrix} D W, \qquad (19)$$

$$\rho_0\left(n + \frac{\nu - \nu' n}{k_1}\right)W = -D L + g\rho_0 \left[\frac{1}{n}\left(\frac{df}{dz}\right)W + \alpha\Theta - \alpha'\Gamma\right] + \rho_0(\nu - \nu' n)(D^2 - k^2)W, \qquad (20)$$

$$n\Theta - \beta W = K(D^2 - k^2)\Theta, \qquad (21)$$

$$n\Gamma - \beta' W = K'(D^2 - k^2)\Gamma, \qquad (22)$$

where $\nu \left(= \frac{\mu}{\rho_0}\right)$ and $\nu' \left(= \frac{\mu'}{\rho_0}\right)$ are respectively the kinematic viscosity and kinematic viscoelasticity.

Elimination of L from equations (19) and (20) gives

$$n\left[\left(n + \frac{\nu - \nu' n}{k_1}\right) - (\nu - \nu' n)(D^2 - k^2)\right](D^2 - k^2)W + gk^2\left(\frac{df}{dz}\right)W + g\alpha n k^2\Theta - g\alpha' n k^2\Gamma = 0. \qquad (23)$$

Equations (21)-(23) in non-dimensional form can be written as

$$[D^2 - a^2 - \sigma p_1]\Theta = -\left(\frac{\beta d^2}{K}\right)W, \qquad (24)$$

$$[\tau(D^2 - a^2) - \sigma p_1]\Gamma = -\left(\frac{\beta' d^2}{K'}\right)W, \qquad (25)$$

$$\sigma\nu(D^2 - a^2)[(1 - A\sigma)(D^2 - a^2) - (\sigma + B - B'A\sigma)]W - \frac{a^2 g d^4}{\nu}\left(\frac{df}{dz}\right)W - g\alpha\sigma\, a^2 d^2\Theta + g\alpha'\sigma\, a^2 d^2\Gamma = 0, \qquad (26)$$

here we have put $\hat{D} = dD, \hat{a} = kd, \hat{\sigma} = \frac{nd^2}{\nu}$ and thereafter dropping the caps for convenience. Also we have put

$$p_1 = \frac{\nu}{K}, \tau = \frac{K'}{K}, A = \frac{\nu'}{d^2}, B = \frac{d^2}{k_1}, R = \frac{g\alpha\beta d^4}{K\nu},$$
$$R' = \frac{g\alpha'\beta' d^4}{K'\nu}, R_2 = \frac{gd^4\left(\frac{df}{dz}\right)}{K\nu}. \qquad (27)$$

Equation (26) with the help of equations (24) and (25) is written as

$$\sigma p_1\left(D^2-a^2\right)\left[D^2-a^2-\sigma p_1\right]\begin{bmatrix}\tau\left(D^2-a^2\right)\\ -\sigma p_1\end{bmatrix}$$
$$\left[(1-A\sigma)\left(D^2-a^2\right)-(\sigma+B-B'A\sigma)\right]W$$
$$-a^2R_2\left[D^2-a^2-\sigma p_1\right]\left[\tau\left(D^2-a^2\right)-\sigma p_1\right]W \quad (28)$$
$$+a^2\sigma p_1 R\left[\tau\left(D^2-a^2\right)-\sigma p_1\right]W-a^2\sigma p_1 R'$$
$$\left[D^2-a^2-\sigma p_1\right]W=0.$$

The equations (24)-(26) and (28) are to be solved using boundary conditions. Here we consider the case where both the boundaries are free, following Chandrasekhar[1], the appropriate boundary conditions for this case are $W = D^2W = 0, \Theta = 0, \Gamma = 0$ at $z = 0$ and $z = 1$. (29)

4. Results and Discussion of Marginal States

(I) Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Hence the substitution of $\sigma = 0$ in equations (21)-(23) gives

$$\left.\begin{aligned}\left(D^2-a^2\right)\Theta &= -\left(\frac{\beta d^2}{K}\right)W\\ \tau\left(D^2-a^2\right)\Gamma &= -\left(\frac{\beta' d^2}{K'}\right)W\\ \left(\frac{df}{dz}\right)W &= 0.\end{aligned}\right\} \quad (30)$$

Integrating equation (30) and using the boundary conditions (29), we see that $W = 0, \Theta = 0, \Gamma = 0$ etc. are the only possible solutions which led to contradiction to the hypothesis that initial state solutions are perturbed.

Therefore, the instability can not set in as stationary convection or in the other words the principle of exchange of stabilities is not valid for our problem.

(II) Oscillatory Convection

Now for the proper solution of equation (28) for W belonging to the lowest mode, we follow Chandrasekhar[1], and find that $W = W_0 \sin \pi z$, where W_0 is constant. Then, from equation (28), we get

$$a^2R-\frac{a^2R'\left[\pi^2+a^2+\sigma p_1\right]}{\left[\tau\left(\pi^2+a^2\right)+\sigma p_1\right]}+\frac{a^2R_2\left[\pi^2+a^2+\sigma p_1\right]}{\sigma p_1}$$
$$=\left[\pi^2+a^2\right]\left[\pi^2+a^2+\sigma p_1\right]$$
$$\left[(1-A\sigma)\left(\pi^2+a^2\right)+\left(\sigma+B-B'A\sigma\right)\right]. \quad (31)$$

As discussed earlier, the principle of exchange of stabilities being not valid for the present problem, the marginal state is governed by $\sigma = i\sigma_2'$ where σ_2' is real. Now letting

$$R_3=\frac{R_2}{\pi^4},\ R_1=\frac{R}{\pi^4},\ R_4=\frac{R'}{\pi^4},\ x=\frac{a^2}{\pi^2},$$
$$\sigma_2=\frac{\sigma_2'}{\pi^2}\ and\ B_1=\frac{B}{\pi^2}. \quad (32)$$

Substituting (32) in equation (31), we get

$$xR_1-\frac{xR_4\left[1+x+i\sigma_2 p_1\right]}{\left[\tau(1+x)+i\sigma_2 p_1\right]}+\frac{xR_3\left[1+x+i\sigma_2 p_1\right]}{i\sigma_2 p_1}$$
$$=[1+x]\left[1+x+i\sigma_2 p_1\right]\begin{bmatrix}\left(1-iA\pi^2\sigma_2\right)(1+x)\\ +\left(i\sigma_2+B_1+B_2 i\pi^2\sigma_2\right)\end{bmatrix} \quad (33)$$

Separating equation (33) in real and imaginary parts, we obtain

$$R_1=\frac{1}{x}\begin{bmatrix}(1+x)^2\left\{1+x+B_1+\sigma_2^2 p_1 A\pi^2\right\}-\\ (1+x)\sigma_2^2 p_1\left\{1+B_2\pi^2\right\}-xR_3\\ +\dfrac{xR_4\left\{\tau(1+x)^2+\sigma_2^2 p_1^2\right\}}{\left\{\tau^2(1+x)^2+\sigma_2^2 p_1^2\right\}}\end{bmatrix}, \quad (34)$$

and

$$A_0\sigma_2^4+A_1\sigma_2^2+A_2=0, \quad (35)$$

where

$$\left.\begin{aligned}A_0&=p_1^3(1+x)\begin{Bmatrix}-\pi^2A(1+x)^2\\ +(1+x)\left(1+B_2\pi^2+p_1\right)+p_1B_1\end{Bmatrix}\\ A_1&=\tau^2p_1(1+x)^3\begin{Bmatrix}-\pi^2A(1+x)^2+(1+x)\\ +B_2\pi^2(1+x)+p_1(1+x)\\ +B_1p_1(1+x)\end{Bmatrix}\\ &\quad+xp_1^2(1+x)\left\{R_4(\tau-1)+R_3\right\}\\ A_2&=xR_3(1+x)^3\tau^2\end{aligned}\right\}. \quad (36)$$

From (35) and (36), the frequency of oscillations σ_2 in marginal state is given by

$$\sigma_2^2 = \frac{-A_1 + \sqrt{\left(A_1^2 - 4A_0A_2\right)}}{2A_0}, \qquad (37)$$

and from (27) and (34), Rayleigh number R is given by

$$R = \pi^4 R_1$$

$$= \pi^4 \left[\frac{1}{x} \left\{ \begin{array}{l} (1+x)^2 \begin{pmatrix} 1+x+B_1 \\ +\sigma_2^2 p_1 A \pi^2 \end{pmatrix} \\ -(1+x)\sigma_2^2 p_1 \left(1+B_1\pi^2\right) \end{array} \right\} \\ -R_3 + R_4 \frac{\left\{\tau(1+x)^2 + \sigma_2^2 p_1^2\right\}}{\left\{\tau^2(1+x)^2 + \sigma_2^2 p_1^2\right\}} \right] \qquad (38)$$

We now discuss the existence of overstable marginal states for various cases:

Case (A): When $R_3 > 0$, i.e. $\frac{df}{dz} > 0$

Since $R_3 > 0$ thus it implies $A_2 > 0$, therefore, if $A_0 > 0$ and $A_1 > 0$ i.e. $1 > \pi^2 A(1+x)$ and $\tau - 1 > 0$ i.e. $1 > \pi^2 A(1+x)$ and $K' > K$, then there will be no real σ_2 resulting non-occurrence of overstable marginal state. But, if R_4 satisfies the inequality

$$R_4 > \frac{1}{(1-\tau)} \left[\frac{\tau^2(1+x)^2}{xp_1} \left\{ \begin{array}{l} \left[1-\pi^2 A(1+x)\right] \\ + B_2\pi^2 + p_1(1+B_1) \end{array} \right\} \\ + R_3 \right], \qquad (39)$$

besides $K' < K$, $A > \frac{1}{\pi^2(1+x)}$ and $A_1^2 - 4A_0A_2 > 0$, then the marginal state may exist even when $R_3 > 0$.

Case (B): When $R_3 < 0$, i.e. $\frac{df}{dz} < 0$

When $R_3 < 0$, the marginal state always exist whatever be the values of other parameters provided $A_1^2 - 4A_0A_2 > 0$ and then σ_2 is given by equation (37).

(III) Nature of Non-Oscillatory Modes

For $R_3 > 0$ i.e. $R_2 > 0$ and $K' > K$, the only modes that may exist are non-oscillatory modes for which $\sigma_2 = 0$ and $\sigma = \sigma_1$ (σ_1 is real). Hence substitution of $\sigma = \sigma_1$ and $W = W_0 \sin \pi z$ in equation (28) gives

$$D_0\sigma_1^4 + D_1\sigma_1^3 + D_2\sigma_1^2 + D_3\sigma_1 + D_4 = 0, \qquad (40)$$

where

$$\left.\begin{array}{l} D_0 = p_1^3\left(\pi^2+a^2\right)\left[-A\left(\pi^2+a^2\right)+1-B'A\right] \\ D_1 = p_1^2\left(\pi^2+a^2\right)^2\left[-A\left(\pi^2+a^2\right)+1+B'A\right] \\ \qquad (1+\tau) + p_1^3\left(\pi^2+a^2\right)\left[\left(\pi^2+a^2\right)+B\right] \\ D_2 = p_1\left(\pi^2+a^2\right)^3\left[\begin{array}{l} \tau\left\{-A\left(\pi^2+a^2\right)+1-B'A\right\} \\ + p_1(1+\tau) \end{array}\right] \\ \qquad + p_1^2\left(\pi^2+a^2\right)^2 B(1+\tau) - a^2 p_1^2 (R_2 + R - R') \\ D_3 = p_1\tau\left(\pi^2+a^2\right)^3\left\{\left(\pi^2+a^2\right)+B\right\} \\ \qquad - a^2 p_1\left(\pi^2+a^2\right)\left\{R_2\tau + R\tau - R'\right\} - a^2 R_2 \tau\, p_1 \\ D_4 = -a^2\left(\pi^2+a^2\right)R_2\tau^2 \end{array}\right\} \qquad (41)$$

Equation (40) is the fourth degree characteristic equation in σ_1 with real coefficients and has four roots, which may be real. The constant term in the characteristic equation being negative, at least two of the roots are real, one positive and one negative. Thus, we have non-oscillatory modes, one of which essentially grows in time making the system unstable.

5. Conclusions

The thermosolutal convection in a layer of heterogeneous Walters B' viscoelastic fluid heated and soluted from below through porous medium is considered in the present paper. The investigation of thermosolutal convection is motivated by its complexities as a double diffusion phenomena as well as its direct relevance to geophysics and astrophysics. Thermosolutal convection problems arise in oceanography, limnology and engineering. Ponds built to trap solar heat and some Antarctic lakes provide examples of particular interest. The main conclusions from the analysis of this paper are as follows:

- The principle of exchange of stabilities is not valid for this problem.
- Frequency of oscillation and the Rayleigh number in the marginal state are given by equations (35) and (34), respectively.
- For density distribution with positive gradient and $1 > \pi^2 A(1+x)$; $K' > K$, the overstable marginal state do not exist and we have only non-oscillatory modes which make the system unstable.
- While for positive density gradient and $K' < K$; $A > \frac{1}{\pi^2(1+x)}$, the overstability may occur for the solute Rayleigh numbers satisfying (39).
- For density distribution with negative gradient, the marginal state and overstable solution exist, irrespective of the values of other parameters.

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