American Journal of Computational and Applied Mathematics: 2011; 1(2): 41-47
DOI: 10.5923/j.ajcam.20110102.08

Application of Multiple Exp-Function Method to Obtain Multi-Soliton Solutions of (2 + 1)- and (3 + 1)-Dimensional Breaking Soliton Equations

M. T. Darvishi[1,*], Maliheh Najafi[2], Mohammad Najafi[2]

[1]Department of Mathematics, Razi University, Kermanshah, 67149, Iran
[2]Young Researchers Club, Kermanshah Branch, Islamic Azad University, Kermanshah, Iran

Abstract The multiple exp-function method is a new approach to obtain multiple wave solutions of nonlinear partial differential equations (NLPDEs). By this method one can obtain multi-soliton solutions of NLPDEs. In this paper, using computer algebra systems, we apply the multiple exp-function method to construct the exact multiple wave solutions of the (2 + 1)- and the (3 + 1)-dimensional breaking soliton equations. By this application, we obtain one-wave, two-wave and three-wave solutions for these equations.

Keywords Multiple Exp-Function Method, Breaking Soliton Equation, Exact Solution, Multi-Soliton Solution

1. Introduction

The study of exact solutions of nonlinear partial differential equations plays an important role in soliton theory and explicit formulas of nonlinear partial differential equations play an essential role in the nonlinear science. Also, the explicit formulas may provide physical information and help us to understand the mechanism of related physical models. Recently, many kinds of powerful methods have been proposed to find exact solutions of nonlinear partial differential equations, e.g., the tanh-method[1], the homogeneous balance method[2], homotopy analysis method[3-8], the $F-$expansion method [9], three-wave method[10-13], extended homoclinic test approach[14-16], the $(\frac{G'}{G})-$expansion method[17] and the exp-function method[18-23].

By these analytic methods, one only can obtain traveling wave solutions for NLPDEs. However, it is known that there are multiple wave solutions to some NLPDEs, for example, multi-soliton solutions to KdV and Toda lattice equations[24] or multiple periodic wave solutions to the Hirota bilinear equations[25,26]. Recently, Ma and Fan[27] explored a key feature of the linear superposition principle that linear equations possess, for Hirota bilinear equations, while aiming to construct a specific sub-class of N-soliton solutions formed by linear combinations of exponential traveling waves. They proved that, a linear superposition principle can apply to exponential traveling waves of Hirota bilinear equations. Applications made to show that the presented linear superposition principle is helpful in generating N-wave solutions to soliton equations, particularly those in higher dimensions. Also, Ma et al.[28] constructed the multiple exp-function method to obtain multiple wave solutions, including one-soliton, two-soliton and three-soliton type solutions to (3+1)-dimensional YTSF equation. One may find another works to find exact solutions of soliton equations in[29-31].

In this paper, we apply the multiple exp-function method to obtain some exact multiple wave solutions for the (2 + 1)- and (3 + 1)-dimensional breaking soliton equations.

The (2+1)-dimensional breaking soliton equation is

$$u_{xt}-4u_{xy}u_x-2u_{xx}u_y-u_{xxxy}=0 \tag{1}$$

this equation describes the (2+1)-dimensional interaction of the Riemann wave propagated along the y-axis with a long wave propagated along the x-axis[32]. Wazwaz[33] introduced an extension to equation (1) by adding the last three terms with y replaced by z. His work, enables us to establish the following (3+1)-dimensional breaking soliton equation

$$\begin{aligned} u_{xt}-4u_x(u_{xy}+u_{xz})-2u_{xx}(u_y+u_z) \\ -(u_{xxxy}+u_{xxxz})=0 \end{aligned} \tag{2}$$

where $u=u(x,y,z,t):\mathbb{R}_x\times\mathbb{R}_y\times\mathbb{R}_z\times\mathbb{R}_t\to\mathbb{R}$

This paper is organized as follows: in the following section we have a brief review on multiple exp-function method. In Sections 3 and 4 we apply the exp-function method on the (2 + 1)-dimensional and the (3 + 1)-dimensional breaking soliton equations, respectively. In

* Corresponding author:
darvishimt@yahoo.com (Mohammad Taghi Darvishi)
Published online at http://journal.sapub.org/ajcam

those sections we obtain one-wave, two-wave and three-wave solutions for our equations. The paper is concluded in Section 4.

2. Methodology

Ma et al.[28] illustrated the multiple exp-function method in three steps. In summary, their steps are: Defining solvable differential equations, transforming nonlinear PDEs and solving algebraic systems. The key point of their approach is to seek rational solutions in a set of new variables defining individual waves. The application of multiple exp-function method yields specific one-wave, two-wave and three-wave solutions to NLPDEs. To explain these fundamental steps in multiple exp-function method, consider a PDE in (1 + 1) dimensions as

$$F(x,t,u_x,u_t,\dots)=0 \tag{3}$$

In the first step, we must introduce a sequence of new variables $\eta_i=\eta_i(x,t), i=1,\dots,n$, by solvable partial differential equations, for example, the linear PDEs,

$$\eta_{i,x}=k_i\eta_i,\quad \eta_{i,t}=-\omega_i\eta_i,\quad i=1,\dots,n, \tag{4}$$

where $k_i, i=1,\dots,n,$ are the angular wave numbers and $\omega_i, i=1,\dots,n$, are the wave frequencies. This step is a starting point to construct exact solutions of nonlinear equations. Solving these linear equations leads to the exponential function solutions,

$$\eta_i=c_ie^{\xi_i},\quad \xi_i=k_ix-\omega_it,\quad i=1,\dots,n, \tag{5}$$

where $c_i, i=1,\dots n,$ are arbitrary constants. Any function η_i describing a single wave and a multiple wave solution will be a combination of all of those single waves. In the second step, we proceed by considering rational solutions in the new variables $\eta_i, i=1,\dots,n$:

$$u(x,t)=\frac{p(\eta_1,\eta_2,\dots,\eta_n)}{q(\eta_1,\eta_2,\dots,\eta_n)},\quad p=\sum_{r,s=1}^{n}\sum_{i,j=0}^{M}p_{rs,ij}\,\eta_r^i\eta_s^j,\quad q=\sum_{r,s=1}^{n}\sum_{i,j=0}^{N}q_{rs,ij}\,\eta_r^i\eta_s^j, \tag{6}$$

where $p_{kl,ij}$ and $q_{kl,ij}$ are constants to be determined from (3). Then we use the differential relations in (4) to express all partial derivatives of u with x and t in terms of $\eta_i, i=1,\dots,n$. Substituting these obtained expressions for partial derivatives into the equation (3) generates a rational function equation in the new variables $\eta_i, i=1,\dots,n$:

$$G(x,t,\eta_1,\dots,\eta_n)=0 \tag{7}$$

Equation (7) is called the transformed equation of the original equation (3). This step makes it possible to compute solutions to differential equations directly by computer algebra systems in the third step. Finally, in the third step we set the numerator of the resulting rational function $G(x,t,\eta_1,\dots,\eta_n)$ to zero. This yields a system of algebraic equations on all variables $k_i,\omega_i,p_{kl,ij},p_{kl,ij}$. By solving this system, with the aid of a mathematical software such as Maple, we determine polynomials p and q and the wave exponents $\xi_i, i=\dots,n$. After this, the multiple wave solution u is computed and given by

$$u(x,t)=\frac{p(c_1e^{k_1x-\omega_1t},\dots,c_ne^{k_nx-\omega_nt})}{q(c_1e^{k_1x-\omega_1t},\dots,c_ne^{k_nx-\omega_nt})} \tag{8}$$

for more details on these steps; cf.[28].

In the following sections we apply multiple exp-function method on the (2 + 1)- and the (3 + 1)-dimensional breaking soliton equations to obtain some multi-wave solutions. It must be noted that all obtained solutions are new ones and for some special values of parameters in these new solutions we can obtain the solutions which have obtained by another methods.

3. One-Wave, Two-Wave and Three-Wave Solutions to the (2 + 1) -Dimensional Breaking Soliton Equation

In this part, we apply the multiple exp-function method to construct the exact multiple wave solutions of the (2 + 1) -dimensional breaking soliton equation

$$u_{xt}-4u_{xy}u_x-2u_{xx}u_y-u_{xxxy}=0 \tag{9}$$

We generate one-wave, two-wave and three-wave solutions of two polynomial functions p and q as follows:

Case i: One-wave solutions

To obtain one-wave solutions, we require the linear conditions

$$\eta_{1,x}=k_1\eta_1,\quad \eta_{1,y}=l_1\eta_1,\quad \eta_{1,t}=-w_1\eta_1 \tag{10}$$

where k_1,l_1 and w_1 are constants which will be determined. Then we try a pair of polynomials of degree one,

$$p(\eta_1)=a_0+a_1\eta_1,\quad q(\eta_1)=b_0+b_1\eta_1 \tag{11}$$

where a_0,a_1,b_0 and b_1 are constants to be determined. By the multiple exp-function method and using the differential relations in (10), we obtain the following solution to the resulting algebraic system with the aid of Maple,

$$a_1=\frac{b_1\left(a_0+2k_1b_0\right)}{b_0},\quad w_1=-k_1^2l_1 \tag{12}$$

and a_0,a_1,b_0 are arbitrary. Thus we can have an exponential function solution to (10),

$$\eta_1=\mathrm{e}^{k_1x+l_1y-w_1t} \tag{13}$$

which gets the following one-wave solutions

$$u=u(x,y,t)=\frac{p}{q}=\frac{a_0+a_1\mathrm{e}^{k_1x+l_1y-w_1t}}{b_0+b_1\mathrm{e}^{k_1x+l_1y-w_1t}} \tag{14}$$

where a_0,a_1 and b_0 are arbitrary constants. It must be noted that, if we set $a_0 = 0, b_0 = 1, b_1 = 1$ and $l_1 = r_1$, the solution (19) of[33] will obtain.

Case ii: Two-wave solutions

In this case, we require the linear conditions,

$$\eta_{i,x} = k_i \eta_i, \quad \eta_{i,y} = l_i \eta_i, \quad \eta_{i,t} = -w_i \eta_i, \quad i = 1,2, \tag{15}$$

where k_i, l_i, w_i, $i = 1,2$, are constants and thus the solutions η_1 and η_2 are defined by

$$\eta_i = \mathrm{e}^{k_i x + l_i y - w_i t}, \quad i = 1,2, \tag{16}$$

where c_1 and c_2 are arbitrary constants.

We suppose that

$$\begin{aligned} p(\eta_1, \eta_2) &= 2[k_1\eta_1 + k_2\eta_2 + a_{12}(k_1 + k_2)\eta_1\eta_2], \\ q(\eta_1, \eta_2) &= 1 + \eta_1 + \eta_2 + a_{12}\eta_1\eta_2 \end{aligned} \tag{17}$$

where a_{12} is a constant to be determined. By the multiple exp-function method and using the differential relations in (15), we obtain the following solution to the resulting algebraic system with the aid of Maple,

$$w_i = -k_i^2 l_i, \quad i = 1,2, \tag{18}$$

and

$$a_{12} = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} \tag{19}$$

Then, we obtain the following two-wave solutions

$$\begin{aligned} u = u(x,y,t) &= \frac{p(\eta_1, \eta_2)}{q(\eta_1, \eta_2)} \\ &= \frac{2[k_1\eta_1 + k_2\eta_2 + a_{12}(k_1 + k_2)\eta_1\eta_2]}{1 + \eta_1 + \eta_2 + a_{12}\eta_1\eta_2} \end{aligned} \tag{20}$$

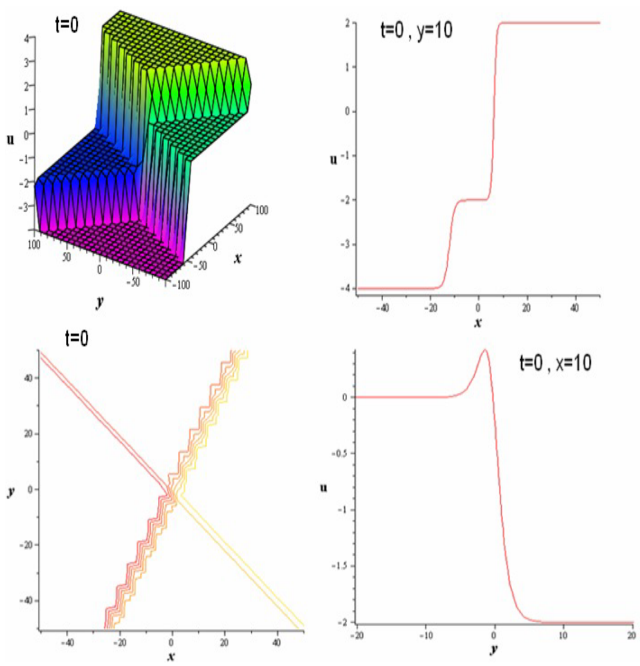

Figure 1. The two-wave solution

For $l_i = r_i, i = 1,2$, solution (20) is as same as solution (24) of[33].

We obtained a wide class of traveling wave solutions to Eq. (9) by setting special values to the arbitrary parameters, we can construct formal two-wave solutions. For instance, taking $k_1 = 1, k_2 = -2, l_1 = 1, l_2 = 1, c_1 = 1$ and $c_2 = 1$ in (20) reveals solitary waves to Eq. (9). Figure 1 shows the plot of solution (20) for these special values of its parameters.

Case iii: Three-wave solutions

Similarly, to obtain the three-wave solutions for equation (9), we require the linear conditions,

$$\eta_{i,x} = k_i \eta_i, \eta_{i,y} = l_i \eta_i, \eta_{i,t} = -w_i \eta_i, i = 1,2,3, \tag{21}$$

where k_i, l_i, w_i, $i = 1,2,3$, are constants and thus the solutions η_1, η_2 and η_3 can be defined by

$$\eta_i = \mathrm{e}^{k_i x + l_i y - w_i t}, \quad i = 1,2,3, \tag{22}$$

where c_1, c_2 and c_3 are arbitrary constants.We suppose that

$$\begin{aligned} p(\eta_1, \eta_2) = 2[&k_1\eta_1 + k_2\eta_2 + k_3\eta_3 + a_{12}(k_1 + k_2)\eta_1\eta_2 \\ &+ a_{13}(k_1 + k_3)\eta_1\eta_3 + a_{23}(k_2 + k_3)\eta_2\eta_3 \\ &+ a_{12}a_{13}a_{23}(k_1 + k_2 + k_3)\eta_1\eta_2\eta_3] \\ q(\eta_1, \eta_2) = 1 &+ \eta_1 + \eta_2 + \eta_3 + a_{12}\eta_1\eta_2 + a_{13}\eta_1\eta_3 \\ &+ a_{23}\eta_2\eta_3 + a_{12}a_{13}a_{23}\eta_1\eta_2\eta_3, \end{aligned}$$

where a_{12}, a_{13} and a_{23} are constants to be determined. By the multiple exp-function method and using the differential relations in (21), we obtain the following solution to the resulting algebraic system with Maple,

$$w_i = -k_i^2 l_i, \quad i = 1,2,3, \tag{24}$$

and

$$a_{ij} = \frac{(k_i - k_j)^2}{(k_i + k_j)^2}, \quad i,j = 1,2,3. \tag{25}$$

Then, we obtain the following three-wave solutions

$$u = u(x,y,t) = \frac{p(\eta_1, \eta_2, \eta_3)}{q(\eta_1, \eta_2, \eta_3)} \tag{26}$$

where p and q are defined by (23) and $\eta_i, i = 1,2,3$ are defined by (22). For $l_i = r_i, i = 1,2,3$ in (26), we meet solution (26) of [32].

Figure 2 shows the plot of solution (26) for some special values of its parameters.

4. One-Wave, Two-Wave and Three-Wave Solutions to the (3 + 1) -Dimensional Breaking Soliton Equation

In this section, we will apply the multiple exp-function method to construct the exact multiple wave solutions for the (3 + 1)-dimensional breaking soliton equation

$$\begin{aligned} u_{xt} - 4u_x(u_{xy} + u_{xz}) - 2u_{xx}(u_y + u_z) \\ - (u_{xxxy} + u_{xxxz}) = 0, \end{aligned} \tag{27}$$

we discuss three cases of two polynomial functions p and q to generate one-wave, two-wave and three-wave solutions as follows:

Case i: One-wave solutions

Again, we require the linear conditions,

$$\eta_{1,x} = k_1\eta_1, \eta_{1,y} = l_1\eta_1, \eta_{1,z} = m_1\eta_1, \eta_{1,t} = -w_1\eta_1 \quad (28)$$

where k_1, l_1, m_1, w_1 are constants. We then try a pair of polynomials of degree one,

$$p(\eta_1) = a_0 + a_1\eta_1, \quad q(\eta_1) = b_0 + b_1\eta_1 \quad (29)$$

where a_0, a_1, b_0 and b_1 are constants to be determined. By the multiple exp-function method and using the differential relations in (28), we obtain the following solution to the resulting algebraic system with the help of Maple,

$$a_1 = \frac{b_1\left(a_0 + 2k_1b_0\right)}{b_0}, \quad w_1 = -\left(m_1 + l_1\right)k_1^2 \quad (30)$$

where a_0, a_1 and b_0 are arbitrary constants. Since we can have an exponential function solution to (28),

$$\eta_1 = \mathrm{e}^{k_1x + l_1y + m_1z - w_1t} \quad (31)$$

By using (31), we obtain the following one-wave solutions for equation (27)

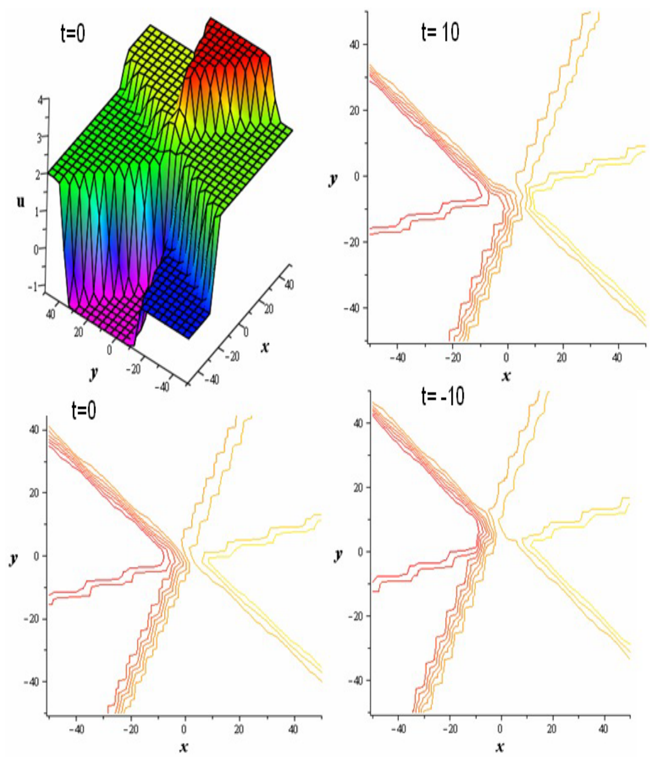

Figure 2. The three-wave solution

$$u = u(x, y, t) = \frac{p}{q} = \frac{a_0 + a_1\mathrm{e}^{k_1x + l_1y + m_1z - w_1t}}{b_0 + b_1\mathrm{e}^{k_1x + l_1y + m_1z - w_1t}} \quad (32)$$

where a_0, a_1 and b_0 are arbitrary constants. For $a_0 = 0$, $b_0 = 1$, $b_1 = -1$, $l_1 = r_1$ and $m_1 = s_1$ our solution (32) is the same solution (49) of [33].

Case ii: Two-wave solutions

In this case, we require the linear conditions,

$$\begin{aligned} &\eta_{i,x} = k_i\eta_i, \eta_{i,y} = l_i\eta_i, \eta_{i,z} = m_i\eta_i, \\ &\eta_{i,t} = -w_i\eta_i, \quad i = 1, 2. \end{aligned} \quad (33)$$

where $k_i, l_i, m_i, w_i, i = 1, 2$, are constants and thus the solutions η_1 and η_2 can be defined by

$$\eta_i = \mathrm{e}^{k_ix + l_iy + m_iz - w_it}, \quad i = 1, 2, \quad (34)$$

where c_1 and c_2 are arbitrary constants.

We suppose that

$$\begin{aligned} &p(\eta_1, \eta_2) = 2[k_1\eta_1 + k_2\eta_2 + a_{12}(k_1 + k_2)\eta_1\eta_2], \\ &q(\eta_1, \eta_2) = 1 + \eta_1 + \eta_2 + a_{12}\eta_1\eta_2, \end{aligned} \quad (35)$$

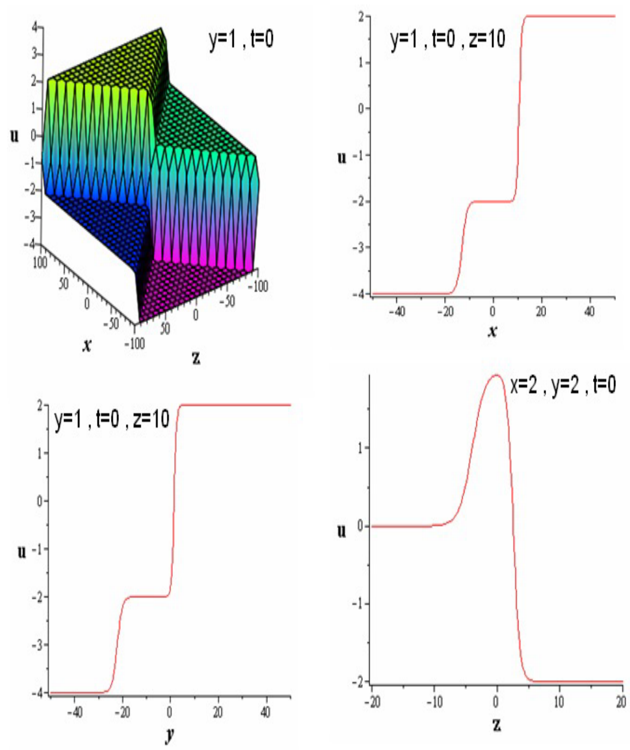

Figure 3. The first two-wave solution

where a_{12} is a constant to be determined. By the multiple exp-function method and using the differential relations in (33), we obtain two solutions to the resulting algebraic system with Maple,

$$w_i = -k_i^3 - k_i^2m_i, \quad i = 1, 2, \quad (36)$$

and

$$a_{12} = \frac{\left(k_1 - k_2\right)^2}{\left(k_1 + k_2\right)^2} \quad (37)$$

when $l_i = k_i, i = 1, 2$, and

$$w_i = -k_i^3 - k_i^2l_i, \quad i = 1, 2, \quad (38)$$

and

$$a_{12} = \frac{\left(k_1 - k_2\right)^2}{\left(k_1 + k_2\right)^2} \quad (39)$$

when $m_i = k_i, i = 1, 2,$ we obtain the following two-wave solutions

$$\begin{aligned} u = u(x, y, t) &= \frac{p(\eta_1, \eta_2)}{q(\eta_1, \eta_2)} \\ &= \frac{2[k_1\eta_1 + k_2\eta_2 + a_{12}(k_1 + k_2)\eta_1\eta_2]}{1 + \eta_1 + \eta_2 + a_{12}\eta_1\eta_2} \end{aligned} \quad (40)$$

Two specific solutions of these two-wave solutions are plotted in Figures 3 and 4.

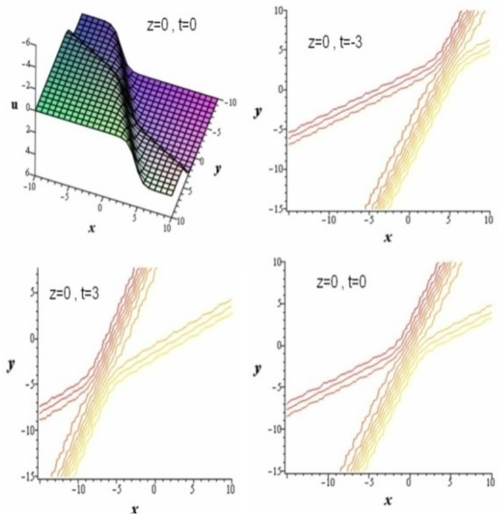

Figure 4. The second two-wave solution

Case iii: Three-wave solutions

To obtain the three-wave solutions for equation (27), we require the linear conditions,

$$\eta_{i,x} = k_i\eta_i, \quad \eta_{i,y} = l_i\eta_i, \quad \eta_{i,z} = m_i\eta_i, \quad \eta_{i,t} = -w_i\eta_i, \quad i = 1,2,3, \tag{41}$$

where k_i, l_i, w_i, $i = 1,2,3$, are constants and thus the solutions η_1, η_2 and η_3 can be defined by

$$\eta_i = \mathrm{e}^{k_i x + l_i y + m_i z - w_i t}, \quad i = 1,2,3, \tag{42}$$

where c_1, c_2 and c_3 are arbitrary constants.

We suppose that

$$\begin{aligned} p(\eta_1,\eta_2) &= 2[k_1\eta_1 + k_2\eta_2 + k_3\eta_3 + a_{12}(k_1+k_2)\eta_1\eta_2 \\ &\quad + a_{13}(k_1+k_3)\eta_1\eta_3 + a_{23}(k_2+k_3)\eta_2\eta_3 \\ &\quad + a_{12}a_{13}a_{23}(k_1+k_2+k_3)\eta_1\eta_2\eta_3] \\ q(\eta_1,\eta_2) &= 1 + \eta_1 + \eta_2 + \eta_3 + a_{12}\eta_1\eta_2 + a_{13}\eta_1\eta_3 \\ &\quad + a_{23}\eta_2\eta_3 + a_{12}a_{13}a_{23}\eta_1\eta_2\eta_3, \end{aligned} \tag{43}$$

where a_{12}, a_{13} and a_{23} are constants to be determined. By the multiple exp-function method and using the differential relations in (41), we obtain the following solution to the resulting algebraic system with the help of Maple,

$$w_i = -k_i^3 - k_i^2 m_i, \quad i = 1,2,3, \tag{44}$$

and

$$a_{ij} = \frac{\left(k_i - k_j\right)^2}{\left(k_i + k_j\right)^2}, \quad i,j = 1,2,3 \tag{45}$$

when $l_i = k_i, i = 1,2,3$, and

$$w_i = -k_i^3 - k_i^2 l_i, \quad i = 1,2,3, \tag{46}$$

and

$$a_{ij} = \frac{\left(k_i - k_j\right)^2}{\left(k_i + k_j\right)^2}, \quad i,j = 1,2,3 \tag{47}$$

when $m_i = k_i, i = 1,2,3$. Then, we obtain the following three-wave solutions for equation (27)

$$\begin{aligned} u = u(x,y,t) &= \frac{p(\eta_1,\eta_2)}{q(\eta_1,\eta_2)} \\ &= \frac{2[k_1\eta_1 + k_2\eta_2 + a_{12}(k_1+k_2)\eta_1\eta_2]}{1+\eta_1+\eta_2+a_{12}\eta_1\eta_2} \end{aligned} \tag{48}$$

Two specific solutions of these three-wave solutions are plotted in Figures 5 and 6.

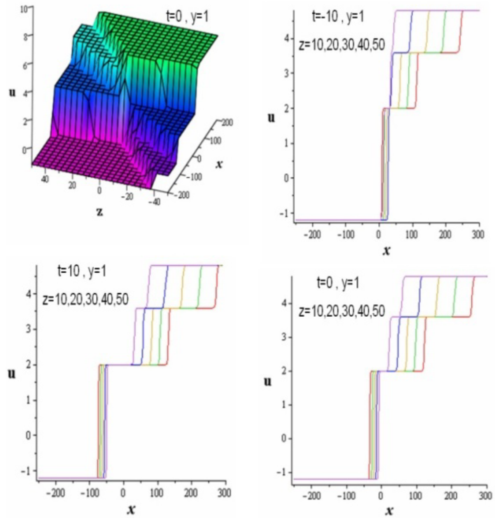

Figure 5. The first three-wave solution

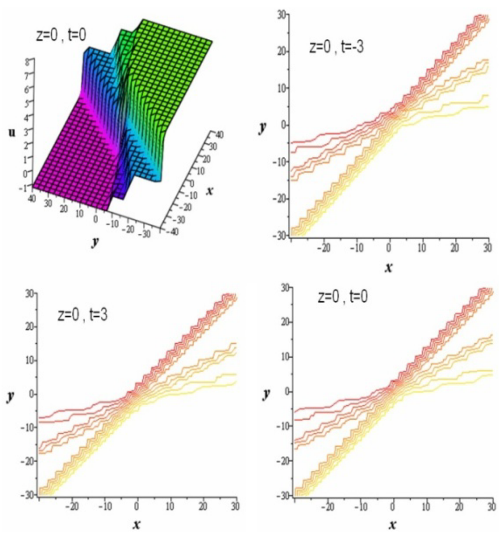

Figure 6. The second three-wave

5. Conclusions

In this paper, we have applied the multiple exp-function method to obtain one-wave, two-wave and three-wave solutions of the (2 + 1)- and the (3 + 1)-dimensional breaking soliton equations. These solutions are new solutions for these equations and for some special values of parameters in these new solutions we can obtain the solutions which have obtained by another methods. The multiple exp-function method is oriented towards the ease of use and capability of computer algebra systems and provides a straightforward and systematic solution procedure that generalizes Hirota's perturbation method. The method can be applied on other nonlinear partial differential equations which have multi-soliton solutions, because the multiple exp-function algorithm is powerful in generating exact solutions of nonlinear equations.

REFERENCES

[1] A. M. Wazwaz. (2005) The tanh method: solitons and periodic solutions for the Dodd-Bullough-Tzikhailov and the Tzitzeica-Dodd-Bullough equations, Chaos, Solitons and Fractals, 25, 55-63

[2] Z. Xiqiang, W. Limin, S. Weijun. (2006) The repeated homogeneous balance method and its applications to nonlinear partial differential equations, Chaos, Solitons and Fractals, 28(2) , 448-453

[3] S. J. Liao. (2004) On the homotopy analysis method for nonlinear problems, Appl. Math. Comput., 147, 499-513

[4] S. J. Liao. (2005) A new branch of solutions of boundary-layer flows over an impermeable stretched plate, Int. J. Heat Mass Transfer, 48, 2529-2539

[5] S. J. Liao. (2009) A general approach to get series solution of non-similarity boundary-layer flows, Commun. Nonlinear Sci. Numer. Simul., 14(5), 2144-2159

[6] M. T. Darvishi, F. Khani. (2009) A series solution of the foam drainage equation, Comput. Math. Appl., 58, 360-368

[7] A. Aziz, F. Khani, M. T. Darvishi. (2010) Homotopy analysis method for variable thermal conductivity heat flux gage with edge contact resistance, Zeitschrift fuer Naturforschung A, 65a(10) , 771-776

[8] F. Khani, M. T. Darvishi, R. S. R. Gorla (2011) Analytical investigation for cooling turbine disks with a non-Newtonian viscoelastic fluid, Comput. Math. Appl., 61(7), 1728-1738

[9] E. G. Fan, Z. Jian. (2002) Applications of the Jacobi elliptic function method to special-type nonlinear equations, Phys. Lett. A, 305 (6), 383-392

[10] M. T. Darvishi, Maliheh Najafi, Mohammad Najafi. (2010) Exact three-wave solutions for high nonlinear form of Benjamin-Bona-Mahony-Burgers equations, International Journal of Mathematical and Computer Sciences, 6(3), 127-131

[11] M. T. Darvishi, M. Najafi. (2012) Some exact solutions of the (2+1)-dimensional breaking soliton equation using the three-wave method, International Journal of Computational and Mathematical Sciences, 6(1), 13-16

[12] M. T. Darvishi, Maliheh Najafi, Mohammad Najafi. (2012) New exact solutions for the (3+1)-dimensional breaking soliton equation, International Journal of Information and Mathematical Sciences, 6(2), 134-137

[13] S. Arbabi Mohammad-Abadi, M. Najafi. (2011) Extend three wave method for the (3+1)-dimensional soliton equation, World Academy of Science, Engineering and Technology, 80, 1342-1345

[14] M. T. Darvishi, Maliheh Najafi, Mohammad Najafi. (2010) New application of EHTA for the generalized (2+1)-dimensional nonlinear evolution equations, International Journal of Mathematical and Computer Sciences, 6(3), 132-138

[15] M. T. Darvishi, M. Najafi. (2011) A modification of extended homoclinic test approach to solve the (3+1)-dimensional potential-YTSF equation, Chin. Phys. Lett., 28(4), art. no. 040202

[16] M. T. Darvishi, M. Najafi. (2012) Some complexiton type solutions of the (3+1)-dimensional Jimbo-Miwa equation, International Journal of Computational and Mathematical Sciences, 6(1), 25-27

[17] M. T. Darvishi, Maliheh Najafi, Mohammad Najafi. (2012) Traveling wave solutions for the (3+1)-dimensional breaking soliton equation by (G'/G)-expansion method and modified F-expansion method, International Journal of Computational and Mathematical Sciences, 6(2), 64-69

[18] J. H. He, M. A. Abdou. (2007) New periodic solutions for nonlinear evolution equations using Exp-function method, Chaos, Solitons and Fractals, 34, 1421-1429

[19] F. Khani, S. Hamedi-Nezhad, M. T. Darvishi, S. W. Ryu. (2009) New solitary wave and periodic solutions of the foam drainage equation using the Exp-function method, Nonlin. Anal.: Real World Appl., 10, 1904-1911

[20] B.-C. Shin, M. T. Darvishi, A. Barati (2009) Some exact and new solutions of the Nizhnik-Novikov-Vesselov equation using the Exp-function method, Comput. Math. Appl., 58(11/12), 2147-2151

[21] F. Khani, M. T. Darvishi, A. Farmani, L. Kavitha. (2010) New exact solutions of coupled (2+1)-dimensional nonlinear system of Schrödinger equations, ANZIAM Journal, 52, 110-121

[22] X. H. Wu, J. H. He. (2008) Exp-function method and its application to nonlinear equations, Chaos, Solitons and Fractals, 38(3), 903-910

[23] M. T. Darvishi, Mohammad Najafi, Maliheh Najafi. (2011) Some new exact solutions of the (3+1)-dimensional breaking soliton equation by the Exp-function method, Nonlin. Science Lett. A., 2(4), 221-232

[24] R. Hirota, The direct method in soliton theory, Cambridge University Press, Cambridge, 2004

[25] Y. Zhang, L. Y. Ye, Y. N. Lv, H. G. Zhao. (2007) Periodic wave solutions of the Boussinesq equation, J. Phys. A: Math. Gen., 40, 5539-5549

[26] W. X. Ma, R. G. Zhou, L. Gao. (2009) Exact one-periodic wave solutions to Hirota bilinear equations in 2+1 dimensions, Mod. Phys. Lett. A, 24, 1677-1688

[27] W. X. Ma E. G. Fan. (2011) Linear superposition principle applying to Hirota bilinear equations, Comput. Math. Appl., 61, 950-959

[28] W. X. Ma, T. W. Huang, Y. Zhang. (2010) A multiple exp-function method for nonlinear differential equations and its application, Phys. Scr., 82, art. no. 065003

[29] W. X. Ma. (2003) Diversity of exact solutions to a restricted Boiti-Leon-Pempinelli dispersive long-wave system, Phys.

Lett. A, 319, 325-333

[30] W. X. Ma, W. Srampp. (2005) Bilinear forms and Backlund transformations of the perturbation systems, Phys. Lett. A, 341, 441-449

[31] W. X. Ma. (2005) Integrability, in: A. Scott (Ed.), Encyclopedia of Nonlinear Science, Taylor & Francis, New York, pp. 450-453

[32] S. H. Ma, J. Peng, C. Zhang. (2009) New exact solutions of the (2+1)-dimensional breaking soliton system via an extended mapping method, Chaos, Solitons and Fractals, 46, 210-214

[33] A. M. Wazwaz. (2010) Integrable (2+1)-dimensional and (3+1)-dimensional breaking soliton equations, Phys. Scr., 81, 1-5